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[9]For $l=0$, for example, one can estimate that the second term of Eq. (4) becomes significant only at those angles for which

$$\frac{d\sigma_1(\theta)}{d\Omega} \gtrsim \left[k_p^{\,2}\frac{d\sigma_1(\theta)}{d\Omega}\frac{(\Delta\theta)^4}{4}\right]\frac{d\sigma_p(\theta)}{d\Omega}.$$

Here $\Delta\theta$ is the half-width of the forward stripping peak yielded by the first term, $d\sigma_1/d\Omega$ is the cross section yielded by the first term, and $d\sigma_p/d\Omega$ is the proton elastic-scattering cross section. Similar estimates are available for $l\neq 0$.

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ANGULAR MOMENTUM EFFECTS IN REACTIONS WITH HEAVY IONS*

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Rearrangement collisions between complex nuclei differ from direct nuclear reactions with simple nuclear projectiles, at least in three aspects: (1) Coulomb effects are stronger. (2) The dimensions of the projectile are not small in comparison with the dimensions of the target nucleus. (3) The transferred particle is usually not in a state of zero orbital angular momentum in the incident particle.

These effects may lead to unusual experimental cross sections. Recently, heavy-ion experiments involving both single-particle and cluster transfer at energies well above the Coulomb barrier actually exhibited several unusual and unexpected features.[1-3] In some cases the angular distribution is smooth, in other cases it is oscillatory.

Dodd and Greider suggested[4] that the smooth behavior is due to recoil effects; however, recent measurements by Bock et al.[3] exhibit a clear oscillatory behavior in many cases, in contrast with Dodd and Greider's predictions. The purpose of this note is to provide a simple explanation for this behavior and to relate the oscillatory angular distribution in a simple way to the angular momenta involved in the reactions.

Our approach is based on the smooth-cutoff diffraction model (SCDM) for direct nuclear reactions.[5] According to this model the transition amplitude for the reaction $A(a,b)B$ (colliding systems A,a and separating systems B,b) has the form of a matrix element between product wave functions:

$$T_{ba}=(\Psi_B\Psi_b\chi_b^{(-)}(\vec{k}_b,\vec{r}_b),V\Psi_A\Psi_a\chi_a^{(+)}(\vec{k}_a,\vec{r}_a)). \quad (1)$$

Here $\Psi_B,\Psi_b,\Psi_A,\Psi_a$ are the internal wave functions for the noninteracting separated particles B,b,A,a. The interaction V is the interaction whose off-diagonal matrix elements are responsible for the transition. $\chi_b^{(-)}$ and $\chi_a^{(+)}$ are the WKB wave functions for the elastic scattering of the pair A,a and B,b, respectively.

For cluster-transfer reactions, particle a is assumed to form a bound state of particles b and c while particle B is assumed to form a bound state of particles A and c, i.e., the reaction is described symbolically as

$$A+a\rightarrow B+b\equiv A+(c+b)\rightarrow(A+c)+b, \quad (2)$$

where closed parantheses denote a bound state. V is taken to be the interaction between b and c. The bound-state wave functions are approximated by

$$\langle B|A\rangle=\langle A+c|A\rangle$$
$$=\sum_{j_fl_f}\theta_{j_fl_f}^{B}\sum_{\mu_fM_cm_f}(l_f\mu_fJ_cM_c|j_fm_f)\times(j_fm_fJ_AM_A|J_BM_B)\langle l_f\mu_f|\langle J_cM_c|, \quad (3a)$$

$$\langle b|a\rangle=\langle b|c+b\rangle$$
$$=\sum_{j_il_i}\theta_{j_il_i}^{a}\sum_{\mu_iM_c'm_i}(l_i\mu_iJ_cM_c'|j_im_i)\times(j_im_iJ_bM_b|J_aM_a)|l_i\mu_i\rangle|J_cM_c'\rangle. \quad (3b)$$

Capital letters stand for spins and magnetic quantum numbers, small letters stand for orbital angular momenta and magnetic quantum numbers. The θ's are the spectroscopic factors for the bound states.

According to the diffraction model, the reaction takes place outside the cores b and B, where the orbital wave functions of the cluster can be well approximated by

$$|l_i\mu_i\rangle=U_{l_i}(r_{cb})Y_{l_i\mu_i}(\Omega_{cb}) \quad (4a)$$

$$|l_f\mu_f\rangle=N_fh_{l_f}^{(1)}(i\beta r_{cA})Y_{l_f\mu_f}(\Omega_{cA}), \quad (4b)$$

where N_{lf} and β are the normalization constant and the wave number, respectively, for the final bound state of the cluster c.

