bered 1 to 6 from the lowest level upwards. I_{34} will then be proportional to the population difference f_{34} between levels 3 and 4 where

$$f_{34} \propto \frac{e^{-E_{13}/kT} - e^{-E_{14}/kT}}{1 + e^{-E_{12}/kT} + e^{-E_{13}/kT} + e^{-E_{14}/kT} + e^{-E_{15}/kT} + e^{-E_{16}/kT}}. \quad (1)$$

Our observed temperature variation is predicted by Eq. (1) when $\Delta = \frac{1}{2}(E_{14}+E_{13}) = 4.8 \pm 0.5$ cm^{-1}. A maximum in the intensity is predicted at a temperature of ~6°K for $h\nu = 0.3$ cm^{-1} (9 Gc/sec), and it is calculated that almost the same depopulation should still occur between 4 and 1.5°K with $h\nu = 0.83$ cm^{-1} (25 Gc/sec).

In view of the discrepancy between our results and those of Kedzie, Lyons, and Kestigian, the same experiment was kindly carried out independently by S. D. McLaughlan and A. F. Fray of the Royal Radar Establishment, Malvern, who examined the relative intensities of a Si-P marker and the $g = 4.3$ resonance in one of our samples using their 9-Gc/sec equipment at 10, 4.2, and 1.4°K. They confirmed that depopulation occurred between 4.2 and 1.4°K and found almost equal intensities at 4.2 and 10°K which is qualitatively consistent with the temperature variation predicted by Eq. (1) with $\Delta = 4.8$ cm^{-1}.

We therefore suggest that the conclusions of Kedzie, Lyons, and Kestigian concerning the detailed nature of the site occupied by Fe^{3+} in $CaWO_4$ are questionable. We hope to publish a full account of observations on this resonance when our own investigations of the center are complete.

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[1]R. W. Kedzie, D. H. Lyons, and M. Kestigian, Phys. Rev. 138, A918 (1965).

[2]T. Castner, Jr., G. S. Newell, W. C. Holton, and C. P. Slichter, J. Chem. Phys. 32, 668 (1960).

[3]R. W. Kedzie and M. Kestigian, Appl. Phys. Letters 3, 86 (1963).

[4]E. A. Gere, unpublished memorandum.

MEASUREMENT OF THE ENERGY LOSS OF GERMANIUM ATOMS TO ELECTRONS IN GERMANIUM AT ENERGIES BELOW 100 keV*

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Knowledge of the relative importance of the mechanisms by which moving atoms lose energy in solids is important in studies of radiation-damage effects and in interpretation and design of experiments in certain areas of nuclear physics. Energy losses at high particle velocities are dominated by the effects of collisions which transfer energy to electrons and at low velocities by the effects of collisions which transfer energy to an entire atom. The theory[1,2] for this energy division is especially straightforward if the particle has the same mass and atomic number as the environment in which it stops because then there are only heavy particles of one type. However, no measurements have previously been made to verify the theory for this particularly simple situation in an energy region where the energy losses to electronic and atomic collisions are comparable, although the calculations of Lindhard, Scharff, and Schiott[1] are in good agreement with measurements of particle ranges in various media. It is the purpose of this Letter to present the results of a direct determination of the energy loss to electron collisions of germanium atoms in a germanium environment.

In a previous investigation[3] of the spectra produced by neutron bombardment of a lithium-drifted germanium-radiation detector, it was found that internal-conversion electrons from the decay of the O^+ first-excited nuclear state of Ge^{72} and gamma rays from the 2^+ first excited state of Ge^{74}, both produced in the sensitive volume of the detector by inelastic neutron scattering, were easily detected. The de-excitation of the excited state is coincident in time (within the integrating time of the amplifying system) with the recoiling germanium atom so that the lines in the pulse-height spectrum are broadened by the amount of electronic excitation produced by the recoiling atom. The lower limit to the observable energy loss is fixed by the intrinsic linewidth of the germanium detector and its associated electronics. In the present work this linewidth was about 5.5 keV. Since the recoil atom is produced in the sensitive volume of the detector, there is no dead-window correction to be made.

