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CAVITY RESONANCES IN ACCELERATED SYSTEMS

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The present note is prompted by the currently revived interest in some classical experimentation dealing with the propagation of electromagnetic waves in rotating systems. Heer[1] has recently published what is probably the first electromagnetic analysis of the mode-splitting phenomena caused by the rotation of a cavity resonator. (It is understood that the cavity rotation is defined or determined by the motion of the beam splitter in the optical case or of the wave-launching coupler in the microwave case.) Except for a wave analysis given by Gordon,[2] who has used the metric to account for medium influences, earlier papers have all been based on a purely kinematic treatment.

By using the transformation properties of the constitutive equations, Heer correctly bypasses the Gordon treatment as unsuitable for applications to general media. In particular, he addresses himself to the case of a cavity filled with a corotating material medium, previously treated kinematically by Harzer[3] and von Laue.[4]

For a cylindrical cavity it is feasible to consider a physical situation where medium and cavity rotate at different angular velocities. Experiments of this type have been performed (Metz and Prunier[5]). It is therefore necessary to impose as a logical requirement that the theory is capable of dealing with any of these situations without further special assumptions. To do this it seems necessary to consider the transformation of the medium part separately from the free-space part of the constitutive equations. The result of this analysis differs slightly from the conclusions obtained by Heer, Harzer, and von Laue. The two approaches coincide for media with an effective magnetic permeability equal to unity.

The constitutive tensor for an arbitrary nonconducting material medium with respect to an accelerated frame of reference can be considered as the result of a superposition of a free-space part $\chi_{(0)}$ and a part $\chi_{(m)}$ solely determined by the electric and magnetic polarization properties of the material medium:

$$\chi^{\lambda\nu\sigma\kappa} = \chi_{(0)}^{\lambda\nu\sigma\kappa} + \chi_{(m)}^{\lambda\nu\sigma\kappa}. \tag{1}$$

The decomposition of the constitutive tensor (1) now leads to two conspicuously distinct situations: (1) The material medium is at rest in the rotating frame; only $\chi_{(0)}$ is affected by the rotation. (2) The material medium is at rest in the inertial system of the laboratory:

$\chi_{(0)}$ and $\chi_{(m)}$ are both affected by the rotation.

Case (1) corresponds to the physical situation encountered in the experiments performed by Harrass,[6] and later by Pogany,[7] while case (2) represents a situation that comes close to an experiment performed by Dufour and Prunier (see reference 5).

It is, of course, possible to leave the cavity stationary and have the medium rotating. It can be shown that the latter situation gives rise to a Fresnel-Fizeau-like effect. For the present purpose we will only discuss case (1).

After the free-space part is transformed from its inertial representation to a cylindrical coordinate system (t, r, φ, z-0, 1, 2, 3) rotating with an angular velocity Ω around the z axis, the constitutive tensor becomes as shown in Table I. Only terms of the first order in $\Omega r/c$ are considered. Note that only the free-space part ϵ_0 occurs in the off-diagonal squares of Table I.

The covariant vector-wave equation for the electromagnetic potential[8] A_κ is

$$\partial_\nu \chi^{\lambda\nu\sigma\kappa} \partial_\sigma A_\kappa = 0. \tag{2}$$

Substituting Table I into (2), and assuming, in accordance with the idealized configuration of a beam in a purely circular path, that $\partial/\partial r$ and $\partial/\partial z$ vanish, one obtains the simple time-asymmetric wave equation in the component A_r:

$$\epsilon\mu \frac{\partial^2}{\partial t^2} A_r - \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} A_r + 2\Omega\epsilon_0 \mu \frac{\partial^2}{\partial\varphi \partial t} A_r = 0. \tag{3}$$

The same equation holds for the component A_z.

Solutions of Eq. (3) satisfying the uniqueness condition for the clockwise and counterclockwise modes give the mode splitting

$$\frac{\Delta\omega}{\omega} = 2\left(\frac{\mu_r}{\epsilon_r}\right)^{1/2} \frac{\Omega R}{c}. \tag{4}$$

The frequency splitting (4) can be converted into an equivalent interferometric fringe shift Δz given by the formula

$$\Delta z = (4A\Omega/c\lambda_0)\mu_r, \tag{5}$$

where $A = \pi R^2$ is the area enclosed by the beam, and λ_0 the free-space wavelength of the light. A slightly more detailed argument shows that in the geometric optic approximation, the result (5) also remains valid for arbitrary noncircular beams.

Table I. Constitutive tensor.

$\chi^{\lambda\nu\sigma\kappa}$		E 01	02	03	B 23	31	12
$-D$	01	$-r\epsilon$	0	0	0	0	$r\Omega\epsilon_0$
	02	0	$-\epsilon/r$	0	0	0	0
	03	0	0	$-r\epsilon$	$-r\Omega\epsilon_0$	0	0
H	23	0	0	$-r\Omega\epsilon_0$	$1/\mu r$	0	0
	31	0	0	0	0	r/μ	0
	12	$r\Omega\epsilon_0$	0	0	0	0	$1/\mu r$

The formula (5) without the factor μ_r was obtained earlier by Harzer and by von Laue. In that form (i.e., $\mu_r = 1$) it is identical with the Sagnac effect in free space because no medium parameters occur. In other words, the presence of a medium with $\mu_r = 1$ does not affect the Sagnac fringe shift.

Pogany[7] has performed a reasonably precise experiment where the beam travels partly through a glass medium. His comparisons with the free-space case show a small discrepancy, which could suggest a very slight diamagnetic susceptibility of his glass medium in the optical range. This result is as yet too inconclusive to be considered experimental support for the validity of Eq. (5). The light beam in Pogany's experiment, outside of the glass medium, traveled through turbulent air of atmospheric pressure. A more precise experiment in an evacuated container using modern photometric methods of interferometric fringe-shift detection is desirable.

We have throughout this analysis assumed the applicability of (absolute time) Galilean rotations, which, at least for first-order effects, seems acceptable. It should, however, be understood that a detailed experimental check should test the validity of the method as well as the applicability of the Galilean rotation.

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