relative velocity of the two particles. In the ultrarelativistic limit $|\vec{v}_1-\vec{v}_2|=2c$.

Since the solution to Eq. (6) is very well known, we may write down the small-angle scattering amplitude. As before,[1] we write the small-angle scattering solution as

$$f_{\rm Coul}(\theta)=[-\eta/2k(\tfrac{1}{2}\theta)^2]\exp 2i(\sigma_0-\eta\ln\tfrac{1}{2}\theta), \qquad (12)$$

where θ is the scattering angle in the center-of-momentum system,

$$\sigma_0=\arg\Gamma(1+i\eta), \qquad (13)$$

and k, η are now given by Eqs. (8)-(9).

The small-angle scattering formula given above reduces to the usual nonrelativistic result at low energies. At very high energies, where $E\gg m_1+m_2$, the phase of the forward-scattering amplitude is reduced in magnitude by a factor of two from the phase which we would obtain for an ultrarelativistic particle of charge Z_1e scattering from an infinitely heavy stationary particle of charge Z_2e.

For the scattering of pions by protons at 8 BeV/c, $\eta\simeq\frac{1}{2}(e^2/\hbar c)$ (1.06); at 14 BeV/c, $\eta\simeq\frac{1}{2}(e^2/\hbar c)$ (1.03).

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REMARKS ON THE B MESON

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The study of the properties of the B meson[1-3] has been considerably impeded by the presence of background. The decay of the B meson into $\omega+\pi$ permits us to try reducing this background by purifying the ω sample. As is well known,[4,5] the ω-decay matrix element is proportional to $(\vec{p}_i\times\vec{p}_j)$. This leads to a density distribution in the ω Dalitz plot that is peaked in the center (i.e., equal energies for the three pions), and vanishes on the periphery (i.e., all three momenta collinear). It is therefore possible, by choosing a sample of events corresponding to the central region of the Dalitz plot, to improve the ω-to-background ratio considerably. Following such a procedure we obtain the suprising result that the "purified" ω sample does not appear to lead to significant $\omega\pi$ enhancement in the B-meson mass band. On the other hand, the "diluted" ω sample, corresponding to events closer to the periphery of the Dalitz plot, leads to a marked enhancement in the 4π mass distribution.

In this note we are reporting on the analysis of π^+p and π^-p interactions at 3.65 and 3.7 BeV/c, respectively, corresponding to the reactions

$$\pi^\pm+p\to\pi^++\pi^-+\pi^0+\pi^\pm+p. \qquad (1)$$

The work on the π^+ interaction was carried out in the Brookhaven National Laboratory 20-in. hydrogen bubble chamber exposed in the Brookhaven-Yale beam[6] at the AGS. The work on the π^- interactions was carried out in the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber exposed in a negative-particle beam[7] at the Bevatron. The π^+p data (1997 events) were analyzed by conventional methods.[8] The π^-p data (930 events) were analyzed on the Lawrence Radiation Laboratory Flying-Spot Digitizer[9] (F-SD) and by associated programs.[10]

For each event corresponding to Reactions (1) two neutral pion triplets $\pi^+\pi^-\pi^0$ can be formed. In the π^+p reaction, for example, we can label the four pions $\pi_a^+, \pi^-, \pi^0, \pi_b^+$. The two neutral mass triplets are then $M_3^a=M(\pi_a^+\pi^-\pi^0)$ and $M_3^b=M(\pi^-\pi^0\pi_b^+)$. To study a π^+, ω interaction we choose events for which at least one mass triplet, let us call it $\pi_a^+\pi^-\pi^0$, lies between two limits, $M_{\rm min}, M_{\rm max}$, corresponding to the "experimental width" of the ω meson.[11]

Thus we can select a sample of events fulfilling the condition

$$760\le M_3^a\le 820\ {\rm MeV}. \qquad (2)$$

This sample then contains the majority of the events corresponding to the channel

$$\pi^++p\to\omega+\pi^++p, \qquad (3)$$

as well as events for which three pions are artificially constrained by condition (2). These latter we call "background events."