The diffraction model furthermore assumes that because of strong absorption, the dominant contribution to the reaction comes from regions where the cores b and A are separated from each other by a distance larger than the sum of their radii.

We now make use of an addition theorem.[7] For $r_{cb}<r_{bA}$,

$$h_{lf}^{(1)}(i\beta|\vec{r}_{cb}-\vec{r}_{bA}|)Y_{lf\mu f}(\Omega_{cA})=(4\pi)^{1/2}\sum_{ll'\mu\mu'}(-1)^{\frac{1}{2}(l_f+l-l')}[(2l_i+1)(2l_f+1)/(2l+1)]^{1/2}(l\mu l_f\mu_f|l'\mu')$$
$$\times(l0l_f0|l'0)h_l^{(1)}(i\beta r_{bA})j_{l'}(i\beta r_{cb})Y_{l\mu}^*(\Omega_{bA})Y_{l'\mu'}(\Omega_{cb}). \quad (5)$$

We substitute Eqs. (3), (4), and (5) into expression (1) and neglect recoil effects in order to reduce the T matrix into a product of two three-dimensional integrals. The differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\frac{m_am_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}\frac{(2J_B+1)}{(2J_A+1)}\sum_{\substack{j_il_il_i'\\ j_fl_fl_f'}}\theta_{j_il_i}^{a}\theta_{j_il_i'}^{(a)}\theta_{j_fl_f}^{B}\theta_{j_fl_f'}N_{lf}N_{lf'}F_{l_i}F_{l_i'}[(2l_i+1)(2l_i'+1)]^{1/2}\sum_{l_f}(l0l_i0|l_f0)$$
$$\times(l0l_i'0|l_f'0)W(ll_ij_fJ_c;l_fj_i)W(ll_i'j_fJ_c;l_f'j_i)\sum_\mu|T_{l\mu}|^2, \quad (6)$$

where

$$F_{l_i}=(4\pi)^{1/2}\int_0^\infty r^2j_{l_i}(i\beta r)V(r)U_{l_i}(r)dr \quad (7a)$$

and

$$T_{l\mu}=\int\chi^{(-)*}(\vec{k}_b,\vec{r})h_l^*(i\beta r)Y_{l,\mu}^*(\Omega)\chi^{(+)}(\vec{k}_a,\vec{r})d^3r. \tag{7b}$$

The most common case is the one when the transferred particle is in definite subshells l_ij_i, l_fj_f in the projectile and final nucleus, respectively. In this case

$$\frac{d\sigma}{d\Omega}=\frac{m_am_b}{(2\pi\hbar^2)^2}\frac{k_b}{k_a}|\theta_{j_il_i}{}^a|^2|\theta_{j_fl_f}{}^B|^2|N_{l_f}|^2|F_{l_i}|^2(2l_i+1)\sum_l(l0l_i0|l_f0)^2W(ll_ij_fJ_c;l_fj_i)^2\sum_{\mu=-l}^{l}|T_{l\mu}|^2. \tag{8}$$

From the triangular rules of the coupling coefficients, one obtains the following selection rules:

$$l+l_i+l_f \text{ even}, \tag{I}$$

$$|l_i-l_f|\le l\le l_i+l_f, \tag{II}$$

$$|j-j_f|\le l\le j_i+j_f. \tag{III}$$

The overlap integrals $T_{l\mu}$ which are defined by Eq. (7b) are the standard zero-range overlap integrals of the smooth-cutoff diffraction model for rearrangement collisions.[5] Under the adiabatic condition, $k_a\sim k_b$, they may be well approximated by analytic expressions. These expressions are described elsewhere[5] for (a) energy well below the Coulomb barrier, and (b) energy well above the Coulomb barrier + large Sommerfeld parameter. In both cases the theory predicts a smooth behavior of the differential cross section in good agreement with the experimental data. Oscillations which may result from angular momenta and diffraction effects are damped completely due to strong Coulomb interaction.