Spectra for the lines of Ge^{72} and Ge^{74} produced by the neutrons incident on the counter are shown in Fig. 1. The gamma-ray line from Ge^{74} is at 596 keV and the conversion-electron line from Ge^{72} at 690 keV. All measurements were made with a Cs^{137} source present to provide a measure of the system resolution and

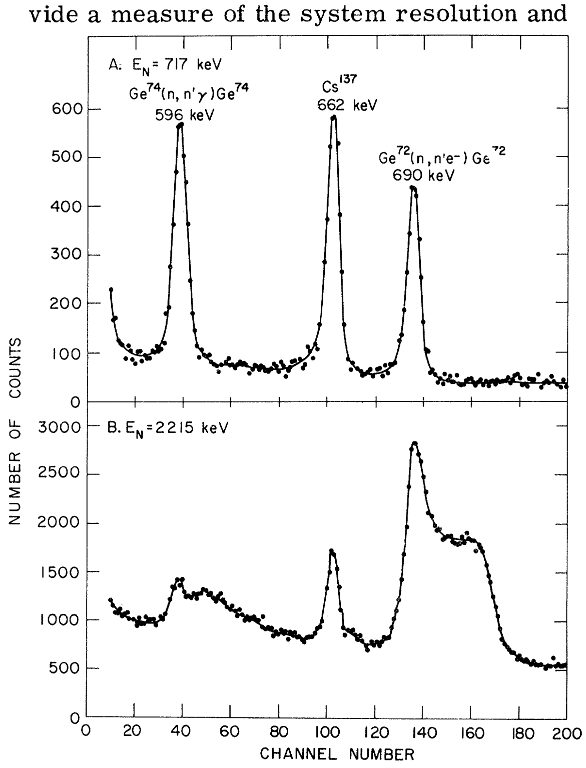

FIG. 1. Pulse-height spectra taken at neutron energies of 717 keV, just above the threshold for the 690-keV Ge^{72} state, and 2215 keV, the highest energy used in the experiment. The broadening of the Ge^{72} and Ge^{74} lines at the higher energy is clear. The energy calibration is 0.96 keV/channel.

stability. Figure 1(a) is a spectrum taken at $E_n = 717$ keV, just above the threshold for production of the 690-keV Ge^{72} state where the energy of the recoils is low. A linewidth comparable to the Cs^{137} line results. Figure 1(b) is a spectrum taken at $E_n = 2215$ keV where the maximum atomic recoil energy is 100 keV. Appreciable broadening of both lines by electronic excitation produced by the recoiling atoms is observed.

Analysis of data such as that shown in Fig. 1 was made in the following way. Line shapes are given by summation of the hole-electron pairs produced in the germanium crystal by the 596- or the 690-keV transition and those hole-electron pairs produced by the atomic recoil. The atomic-recoil energy is determined by the angle at which the inelastically scattered neutron is emitted and ranges from nearly zero for a neutron emitted at zero degrees to a maximum value for a neutron emitted at 180°. The number of recoil atoms in a given pulse-height interval is thus related to the angular distribution of the inelastically scattered neutrons and the detector response to the recoils. Observed line shapes were found by combining the detector resolution function with this distribution. At the high-energy end point, the line shape will be determined by a combination of the detector energy resolution and fluctuations in the energy loss of the recoils to electronic excitation.[2] In order to extract the value for the energy loss of the recoil atoms to electronic excitation, the end points of the distribution are approximated by the half-maximum points. This method does not take into detailed account the effect of the detector resolution, but it is a good approximation. We estimate an uncertainty of ±0.7 keV in measurement of the linewidth. The energy scale for the electronic excitation rests on the assumption that the energy loss per hole-electron pair is independent of the energy of the stopping electrons. At a fixed incident neutron energy, the detector bias voltage was varied from 300 to 800 V without any discernible change in the width of the distributions, which indicates that charge recombination is not playing a significant role in this experiment.

Figure 2 shows the results of the present work plotted in the universal curves used by Lindhard et al.[1,2] In their notation, ϵ is a dimensionless measure of the recoil energy

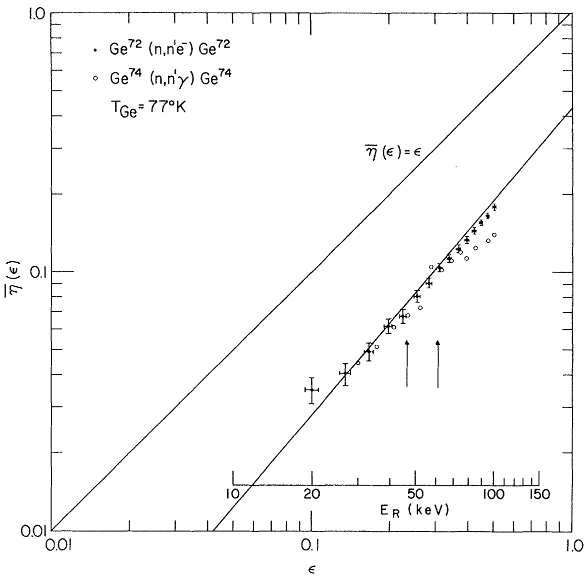

FIG. 2. The quantity $\bar{\eta}(\epsilon)$, which is related to the fractional electronic energy loss by $\bar{\eta}(\epsilon)/\epsilon$, plotted as a function of the dimensionless measure of the germanium recoil energy, ϵ. The definitions of $\bar{\eta}(\epsilon)$ and ϵ are given in the text. The solid line marked $\bar{\eta}(\epsilon) = \epsilon$ is the limit for which the recoil atom loses all its energy to electronic excitation. The other line gives the theoretical prediction from the work of Lindhard et al.,[1,2] and was calculated from Eq. (5.2) and Fig. 9 of reference 2 with $R = 0.157$. According to Lindhard et al., this procedure should reproduce the exact computer calculations with an "accuracy of some percent." This was verified in the present case by comparison with Fig. 3 of reference 2. The arrows in the figure indicate positions of thresholds for higher excited states in Ge^{74} which can feed the 596-keV level. This point is discussed further in the text.

