tion before acceleration.

It seemed reasonable to assume that the depolarization occurred either in the inclined field-acceleration tube[3] or in the center electrode ("stripper") of the accelerator where the beam passes through a 75-cm-long gas-loaded canal to remove both electrons from the negative ions. The second possibility was tested first. When the gas stripper was replaced by a thin (~15-μg/cm^2) carbon foil, a pronounced increase in beam polarization was observed (solid dots, Fig. 1). Under these conditions, no depolarization seems to occur within the accuracy (about $\pm 10\%$) with which the analyzing power of the analyzing reactions is known.

For protons the polarization of the accelerated beam can be measured much more accurately than for deuterons because p-α scattering provides an accurate polarization analyzer.[4] Measurements were made of the proton-beam polarization for proton energies between 4 and 12 MeV. The polarization was found to be 0.46 ± 0.01 independent of beam energy if an unpolarized beam component of 10% resulting from rest gas in the ionizer is subtracted. Because of various small effects which reduce the polarization of the beam from the ion source we would have expected a proton polarization of 0.47 ± 0.01, provided no depolarization takes place in the formation and the stripping of the negative ions. It is therefore concluded that with a foil stripper the polarization of the beam after acceleration is $0.98^{+0.02}_{-0.03}$ of the beam polarization at injection.

The depolarization observed with the gas stripper suggests that the stripping of the negative hydrogen ions is a two-step process. If this is the case, the proton polarization after the first electron is removed changes periodically between the initial value and zero on account of the Larmor precession. Thus the polarization is reduced by a factor two if the second electron gets removed at a random time equal to or larger than the Larmor precession time. Presumably the depolarization can be removed by applying a magnetic field. The above considerations also suggest that the successive pickup and removal of electrons that occurs when protons travel through matter is an important depolarization process in those cases where the material traversed is of low density.

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QUANTUM NUMBERS OF RELATIVISTIC ROTATIONAL STATES*

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Several years ago, it was suggested that all or part of the elementary-particle spectrum may be simply a system of rotational rest-energy levels, and that the way to test this hypothesis was to add to the Dirac or Kemmer or other irreducible relativistic wave equations an extra term, analogous to the Pauli term, which represented rotational energy in a Lorentz-invariant manner.[1] The requirement that the resulting theory should correspond to the relativistic classical theory of a symmetric top led to the free-particle wave equation

$$[i\gamma_\mu p_\mu + Mc - (i\hbar/4Ic)\gamma_{\mu\nu}J_{\mu\nu}]\psi = 0, \qquad (1)$$

where M and I are assumed to be constant parameters, $\gamma_{\mu\nu} = \frac{1}{4}(\gamma_\mu, \gamma_\nu)$, and $J_{\mu\nu} = -i\hbar(\gamma_{\mu\nu} + \Gamma_{\mu\nu})$. The operators $\gamma_{\mu\nu}$, $\Gamma_{\mu\nu}$ satisfy the commutation relations

$$(\gamma_{\mu\nu}, \Gamma_{\sigma\tau}) = 0,$$

$$(\gamma_{\mu\nu}, \gamma_{\sigma\tau}) = -(\gamma_{\mu\sigma}\delta_{\nu\tau} + \gamma_{\nu\tau}\delta_{\mu\sigma} - \gamma_{\mu\tau}\delta_{\nu\sigma} - \gamma_{\nu\sigma}\delta_{\mu\tau}), \qquad (2)$$

$$(\Gamma_{\mu\nu}, \Gamma_{\sigma\tau}) = -(\Gamma_{\mu\sigma}\delta_{\nu\tau} + \Gamma_{\nu\tau}\delta_{\mu\sigma} - \Gamma_{\mu\tau}\delta_{\nu\sigma} - \Gamma_{\nu\sigma}\delta_{\mu\tau}). \qquad (3)$$

Equation (1) couples the Poincaré group to the internal quantum numbers of the spinning top in a nontrivial way, but does not imply any a priori assumption about isospin or unitary symmetry.