We now consider the case where the energy is well above the Coulomb barrier and the Sommerfeld parameter is not too large. In this case the $T_{l\mu}$ may be well approximated by

$$|T_{l\mu}|^2\approx\frac{4\pi^4\delta^2(2L_0+1)^2}{2l+1}B_{L_0}{}^2|Y_{l\mu}(\pi/2,0)|^2(\theta/\sin\theta)[\{|A_+|^2+|A_-|^2\}\{J_{|\mu|}{}^2[(L_0+\tfrac{1}{2})\theta]+J_{|\mu|-1}{}^2[(L_0+\tfrac{1}{2})\theta]$$
$$+2R_e(A_+A_-^*)\{J_{|\mu|}{}^2[(L_0+\tfrac{1}{2})\theta]-J_{|\mu|-1}{}^2[(L_0+\tfrac{1}{2})\theta]\}$$
$$+4\mathrm{Im}(A_+A_-^*)J_{|\mu|}[(L_0+\tfrac{1}{2})\theta]J_{|\mu|-1}[(L_0+\tfrac{1}{2})\theta], \tag{9}$$

where

$$A\pm=\csc[\pi\delta\Delta/2\mp i\pi\delta(\theta\pm\theta_0)]. \tag{10a}$$

L_0 and θ_0 are the angular momentum and the classical deflection angle, respectively, for a grazing collision. δ is the diffuseness in l space of the reflection coefficients η_l, for the elastic scattering of the colliding particles.

$$|\eta_l|=\{1+\exp[(L_0-1)/\delta]\}^{-1}. \tag{10b}$$

The other constants which appear in Eqs. (9) and (10a) have the following meanings:

$$\Delta/2=\beta/k,\quad B_{L_0}=\frac{1}{2\beta k^2}\left(\frac{\pi}{L_0\Delta}\right)^{1/2}\exp(-L_0\Delta/2). \tag{10c}$$

In the asymptotic region $\theta\gg|\mu|/L_0$, the Bessel functions may be approximated by their asymptotic behavior and Eq. (9) is further reduced to[5]

$$\sum_\mu|T_{l\mu}|^2=4\pi^2\delta^2(2L_0+1)B_{L_0}{}^2(\sin\theta)^{-1}\Big[|A_+|^2+|A_-|^2+(-1)^l2\{\mathrm{Re}(A_+A_-^*)\sin[(2L_0+1)\theta]$$
$$+\mathrm{Im}(A_+A_-^*)\cos[(2L_0+1)\theta]\}\Big]. \tag{11}$$

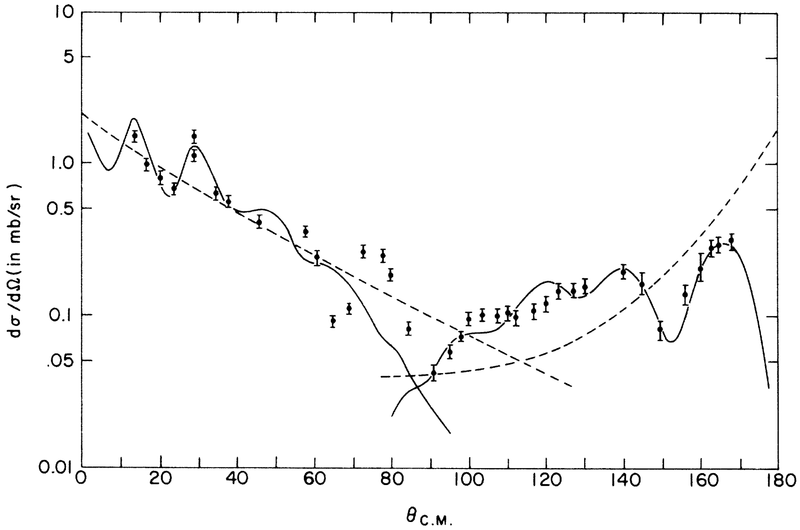

FIG. 1. Differential cross section for the reaction[3] $B^{11}(O^{16}, N^{15})C^{12}$. $E_{lab} = 30$ MeV. The solid curves were calculated from Eqs. (8) and (9) for proton and α-particle transfers. The parameters are listed in Table I. The dashed lines are Dodd and Greider's predictions for the same transitions.

