DETERMINATION OF MIXING PARAMETERS OF THE K^0-$\bar{K}^0$ SYSTEM BY MEANS OF STRONG INTERACTIONS

Frank S. Crawford, Jr.

Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 15 November 1965)

The purpose of this paper is to call attention to a class of experiments which provides, using strong interactions only, a complete determination of the mixing parameters p, q, p', and q' that correspond to the linear relations among $|K^0\rangle$, $|\bar{K}^0\rangle$, $|K_S\rangle$, and $|K_L\rangle$. (We do not assume CPT invariance.[1,2]) We assume that these linear relations can be written in the form

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle,$$

$$|K_L\rangle = p'|K^0\rangle - q'|\bar{K}^0\rangle. \tag{1}$$

In any consistent theory of neutral K mesons, the values of p, q, p', and q' that we so determine must agree with those determined by means of the weak decays. It is worthwhile to determine p, q, p', and q' without making explicit use of the weak-decay experiments, in order to establish this consistency, if it exists.[3]

We consider two specific experiments. In each experiment we begin with a known number of neutral K mesons of known strangeness S, "born" at $t=0$ in a liquid-hydrogen bubble chamber in one or the other of the two reactions

$$S=+1:\ \pi^- + p \rightarrow \Lambda + K^0 \tag{2}$$

or

$$S=-1:\ K^- + p \rightarrow \bar{K}^0 + n. \tag{3}$$

The experiment in each case consists of determining the relative time dependences and the absolute counting rates for both of the secondary reactions

$$S=+1:\ K^0 + p \rightarrow K^+ + n \tag{4}$$

and

$$S=-1:\ \bar{K}^0 + p \rightarrow \Lambda + \pi^+. \tag{5}$$

In the remainder of this paper we first show how either one of the two experiments (2), (4), and (5), or (3), (4), and (5) can be used to determine p, q, p', and q'. We then discuss present knowledge of p, q, p', and q' as obtained from the weak decays.

The inverse of Eq. (1) is

$$|K^0\rangle = (pq'+p'q)^{-1}[q'|K_S\rangle + q|K_L\rangle],$$

$$|\bar{K}^0\rangle = (pq'+p'q)^{-1}[p'|K_S\rangle - p|K_L\rangle]. \tag{6}$$

Of the eight real parameters p, q, p', and q', only three are independent of conventions of normalization and phase.[2,4] They may be expressed in terms of the two real parameters

$$(p/q)/(p'/q') \equiv z \equiv x+iy \equiv |z|\exp i\varphi, \tag{7}$$

and one (or the other) of the ratios $|p/q|$ or $|p'/q'|$. Alternatively, they may be expressed in terms of $|p/q|$, $|p'/q'|$, and φ. CP invariance[5] (with a particular phase convention) implies $p=q=p'=q'$, whether or not T or CPT invariance holds, and hence implies (independent of phase convention) $x=+1$, $y=0$, and $|p/q|=|p'/q'|=1$; however, we know that CP does not hold.[6] CPT invariance[5] (with a phase convention) implies $p'=p$ and $q'=q$, and hence (independent of phase convention) $x=+1$, $y=0$, and $|p/q|=|p'/q'|$ not necessarily equal to 1.[7,8] T invariance[2,9] (and a phase convention) implies $p'=q$ and $q'=p$; then $z=p^2/q^2=q'^2/p'^2$, with no a priori constraint on the magnitude or phase of z.

Let $\psi(t)$ denote the development in time (in the K rest frame and in vacuum[10]) of a neutral K amplitude that is given at $t=0$ by $\psi(0)=|K^0\rangle$, corresponding to Reaction (2). Similarly let $\bar{\psi}(t)$ correspond to $\bar{K}^0$ production at $t=0$ via Reaction (3), with $\bar{\psi}(0)=|\bar{K}^0\rangle$. Assume[2] that the Weisskopf-Wigner time dependence holds for K_S and K_L, and define the time-dependent factors $f_S \equiv \exp(-im_S t - \frac{1}{2}\lambda_S t)$, $f_L \equiv \exp(-im_L t - \frac{1}{2}\lambda_L t)$, where m_S, m_L, λ_S, and λ_L are the masses and inverse lifetimes of K_S and K_L. Then we find

$$\psi(t) = (1+z)^{-1}[(zf_S + f_L)|K^0\rangle$$

$$+ (q'/p')(f_S - f_L)|\bar{K}^0\rangle] \tag{8}$$

and

$$\bar{\psi}(t)=(1+z)^{-1}[(p/q)(f_S-f_L)|K^0\rangle+(f_S+zf_L)|\bar{K}^0\rangle]. \quad (9)$$