given by

$$\epsilon = E_R \cdot \frac{am_2}{Z_1 Z_2 e^2 (m_1 + m_2)},$$

where $m_1 \approx m_2 \approx 72$, $Z_1 = Z_2 = 32$, and $a = 0.8853 \times (\hbar^2/me^2)Z_1^{2/3} = 1.47 \times 10^{-9}$ cm for the present work. The quantity $\bar{\eta}(\epsilon)/\epsilon$ is that fraction of the recoil-atom energy which goes into electronic excitation and is related to the fractional atomic-energy loss $\bar{\nu}(\epsilon)/\epsilon$ by $\bar{\eta}(\epsilon) + \bar{\nu}(\epsilon) = \epsilon$. The upper solid line shows the high-energy limit for $\bar{\eta}(\epsilon)$, and the other solid line shows the results of numerical computations by Lindhard et al. for the parameter, $k = 0.133Z^{2/3}A^{-1/2}$, which is equal to 0.517 for germanium atoms stopping in germanium, ignoring a small isotope effect.

A summary of the experimental results is given in Table I and Fig. 2. Agreement between theory and experiment is remarkable for the case of the 690-keV transition in Ge^{72}. Our values for $\bar{\eta}(\epsilon)$ for Ge^{72} are systematically lower than the theoretical values by less than 10%. Agreement is only qualitative for the Ge^{74} case. Such results are to be expected because of the better counting statistics for Ge^{72} and because the detailed shape of the 596-keV line is obscured by contributions from inelastic neutron scattering to higher excited states in Ge^{74} followed by nuclear transitions to the 596-keV level. The subsequent decay of the 596-keV state is then in coincidence with an atomic recoil having an energy less than that associated with the direct production of the 596-keV state. In Ge^{72} the 690-keV level is not fed appreciably by higher states and such effects do not exist for the range of neutron energies considered here.

We are indebted to Dr. M. Goldhaber and Dr. A. Schwarzschild for several stimulating discussions.

Table I. Summary of results for $Ge^{72}(n,n'e^-)Ge^{72}$. E_N is the incident neutron energy, E_R is the maximum recoil energy for the residual Ge^{72} nucleus, ϵ is a dimensionless measure of E_R discussed in the text, $\bar{\eta}(E)$ is the average amount of energy given to electronic excitation by the Ge^{72} recoil atom, and $\bar{\eta}(\epsilon)$ is a measure of the same quantity in dimensionless form. Estimated uncertainties in E_N, E_R, ϵ, $\bar{\eta}(E)$, and $\bar{\eta}(\epsilon)$ are the same for all energies, while the uncertainty in $\bar{\eta}(E)/E_R$ is given for each energy.

E_N[a] (keV)	E_R[b] (keV)	ϵ[c]	$\bar{\eta}(E)$[d] (keV)	$\bar{\eta}(E)/E_R$	$\bar{\eta}(\epsilon)$[e]
803	20.2	0.101	6.7	0.347±0.042	0.035
906	26.9	0.135	8.1	0.299±0.029	0.040
1008	33.1	0.166	9.8	0.295±0.024	0.049
1113	39.3	0.197	12.3	0.313±0.020	0.062
1212	45.0	0.225	13.4	0.298±0.017	0.067
1312	50.7	0.254	15.9	0.314±0.016	0.080
1412	56.3	0.282	18.0	0.320±0.014	0.090
1512	61.9	0.310	20.5	0.331±0.013	0.103
1612	67.5	0.338	22.4	0.332±0.012	0.112
1712	73.1	0.366	24.5	0.335±0.011	0.122
1810	78.5	0.393	26.4	0.336±0.010	0.132
1910	84.1	0.420	28.7	0.341±0.010	0.143
2007	89.4	0.447	30.8	0.344±0.009	0.154
2105	94.9	0.474	32.5	0.343±0.009	0.163
2203	100.3	0.501	35.3	0.352±0.008	0.177

[a]Error is ±25 keV.
[b]Error is ±1.2 keV.
[c]Error is ±0.006.
[d]Error is ±0.7 keV.
[e]Error is ±0.004.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]J. Lindhard, M. Scharff, and H. E. Schiott, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 33, No. 14 (1963).