Expression (11) consists of two contributions: a smooth part and an oscillatory interference term proportional to

$$(-1)^l = (-1)^{l_i + l_f}$$

[see selection rule (I)]. The condition for Coulomb damping of the oscillations is $2\pi\delta\theta_0 \gg 1$. If this condition is not satisfied we may conclude the following:

(a) For a transfer of a cluster from a definite subshell $l_i j_i$ into a definite subshell $l_f j_f$, the angular distribution should exhibit an oscillatory structure. Transitions with the same parity (of $l_i + l_f$) are in phase. Transitions with different parity are out of phase.

(b) If the sum over l in Eq. (6) involves both odd and even l values, the interference terms which are out of phase tend to cancel each other. This situation can result only from a configuration mixing for the transferred particle.

(c) Mixing of l values with different parity may result also from a transfer of a particle accompanied by a core excitation.

Comparison with experiments. — To illustrate our results we apply Eqs. (8) and (9) to the reaction $B^{11}(O^{16}, N^{15})C^{12}$ (see Figs. 1 and 2).

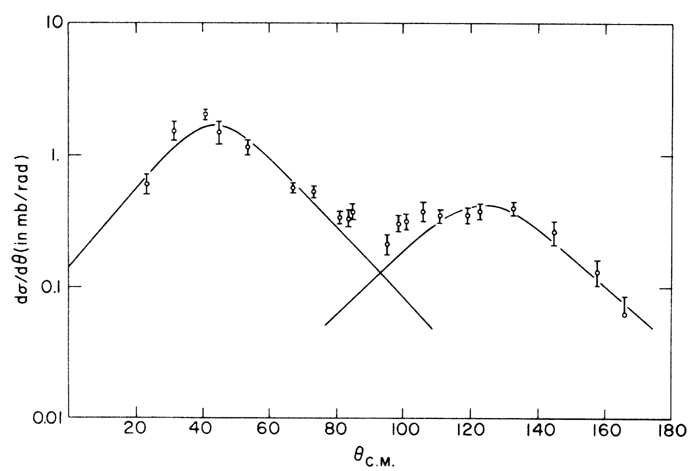

FIG. 2. Differential cross section for the reaction[3] $B^{11}(O^{16}, N^{15}(6.33\ \text{MeV}))C^{12}$. $E_{lab} = 30$ MeV. The solid curves were calculated from Eqs. (8) and (11), neglecting the interference term, for proton and α-particle transfers. The parameters are listed in Table I.

Table I. Parameters for the reaction $B^{11}(O^{16}, N^{15})C^{12}$.

Reaction	E_{lab} (MeV)	n	L_0	δ	θ_0	R	r_0	d
$B^{11}(O^{16}, N^{15})C^{12}$	30	4.6	10.5	0.62	35	8.3	1.75	0.25
$B^{11}(O^{16}, C^{12})N^{15}$	30	4.6	7.0	0.47	48	6.6	1.40	0.20
$B^{11}(O^{16}, N^{15*})C^{12}$	30	4.6	10.5	0.62	47	8.3	1.75	0.25
$B^{11}(O^{16}, C^{12})N^{15*}$	30	4.6	8.5	0.47	58	7.9	1.67	0.20

The forward peak of the angular distribution is associated with a proton transfer, the backward peak with α-particle transfer. The parameters L_0, δ, θ_0 which determine the angular distribution[8] were chosen to give a rough fit to experimental data. The binding potentials were chosen to be of a Woods-Saxon form. Their parameters were fitted to give the correct binding energies and the absolute values of the cross sections. All the parameters are summarized in Table I. As one expects, the "effective radius" for the α-particle transfer is smaller than for the proton transfer.

The deep minimum around 65° in the differential cross section is not predicted by the theory. It was suggested[3] that it may be a j-dependence effect similar to the "Lee-Schiffer effect."[9]

Deviations between the experimental results and our theoretical predictions around 90° may result from interference between the proton and α-particle transfers, and also from a compound nucleus formation.

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[8]L_0 and δ are related to the sum of radii and the diffuseness of the colliding particles[5] through

$$L_0 = \bar{k}\bar{R}[1-(2\bar{n}/\bar{k}\bar{R})]^{1/2}$$

$$\delta = \bar{k}\bar{d}[1-(\bar{n}/\bar{k}\bar{R})][1-(2\bar{n}/\bar{k}\bar{R})]^{-1/2}.$$

The bars on the Coulomb parameter n, the diffuseness d, the sum R of the radii, and the center-of-mass momentum k indicate an average over initial and final states.

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