Although experimentally we have no way of distinguishing between "true ω events" and background events on an individual basis, we can get samples enriched in true ω events by utilizing the properties of the ω-decay matrix element. In Fig. 1 we illustrate the normalized Dalitz plot for the $\pi_a^+\pi^-\pi^0$ system for events selected according to condition (2). Here $x = (T_{\pi^+}-T_{\pi^-})/\sqrt{3}Q$ and $y = T_{\pi^0}/Q$ are the usual Dalitz coordinates. We have chosen to examine two regions of the Dalitz plot, which are delineated by a curve of equal probability corresponding to the square of the ω matrix element, λ, given by

$$\lambda = \left|\frac{\vec{p}_+ \times \vec{p}_-}{Q^2}\right|^2. \tag{4}$$

The separation between the two regions is given by the dashed curve in Fig. 1(a). This corresponds to a value of $\lambda = 0.07$. From integrating expression (4) over the Dalitz plot, we find (for a pure 1^{--} state) that 50% of all ω's should be contained in the central region. The contribution of the background events to the central region will be proportional to the area, which amounts to only 29% of the total area. We assume here that the background events are uniformly distributed (i.e., according to phase space). In order to estimate the experimental ratio ω/background in the respective regions we employ the following method. We include all $\pi^+\pi^-\pi^0$ mass triplets in a normalized Dalitz plot. Since the curve delineating the two regions can be expressed in terms of the x and y Dalitz coordinates, one can apply the analogous separation to all mass triplets, as was done for the ω band.[12] The resulting two $\pi^+\pi^-\pi^0$ mass plots are shown in Fig. 2(a) for the central region and Fig. 2(b) for the peripheral region.

We have estimated the background underneath

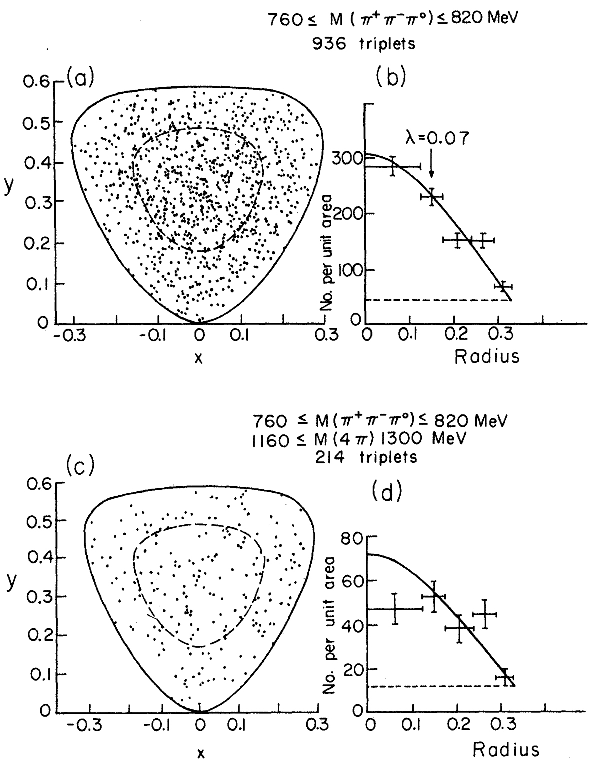

FIG. 1. (a) Normalized Dalitz plot for all mass triplets with $M(\pi^+\pi^-\pi^0)$ between 760 and 820 MeV. (b) Radial density distribution for the data in (a). The radius quoted refers to radius measured along the y axis. The areas are delineated by a curve of equal probability for the ω-decay matrix element. The dashed line represents the background estimate considered as isotropic over the Dalitz plot. (c) Same as (a) with the additional condition that $1160 \leq M(4\pi) \leq 1300$ MeV. (d) Same as (b) with the additional condition given in (c).