We now consider separately the two experiments corresponding to (2) and (3).

K^0 production via Reaction (2).—The relative time dependence for hyperon production (5) is given by the familiar factor[11]

$$|f_S-f_L|^2=\exp(-\lambda_S t)+\exp(-\lambda_L t)-2\{\exp[-\tfrac{1}{2}(\lambda_S+\lambda_L)t]\}\cos mt, \quad (10)$$

where $m=m_S-m_L$. Equation (10) leads to a determination of $|m|$ that is independent[1] of the relations between p, q, p', and q'. On the other hand, the relative time dependence for charge exchange (4) is given by

$$|zf_S+f_L|^2=(x^2+y^2)\exp(-\lambda_S t)+\exp(-\lambda_L t)+2\{\exp[-\tfrac{1}{2}(\lambda_S+\lambda_L)t]\}\times(x\cos mt+y\sin mt) \quad (11)$$

and can be used to find x and y.[12] The absolute counting rate for hyperon production (5) can be used to determine $|q'/p'|$ as follows. The cross section for (5) is equal (assuming charge independence) to twice that for $K^-+p\to\Lambda+\pi^0$, which is known.[13] The absolute counting rate for (5) obtained from a known number of initial K^0 can therefore be predicted from the cross section, the density of liquid hydrogen, and the $\bar{K}^0$ flux factor $|q'/p'|^2|1+z|^{-2}$ obtained from Eq. (8). The absolute counting rate for charge exchange (4) provides a consistency check (or an overdetermination) for x and y as follows. The cross section for (4) is equal (assuming charge symmetry) to that for $K^++n\to K^0+p$ at the same momentum. The latter cross section is obtained from K^+ charge exchange in deuterium.[14] Thus the yield for (4) gives the K^0 flux factor $|1+z|^{-2}=[(1+x)^2+y^2]^{-1}$.

$\bar{K}^0$ production via Reaction (3).—The relative time dependence for charge exchange (4) is given by Eq. (10) and determines $|m|$, independently of p, q, p', and q'. The relative time dependence for hyperon production (5) is given by

$$|f_S+zf_L|^2=\exp(-\lambda_S t)+(x^2+y^2)\exp(-\lambda_L t)+2\{\exp[-\tfrac{1}{2}(\lambda_S+\lambda_L)t]\}\times(x\cos mt-y\sin mt) \quad (12)$$

and determines x and y.[12] The absolute counting rate for charge exchange (4) determines the K^0 flux factor $|p/q|^2|1+z|^{-2}$ and hence gives $|p/q|^2$. The absolute counting rate for hyperon production (5) gives $|1+z|^{-2}$ and hence a check on z.

To summarize, either one of the two experiments consisting of (2) followed by (4) and (5) or (3) followed by (4) and (5) can provide a complete determination of the relations among p, q, p', and q' as defined in Eq. (1). They can therefore provide tests of CP, CPT, and T invariance, as well as tests of consistency with values of p, q, p', and q' obtained from the weak decays. (No explicit use was made of experiments involving the weak decays.)