[2]J. Lindhard, V. Nielsen, M. Scharff, and P. V. Thomsen, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 33, No. 10 (1963).

[3]C. Chasman, K. W. Jones, and R. A. Ristinen, to be published.

NONLINEAR TWO-FLUID EQUATIONS FOR A SUPERCONDUCTOR

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The recent interest in dissipative effects in type-II superconductors has pointed up the need for two-fluid equations which are nonlinear in the velocities and describe the motion of the normal electrons without the contradiction encountered in the London theory.[1] The purpose of the present Letter is to give the phenomenological equations of motion which are obtained from Eckart's[2] variational principle as applied to the two-fluid model of a superconductor. The derivation follows closely that used by Zilsel[3] to obtain the two-fluid equations for liquid-helium II, but with the addition of the electromagnetic terms; it differs from Cook's[4] derivations of the London equations from Eckart's principle in that it includes the normal electrons. The resulting equations yield a source term for the normal electrons and the explicit form of the osmotic-pressure term suggested by London[1] and recently applied to extreme type-II superconductors by Vijfeijken and Staas.[5]

Eckart's principle assumes that the dynamics of a continuous system are such that

$$\delta\int_{t_1}^{t_2}\int \mathcal{L}\, d^3r dt = 0, \qquad (1)$$

where $\mathcal{L}$ is the Lagrangian density in a given region whose boundaries are fixed. There is considerable evidence that this variational principle, which is more restrictive than Hamilton's principle, gives the correct description of superfluids. The Lagrangian density is assumed to be[6]

$$\mathcal{L} = \tfrac{1}{2}x\rho\vec{V}_n^{\,2} + \tfrac{1}{2}(1-x)\rho\vec{V}_s^{\,2} + \frac{e}{mc}x\rho\vec{V}_n\cdot\vec{A} + \frac{e}{mc}(1-x)\rho\vec{V}_s\cdot\vec{A} - \frac{e}{m}\rho\Phi - \rho U(\rho, S, x) + \frac{\epsilon}{8\pi}\left[-\nabla\Phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right]^2 - \frac{1}{8\pi\mu}(\nabla\times\vec{A})^2. \qquad (2)$$

Here $x = \rho_n/\rho$ and $1-x = \rho_s/\rho$ are the normal and superfluid mass fractions and $\vec{V}_n$ and $\vec{V}_s$ are the normal and superfluid velocities. S is the entropy and U is the internal energy, both per unit mass. Φ and A are the scalar and vector potential and e/m is the charge-to-mass ratio of the carriers. Equation (1) is to be restricted by conservation of mass (which for electronic carriers also implies conservation of charge) and conservation of entropy:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho[x\vec{V}_n + (1-x)\vec{V}_s] = 0, \qquad (3)$$

$$\frac{\partial(\rho S)}{\partial t} + \nabla\cdot[\rho S\vec{V}_n] = 0. \qquad (4)$$

Equation (4) also contains the assumption that entropy is carried only by the normal electrons. Conservation of entropy is slightly relaxed later by including a linear frictional force in the normal fluid equation. Lin's[7] constraint, which is necessary for a classical fluid, and which Whitlock[8] used to derive the one-fluid plasma equations, is assumed not to apply to a quantum fluid where the particle trajectory is not a defined concept. The variational principle with these two constraints is therefore

$$\delta\int_{t_1}^{t_2}\int\{\mathcal{L} - \alpha(\partial\rho/\partial t + \nabla\cdot\rho[x\vec{V}_n + (1-x)\vec{V}_s]) - \beta[\partial(\rho S)/\partial t + \nabla\cdot\rho S\vec{V}_n]\}d^3r dt = 0. \qquad (5)$$

Variation with respect to $\rho, S, x, \vec{V}_n, \vec{V}_s, \Phi, \vec{A}$ lead to the following relations:

$$\delta\rho:\ \tfrac{1}{2}x\vec{V}_n^{\,2} + \tfrac{1}{2}(1-x)\vec{V}_s^{\,2} + \frac{e}{mc}x\vec{V}_n\cdot\vec{A} + \frac{e}{mc}(1-x)\vec{V}_s\cdot\vec{A} + \frac{e}{m}\Phi - U - \frac{1}{\rho}P + \partial\alpha/\partial t + [x\vec{V}_n + (1-x)\vec{V}_s]\cdot\nabla\alpha + S(\partial\beta/\partial t) + S\vec{V}_n\cdot\nabla\beta = 0, \qquad (6)$$

$$\delta s:\ -T + \partial\beta/\partial t + \vec{V}_n\cdot\nabla\beta = 0, \qquad (7)$$