The solutions of Eq. (1) have been examined for the special case in which $\gamma_{\mu\nu}$, $\Gamma_{\mu\nu}$ were Dirac or Kemmer operators.[2] In this Letter we report some of the properties of Eq. (1) when the γ_μ are Dirac operators

$$\gamma_i = \rho_2\sigma_i, \quad \gamma_4 = \rho_3, \quad \gamma_5 = -\rho_1,$$

but the $\Gamma_{\mu\nu}$ are general spin operators which satisfy Eq. (3):

$$(\Gamma_{23}, \Gamma_{31}, \Gamma_{12}) = i(\vec{L} + \vec{L}'), \quad (\Gamma_{14}, \Gamma_{24}, \Gamma_{34}) = i(\vec{L} - \vec{L}'),$$
$$\vec{L}\times\vec{L} = i\vec{L}, \quad \vec{L}'\times\vec{L}' = i\vec{L}', \quad (\vec{L}, \vec{L}') = 0,$$
$$\vec{L}^2 = l(l+1), \quad \vec{L}'^2 = l'(l'+1), \quad l, l' = 0, \tfrac{1}{2}, 1, \cdots.$$

The spin is therefore the sum of three commuting operators

$$\vec{J} = \hbar(\tfrac{1}{2}\vec{\sigma} + \vec{L} + \vec{L}') \qquad (4)$$

reminiscent of the superposition of three "quarks," and the rest-energy operator is given from (1) by

$$H\psi = mc^2\rho_3[1 + a\{(1+\rho_1)\vec{\sigma}\cdot\vec{L} + (1-\rho_1)\vec{\sigma}\cdot\vec{L}'\}]\psi = E\psi, \qquad (5)$$

where $\vec{m}$ and $\vec{a}$ are related to the constants M and I. For fermions, $l'-l$ is an integer, and since it conserved even for $p \neq 0$ we identify it with the electric charge.

Equation (1) defines a conserved current $\bar{\psi}\gamma_\mu\psi$ where $\bar{\psi} = \psi^+\gamma_4\eta_4$, and

$$(\eta_4, \gamma_\mu) = 0, \quad \eta_4\vec{L} = \vec{L}'\eta_4, \quad \eta_4\vec{L}' = \vec{L}\eta_4.$$

From (5), if ψ is an eigenstate of H belonging to the eigenvalue $+E$, it follows that $\rho_2\eta_4\psi$ is an eigenstate of H belonging to the eigenvalue $-E$. The values of E^2 separate into two distinct classes:

$$(E/mc^2)^2 = 1 - a + 2aJ + O(a^2), \qquad (6)$$

$$(E/mc^2)^2 = 1 - 3a - 2aJ + O(a^2), \qquad (7)$$

as is evident from Fig. 1, which shows the rest-energy trajectories (l' = const.) derived[3] from (5), as functions of J, for the choice $m = m_\Lambda$, $a = 0.27$.

The levels of type (6) consist of two rotational bands characterized by $J = l + l' \pm \frac{1}{2}$. The higher of these has rest energies given by

$$(E/mc^2)^2 = (1 + 2al)(1 + 2al').$$

It therefore begins on a spin-$\frac{1}{2}$ singlet ($l = 0$, $l' = 0$, $Q = 0$) and includes a spin-$\frac{3}{2}$ triplet with $Q = 0, \pm 1$. The lower band of type (6) includes a spin-$\frac{1}{2}$ doublet $(l', l) = (1, 0)(\frac{1}{2}, \frac{1}{2})$, $Q = +1, 0$, masses $m(1-4a^2)^{1/2}$ and $m[a + (1 - 2a - 2a^2)^{1/2}]$, respectively, and a spin-$\frac{3}{2}$ quartet with $Q = -1$, 0, +1, +2, together with the corresponding anti-