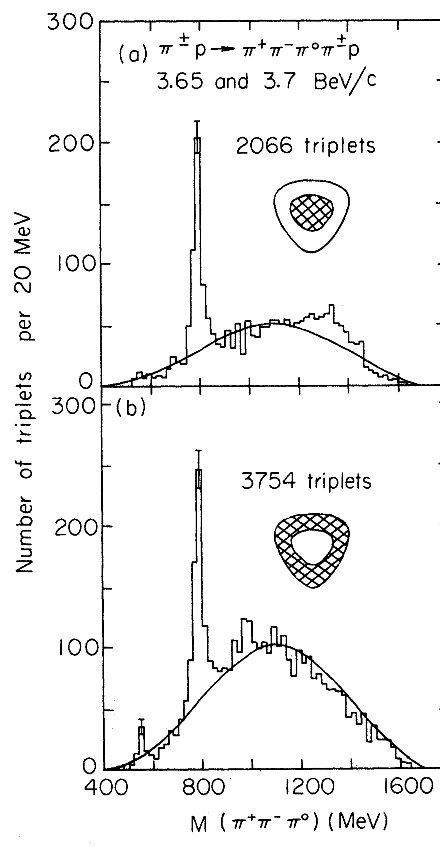

FIG. 2. The three-pion mass distributions. The curves shown correspond to phase space. (a) Pion triplets with Dalitz coordinates x, y lying in the "central region" as defined in the text. (b) Pion triplets with Dalitz coordinates lying in the "peripheral region".

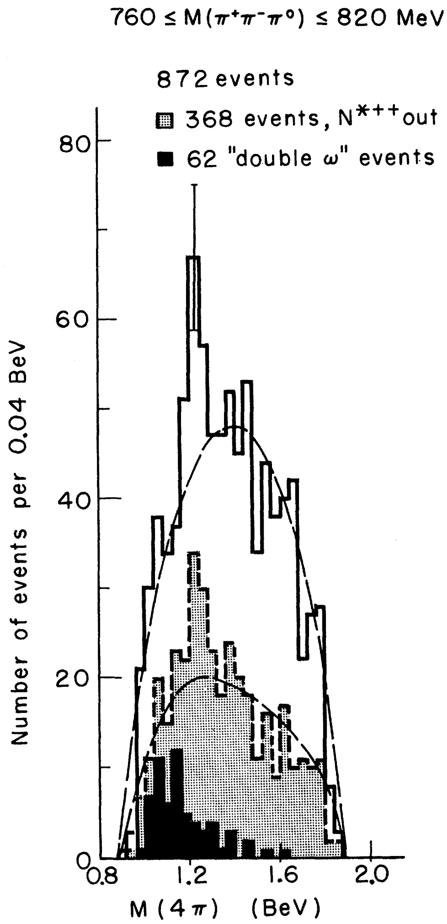

FIG. 3. The four-pion mass distributions, where one neutral three-pion mass is given by condition (2) in the text. The curves shown correspond to phase space. Computed by a Monte-Carlo method as described in the text.

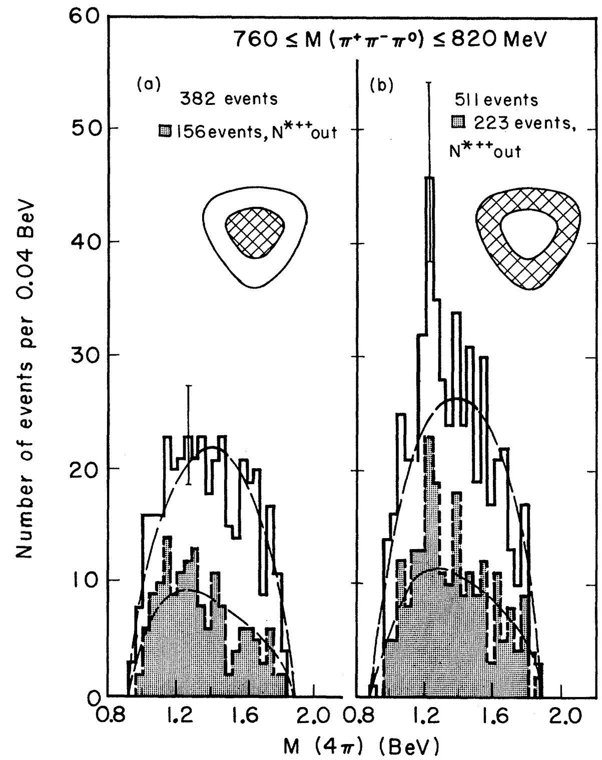

FIG. 4. Same as in Fig. 3 with the two regions of the Dalitz plot given separately. (a) Pion triplets in the central region of the Dalitz plot. (b) Pion triplets in the peripheral region of the Dalitz plot.

the ω peak by comparison with the mass regions below and above the ω mass. For the entire data we obtain the ratio "true ω"/background, $R = 2.9 \pm 0.2$, whereas for the central and peripheral regions the ratios are $R_c = 5.7 \pm 0.8$ and $R_p = 2.1 \pm 0.2$, respectively. The values of R_c and R_p are in good agreement with the values computed from the ω matrix element and R for the entire data.