Experiments that involve coherent regeneration of K_S do not provide a complete determination of p, q, p', and q', and we do not discuss them.[15]

We now ask what is known about the relations among p, q, p', and q' from presently published experiments on neutral K decay. For our purposes it suffices to consider decay into $\pi^++\pi^-$. Let a_S and a_L denote amplitudes for decay into $\pi^++\pi^-$ by K_S and K_L. Then the decay rate $I(t)$ into $\pi^++\pi^-$ that develops from a number N_0 of K^0 mesons produced at $t=0$, and the corresponding rate $\bar{I}(t)$ from a number $\bar{N}_0$ of $\bar{K}^0$ mesons at $t=0$, are given according to (6) by

$$I(t)=N_0|pq'+p'q|^{-2}|q'a_Sf_S+qa_Lf_L|^2,$$

$$\bar{I}(t)=\bar{N}_0|pq'+p'q|^{-2}|p'a_Sf_S-pa_Lf_L|^2. \quad (13)$$

The magnitude of a_L is about 2×10^{-3} times the magnitude of a_S.[6] Therefore in experiments for which t never gets beyond ~$10\tau_1$, the decay rate into $\pi^++\pi^-$ is to within a correction of order 1% (due to the interference term between a_S and a_L) given by the contribution from K_S alone.[16] According to (13) we then have

$$\frac{N(\pi^+\pi^-)/N_0}{\bar{N}(\pi^+\pi^-)/\bar{N}_0}=\left|\frac{q'}{p'}\right|^2, \quad (14)$$

where $N(\pi^+\pi^-)$ and $\bar{N}(\pi^+\pi^-)$ are the time integrals of $I(t)$ and $\bar{I}(t)$ from $t=0$ to ~$10\tau_1$. Thus

$|q'/p'|$ can be obtained by comparing the branching ratio for K_S decay into $\pi^+ + \pi^-$ when the initial meson is K^0 to that when it is $\bar{K}^0$. The K^0 branching ratio has been determined in many experiments. The $\bar{K}^0$ branching ratio is not easily determined (because $\bar{N}_0$ is not easily determined), and there is no published value. However, a ratio that is closely related to (14) has been determined by Armenteros et al.[17] and by Baltay et al.[18] They stop antiprotons in hydrogen and determine the ratio

$$r \equiv \frac{\mathrm{Rate}(\bar{p}+p \to K_S + K^- + \pi^+)}{\mathrm{Rate}(\bar{p}+p \to K_S + K^+ + \pi^-)} = \frac{N_0 |q'|^2}{\bar{N}_0 |p'|^2}, \qquad (15)$$

where K_S stands for the $\pi^+\pi^-$ decay. In order that r as given by (15) be a true measure of $|q'/p'|^2$ as given by (14), we must assume that the number N_0 of strong interactions $\bar{p}+p \to K^0 + K^- + \pi^+$ is the same as the number $\bar{N}_0$ of strong interactions $\bar{p}+p \to \bar{K}^0 + K^+ + \pi^-$, that is, assume that C is conserved in the strong interactions of neutral K mesons.[19] In that case we find (adding the results of Refs. 17 and 18, and including the case of an extra π^0 from Ref. 18)

$$\left|\frac{q'}{p'}\right|^2 = r = \frac{560+438+953}{585+413+957} = 1.00 \pm 0.032. \qquad (16)$$

Thus $|q'/p'|$ is 1.00 ± 0.016.

The remaining two parameters x and y [or their equivalents $|q/p|$ and the phase of $z = (p/q)/(p'/q')$] remain undetermined. Rather than consider other decay experiments, we now use a powerful inequality given by Lee and Wolfenstein.[2] Independent of whether or not CPT invariance holds or whether or not T invariance holds, the single requirement of unitarity (i.e., conservation of probability) plus the two experimental results $\lambda_L/\lambda_S \approx 1/655$ and $|m_S - m_L| \approx 0.6\lambda_S$ leads to the result

$$|p^*p' - q^*q'|^2 \leq \frac{4\lambda_S\lambda_L}{(\lambda_S+\lambda_L)^2 + 4(m_S-m_L)^2} \approx \frac{1}{400}. \qquad (17)$$

When (17) and the experimental result (16) are combined, one finds $0.9 \leq |q/p| \leq 1.1$ and that the phase φ of z satisfies -0.1 rad $\leq \varphi \leq +0.1$ rad. According to P. Eberhard,[20] these limits can be further narrowed by a factor of about three by applying the unitarity condition to all of the experimentally determined branching ratios and using the result (16).