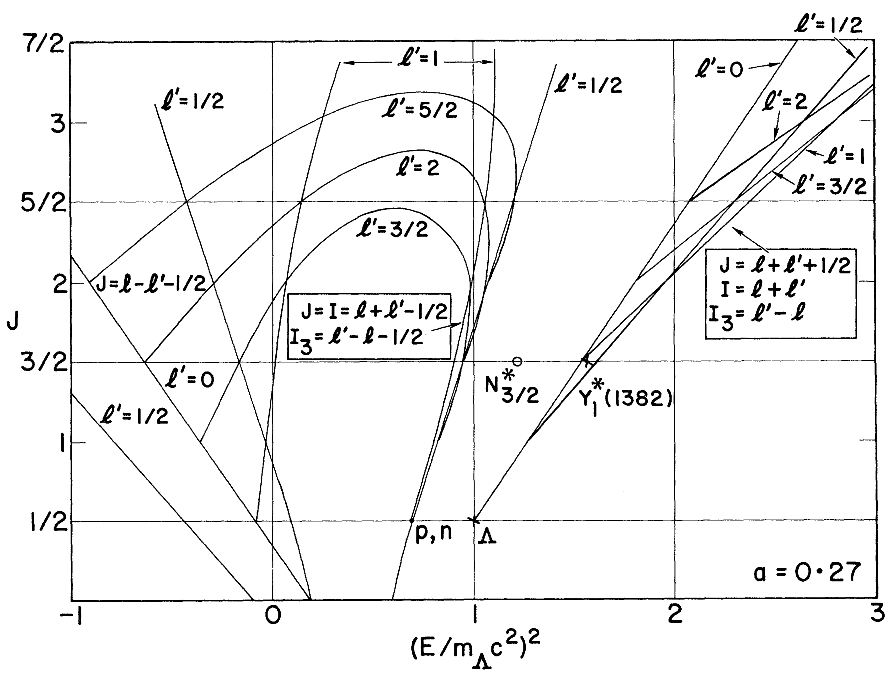

FIG. 1. Trajectories of J against E^2 for generalized Dirac equation (5) with $a = 0.27$.

Table I. Quantum numbers for generalized Dirac equation, $J = 1/2, 3/2$.

l'	J	l for $J=1/2$	$Q=l'-l$	$l'+l$	Particle	l for $J=3/2$	$Q=l'-l$	$l'+l$	Particle
0	$l+1/2$	0	0	0	Λ	1	−1	1	Y_1^{*-}
1/2	$l+1$					1/2	0	1	Y_1^{*0}
	l	1/2	0	1	n	3/2	−1	2	$N_{3/2}^{*-}$
1	$l+3/2$					0	1	1	Y_1^{*+}
	$l+1/2$	0	1	1	p	1	0	2	$N_{3/2}^{*0}$
3/2	$l+2$								
	$l+1$					1/2	1	2	$N_{3/2}^{*+}$
2	$l+5/2$								
	$l+3/2$					0	2	2	$N_{3/2}^{*++}$

particles. The above choice of the two parameters m, a brings the masses of these states into reasonable agreement with the $\Lambda, Y_1^*, n, p, N_{3/2}^*$. There are no states of type (6) below the np. The branches of type (7) have imaginary mass or mass small compared with the np. The "missing" $l'=0$ trajectory is one of these, since it includes a state $l=1$, $Q=-1$, $J=\frac{1}{2}$, mass $m(1-4a)^{1/2}$.

The quantum numbers derived for the above baryons are listed in Table I. For $B=+1$ they are related to more familiar quantum numbers as follows:

$$\left.\begin{aligned} J-2l-\tfrac{1}{2} &= Q-S-1 = Q-Y \\ J-2l'-\tfrac{1}{2} &= -(Q+S+1) = -(Q+Y) \\ l'-l &= \tfrac{1}{2}Y+I_3 \\ l'+l &= \tfrac{1}{2}Y+I \end{aligned}\right\} . \qquad (8)$$

Hence the isospin is simply the maximum value of $l=\frac{1}{2}(I-I_3)$. For $B=-1$, I is the maximum value of $l'=\frac{1}{2}(I+I_3)$.

Charge conservation in any transition from one particle state to another may therefore be expressed by the selection rule

$$\Delta l - \Delta l' = q, \qquad (9)$$

where q is the charge of the emitted particle, and the selection rule for strong interactions in which a boson of strangeness S is emitted by a baryon may be expressed thus:

$$\Delta J - \Delta(l+l') = S, \qquad (10)$$

where ΔJ is the increase of the intrinsic spin of the baryon. Assignment of quantum numbers $l'=\frac{1}{2}(S+q)$, $l=\frac{1}{2}(S-q)$ to the emitted bosons then leads to the over-all selection rule $\frac{1}{2}\Delta J = \Delta l' = \Delta l$. Analysis of the generalized Kemmer equa-