We note here that the η meson ($J^{PG} = 0^{-+}$) stands out clearly above background in the peripheral region and is hardly discernible in the central region. This can be understood in terms of the well-known feature that the highest density for η decay occurs at low π^0 kinetic energies.

In Fig. 3 we show the 4π mass distribution for events selected according to condition (2). The shaded histogram shows the distribution for events without N^{*++} formation.[13] The dark shaded region corresponds to "double ω" events, i.e., events for which both pion triplets fall in the ω mass band.[14] It is noteworthy that these cluster below the "B-meson" mass band. The curves shown correspond to phase-space calculations by a Monte-Carlo method,[15] which takes into account reflections due to resonance formations, viz., ωN^*, $\omega p\pi$, $\pi\pi\pi N^*$, and $\pi\pi\pi\pi p$ in the experimentally observed ratios. As may be noted, the "B meson" appears as a well-defined peak in our data.

We now proceed to calculate the 4π mass distribution with pion triplets from the two regions (see Fig. 4). Here we note the surprising effect that the B enhancement appears to be primarily associated with pion triplets from the peripheral region [see Fig. 4(b)]. The observed peak occurs at $E = 1220$ MeV and has a full width at half-maximum of approximately 80 MeV. On the other hand, the four-pion mass distribution corresponding to the highly purified ω mesons from the central region does not show a significant peak at the B-meson mass band.

We may now reverse the argument and ask what type of Dalitz density distribution is obtained for "ω" mesons produced in the decay of the B meson. For this purpose we chose the central band of the B meson by selecting a mass between 1160 and 1300 MeV in the four-pion mass spectrum. The Dalitz plot for the $\pi^+\pi^-\pi^0$ mass triplets fulfilling condition (2) and the radial density distribution plot, are shown in Figs. 1(c) and 1(d). As may be noted, the data deviate from the calculated curve corresponding to the ω matrix element and 25.4% background. The probability that the experimental distribution corresponds to a pure 1^{--} matrix element together with the average background is $\leq 0.5\%$. This result is not independent of the observation shown in Fig. 4, and reflects the statistical reliability of the effect we have observed.

We can now summarize what we have learned about the B meson.

(a) When all available data[1-3,16] are considered there is good evidence for an enhancement in the four-pion mass distribution if the neutral pion triplet is restricted by condition (2). This enhancement, with $E_B = 1220$ MeV and $\Gamma_B \approx 120$ MeV, has been called the B meson.

(b) From our present work it appears, however, that the B meson may not be primarily associated with the $J^{PG} = 1^{--}$ ω meson. Furthermore, the observed B peak appears to be somewhat narrower than indicated in the combined results of earlier experiments.

(c) There does not appear to be any 2π or $K\bar{K}$ decay mode for the B meson.[17,18] This rules out the spin-parity $J^P = 1^-$ and 3^- assignments for the B meson.

It is not clear to us at present what the correct interpretation of the B peak is. Some possibilities we have considered are the following:

(a) Distortion of the matrix element due to final-state interactions.—Here we consider the possibility that the effect we observed is due to $B \rightarrow \pi + \omega$ but that the ω matrix element is distorted because of the B-decay process. The effects of Bose statistics will be more pronounced for the "double ω" events. As was pointed out above, however, these occur primarily outside the B peak (see Fig. 3). Furthermore, the ratio of Γ_ω/Γ_B is such as to make distortion effects unlikely.

(b) A kinematical enhancement.—Such an enhancement can come about, for example, by the presence of two constraints.[19] One constraint is the artificial one introduced for the "background events" by condition (2). The other constraint would have to be a natural one due to the presence of a two- or three-particle resonance other than the ω.[20] There is no convincing evidence for the occurrence of such a mechanism for the B peak in our data. We cannot rule it out, however.