Thus from the weak decays alone and without any special assumptions[21] as to CPT (or any other) invariance, we have the clear set of results $|q'/p'| = 1.00 \pm 0.016$,[19] $(\approx 0.97) \leq |q/p| \leq (\approx 1.03)$, and $(\approx -0.03) \leq \varphi \leq (\approx +0.03)$. It remains to be seen whether the proposed determinations using the strong interactions will give the same answers.[3]

I am grateful for conversations with Sheldon L. Glashow, Sidney Coleman, Philippe Eberhard, and Michael Nauenberg.

---

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]R. G. Sachs, Phys. Rev. 129, 2280 (1963).

[2]T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1490 (1965).

[3]If there were a third neutral K, as discussed, for example, by P. K. Kabir and R. R. Lewis, Phys. Rev. Letters 15, 711 (1965), then Eq. (1) would be incomplete. Then we might expect large discrepancies between the values of the parameters obtained via the strong interactions and those obtained via the weak decays, under the (false) assumption that Eq. (1) is complete.

[4]Strangeness conservation in strong interactions is assumed, and guarantees that $|K^0\rangle$ and $|\bar{K}^0\rangle$ are orthogonal. The most natural normalization conditions are therefore $|p|^2 + |q|^2 = 1$ and $|p'|^2 + |q'|^2 = 1$. (If CP, C, or P were conserved in weak interactions, $|K_S\rangle$ and $|K_L\rangle$ would also be orthogonal.) The relative phase of $|K^0\rangle$ and $|\bar{K}^0\rangle$ and the absolute phases of $|K_S\rangle$ and $|K_L\rangle$ can be chosen by convention. These five conditions reduce p, q, p', and q' to three real parameters to be determined by experiment.[2] Our parameters x, y, and $|p/q|$ (or $|p'/q'|$) are independent of phase conventions; they are directly measurable. (They are closely related to the parameters A, B, and C of Ref. 2).

[5]T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957).

[6]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters 15, 73 (1965).

[7]The best-known prediction of CPT invariance is that the masses and lifetimes of weakly decaying particle and antiparticle must be the same. [T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956).] But this result, like most other predictions following from CPT invariance, is guaranteed by CP invariance alone, whether or not CPT invariance holds.[1] Thus CPT invariance is most easily susceptible to test in those interactions where CP invariance is now believed to fail, namely in the weak interactions of neutral K mesons. Sachs[1] discusses a number of tests of CP and CPT invariance involving the decays of neutral K mesons and also involving elastic scattering and regenerative scattering of neutral K's from nuclei. The experiments we consider in this paper have not been previously discussed as possible tests of CP, CPT, and T invariance, although they are well known as possible experiments to determine the magnitude of the mass difference

$|m_S - m_L|$.

[8]T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964), give a systematic phenomenological analysis of neutral K decay, assuming CPT invariance.

[9]S. Coleman and S. L. Glashow, to be published.

[10]The nuclear mean free path in liquid hydrogen is several hundred times larger than the mean K_S decay distance, for the experiments we envisage. Therefore we assume that we can neglect coherent regeneration in the liquid hydrogen, and we take the K amplitude in liquid hydrogen to be the same as it would be in vacuum. That there is no "anomalous" (large) regeneration in liquid hydrogen has been shown by Christenson et al. (Rev. 15).

[11]W. F. Fry and R. G. Sachs, Phys. Rev. 109, 2212 (1958).

[12]The sign of y is determined only relative to that of $m_S - m_L$. However, the sign of $m_S - m_L$ can still be determined by using the method of elastic scattering of neutral K's from hydrogen, followed by $\pi^+\pi^-$ decay, as suggested by U. Camerini, W. F. Fry, and J. Gaidos, Nuovo Cimento 28, 1096 (1963). The generalization of their formulas to encompass $x \neq 1$ and $y \neq 0$ is straightforward.

[13]For $\bar{K}$ momenta between 300 and 500 MeV/c, this cross section has been measured by M. B. Watson, M. Ferro-Luzzi, and R. D. Tripp, Phys. Rev. 131, 2248 (1963).