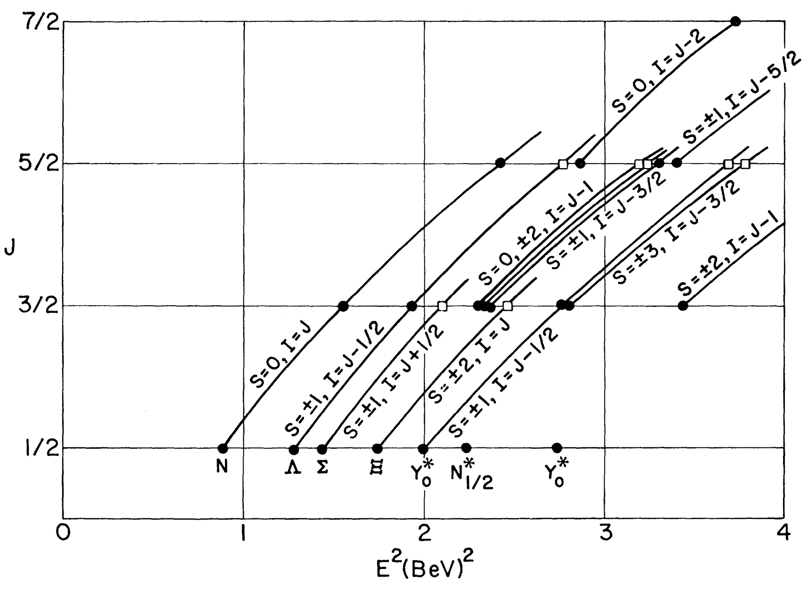

FIG. 2. Lines of constant $J-I$ and S. Empty boxes represent expected states.

tion, in which the γ_μ in Eq. (1) are Kemmer operators, similarly leads to rotational bands $J=l+l'\pm 1, l+l'$, etc.

It is evident that use of an energy term proportional to $J(J+1)$ to phenomenologically describe "rotational bands" of elementary-particle spectra is misleading. Calculated relativistically, the rotational levels have a much more complex structure and admit widely separated mass values as well as closely spaced multiplets. The difference between the relativistic and nonrelativistic theories becomes significant when the radius of the rotating particle is no longer large compared with its Compton wavelength. The main feature of the rotational spectrum of such a particle is the fact that both $J-I$ and S are constant along each energy-level trajectory. To see if this is a general property of baryon energy levels, in Fig. 2 we have plotted the observed spectrum on the same graph as Fig. 1 and drawn lines of constant $J-I$ and S, although some of the J values, in particular that of $N_{5/2}^*(1560)$, are not firmly established. If states corresponding to the empty square boxes are found, there may indeed be a fundamental significance to such trajectories.

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SU(9) SYMMETRY OF BARYONS AND MESONS

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The internal-symmetry group for baryons and mesons, which is only broken by electromagnetism and weak interactions, is

$$\mathcal{G}_0 = B\otimes Y\otimes \mathrm{SU}(2)_I,$$

where B and Y correspond to baryon number and hypercharge gauge groups and $\mathrm{SU}(2)_I$ to the isospin group. In the eightfold way, one looks at a larger group

$$\mathcal{G}_1 = B\otimes \mathrm{SU}(3)_{I,Y},$$

which includes $\mathcal{G}_0$, but is broken by more interactions than is $\mathcal{G}_0$.[1] The group of invariance of the eightfold-way theory, including space-time properties, is thus

$$G_1 = \mathcal{G}_1\otimes \mathcal{L} = B\otimes \mathrm{SU}(3)_{I,Y}\otimes \mathcal{L},$$

where $\mathcal{L}$ is the Poincaré group. A useful subgroup of G_1 is

$$\mathcal{G}_2 = B\otimes \mathrm{SU}(3)_{I,Y}\otimes \mathrm{SU}(2)_J,$$

where $\mathrm{SU}(2)_J$ is the SU(2) group of angular momentum. Recently Gürsey, Pais, Radicati, and Sakita proposed an SU(6) symmetry, which contains $\mathcal{G}_2$ and is broken by more interactions than is $\mathcal{G}_2$ symmetry.[2] The group in question is

$$\mathcal{G}_3 = B\otimes \mathrm{SU}(6)_{I,Y;J}.$$

This "marriage" of the internal properties isospin and hypercharge with space-time properties of angular momentum has been very successful. In this way one has through the introduction of a succession of higher symmetry groups succeeded in providing a unified description of the spin, isospin, and hypercharge properties of hadrons. Here we wish to propose an SU(9)-symmetry model for baryons and mesons in which we extend the fusion achieved in $\mathcal{G}_3$ one step further to include the baryon gauge group B.

The SU(9) group has 80 generators $A_\mu{}^\nu$ with $\mu,\nu=1,2,\cdots,9$ and $\sum_\lambda A_\lambda{}^\lambda=0$ satisfying the commutation rules

$$[A_\mu{}^\nu, A_\beta{}^\alpha] = \delta_\beta{}^\nu A_\mu{}^\alpha - \delta_\mu{}^\alpha A_\beta{}^\nu. \qquad (1)$$

We shall also give our identification of some physical operators.

$$\mathrm{SU}(3)_{I,Y}:\ A_i{}^j + A_{i+3}{}^{j+3} + A_{i+6}{}^{j+6}, \qquad (2)$$

$$i,j=1,2,3;$$