(c) Decay into four pions.—We can make the assumption that the B meson is indeed a resonance and that the $\omega\pi$ decay mode is rigorously forbidden. This would occur in the case of a $J^{PG} = 0^{++}$ assignment for the B meson, for example. Such a meson could decay into four pions. The problem, however, is to explain why three of these pions appear to be bunched in the vicinity of the ω-meson mass, since on simple phase-space considerations only 12% of the three-pion-mass triplets would fall into the mass band given by condition (2).

(d) Decay into a π and a definite three-pion state with $J^P \neq 1^-$.—If the B meson, considered again as a 0^{++} particle, decays into a pion and a definite three-pion state with mass similar to that of the ω meson, the quantum numbers of such a three-pion state would be $I = 0$ and $J^{PG} = 0^{--}, 1^{+-}, 2^{--}$, etc. We note here that for the above J^{PG} assignments to the three-pion system the density distribution goes to zero at the center of the Dalitz plot. Our data alone do not have the statistical accuracy, however, nor do we have any direct evidence to ascertain the existence of such a three-pion state.

This work would not have been possible without the help of many people. While space limitation does not permit us to thank all of them individually, we do wish to single out a few. We wish to thank Luis W. Alvarez for making the 72-in. bubble chamber available to us; the Bevatron crew under Edward J. Lofgren and Walter D. Hartsough; the bubble chamber crew under Robert Watt and Glen Eckman; the F-SD crew under Howard S. White and our own scanning staff at Berkeley; R. Shutt for making the 20-in. bubble chamber available to us; the AGS crew under K. Green and J. Spiro; and the 20-in. bubble chamber crew, and in particular H. Brown, for helping with our run at Brookhaven. Finally, we wish to acknowledge the help of John L. Brown and George H. Trilling in various stages of this work.

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[11]In experiments with stopping antiprotons the reaction $\bar{p}+p\rightarrow K^{+}+K^{-}+\omega$ gives rise to stopping K^{+} and K^{-} mesons. This allowed the measurement of $\Gamma_{\omega}=9.5\pm2.1$ MeV, based on range measurement [N. Gelfand, D. Miller, N. Nussbaum, J. Ratau, J. Schulz, J. Steinberger, T. H. Tan, L. Kirsch, and R. Plano, Phys. Rev. Letters 11, 436 (1963)]. In our present experiment, and all others of similar type, where no range measurements are available, the experimental resolution for the one-constraint fits involved here is considerably larger than Γ_{ω}. This resolution problem is thus the reason for picking a 60-MeV width in which to include the majority of the true ω events formed.

[12]Here we must note that we are using the same "constant probability curve" computed for the ω mass, $M(3\pi)=782$ MeV, as the dividing line for all three-pion mass values. Actually, the area of the normalized Dalitz plot changes slightly as the boundary changes shape from a circle in the nonrelativistic region to a triangular figure in the highly relativistic region. Thus, for example, when the three-pion mass corresponds to the η, the "central region" of the Dalitz plot, as defined above, amounts to 25% of the total area.

[13]The N^{*++} band was taken as 1130 to 1300 MeV in the $\pi^{+}p$ reaction and 1170 to 1270 MeV in the $\pi^{-}p$ reaction.

[14]In these data we find 62 "double-ω" events. In the entire 4π mass plot these events are counted once only (see Fig. 3). In the two separate regions, however, for 21 of these double-ω events, one pion triplet falls in the central region, the other in the peripheral one. These events are included in both Figs. 4(a) and 4(b). They do not cluster in the B peak.

[15]These calculations were carried out by J. Dash by use of a modification of the program FAKE due to G. A. Lynch, Lawrence Radiation Laboratory Report No. UCRL-10335, July 1962 (unpublished).

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[19]An example of such a "kinematical enhancement" was observed in the present data. If we artificially constrain M_3^{b} to lie between 510 and 710 MeV, we observe a marked peak at 1010 MeV in the 4π mass distribution. This comes about because of the presence of the natural constraint in M_3^{a}, the ω meson. This example is discussed in reference 3.

[20]Here one may comment that if such a mechanism were present, one would expect a peak similar to the B, but displaced in mass, to occur when we constrain background events outside the ω mass as well. We have looked into the possibility, but find that any such effect, if it occurs, would be masked because of the overwhelming number of ω mesons present in the data.