[14]For K momenta less than 810 MeV/c, see V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. 134, B1111 (1964).

[15]In a regeneration experiment like that of J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. 140, B74 (1965), the intensity of regenerated K_S is proportional to $[|q'/q|^2/|1+z|^2]|f_S - f_L|^2$, where the time t in the terms f_S and f_L is the time the beam takes to traverse the slab of regenerator. We see that the dependence on thickness is independent of p, q, p', and q', and provides a determination of $|m_S - m_L|$ that is independent of these parameters, provided the $\pi^+\pi^-$ decay from K_L can be neglected compared to that from regenerated K_S. The absolute intensity does depend on $|q'/q|$ and on z, as well as on the optical-model parameters, but is difficult to determine experimentally.

[16]If q'/q or p'/p were pathologically small, we could not make such a statement, i.e., we could not neglect $qa_L f_L$ and $pa_L f_L$ in Eq. (13) for reasonably short times t. But we know this is not the case: q'/q is not pathologically small, because all published determinations of the lifetime of K_S start with a K^0 at $t=0$, and the only time dependence seen in the first 10 K_S mean lives is that of the exponential $|f_S|^2 = \exp(-\lambda_S t)$; p'/p is not pathologically small, because if one starts with a $\bar{K}^0$ at $t=0$, one also gets the time dependence $\exp(-\lambda_S t)$, with the same value for λ_S as in the published lifetimes that start with K^0. (For example, A. Barbaro-Galtieri, private communication and to be published, finds the normal K_S lifetime in an analysis of 20 000 $\pi^+\pi^-$ decays arising from $\bar{K}^0$.) Thus we need not actually use the smallness of $|a_L/a_S|$ as determined in Ref. 6.

[17]R. Armenteros, D. N. Edwards, T. Jacobsen, L. Montanet, J. Vandermeulen, Ch. D'Andlau, A. Astier, P. Baillon, J. Cohen-Ganouna, C. DeFoix, J. Siad, and P. Rivet, Phys. Letters 17, 170 (1965).

[18]C. Baltay, N. Barash, P. Franzini, N. Gelfand, L. Kirsch, G. Lütjens, J. C. Severiens, J. Steinberger, D. Tycko, and D. Zanello, Phys. Rev. Letters 15, 591 (1965).

[19]In Refs. 17 and 18 the K^0 and $\bar{K}^0$ are found by scanning for decays. Thus N_0 and $\bar{N}_0$ are not determined. Therefore these experiments do not rule out the rather pathological possibility that $N_0/\bar{N}_0$ and $|q'/p'|^2$ are both different from unity but conspire in their product to give $r=1$. This possibility can be eliminated by a measurement of the branching ratio for $\bar{K}^0$ into $\pi^+ + \pi^-$.

[20]Philippe Eberhard, private communication and to be published.

[21]Wu and Yang[8] show that the smallness of $|a_L/a_S|$ and the validity of the $|\Delta I| = \frac{1}{2}$ rule imply $p \approx q$, if CPT invariance holds. Lee and Wolfenstein[2] show that p, q, p', and q' are closely equal even if CPT invariance does not hold (or if there is an equivalent cosmological force) provided that the CP-noninvariant Hamiltonian, H_F, is very small. S. L. Glashow and S. Weinberg, Phys. Rev. Letters 14, 835 (1965), show that the 2π decay rates imply $|p/q|^2 - 1 \leq 1/25$, provided CPT invariance holds. Coleman and Glashow[9] show that T invariance implies (with their phase convention) $z = p^2/q^2 = q'^2/p'^2$. The experimental result (16) therefore implies $|q/p| = 1.00 \pm 0.016$ if T invariance is good. They further show that the rates for $K_S \to 2\pi$ and $K_L \to \pi^+ + \pi^-$ imply that z is nearly real if T invariance is good. M. Nauenberg (private communication) shows that the smallness of $|a_L/a_S|$ implies that any CPT-invariance violation must be small, namely, $|q/p||1-z| \leq 4|a_L/a_S| \approx 8\times 10^{-3}$, unless there are accidental cancellations.