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[12]If the target was irradiated in vacuum and the helium introduced after the beam burst, the yield was approximately the same; however, the total transport time was necessarily longer.

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[14]The spectrum also contained a long-lived 511-keV peak, presumably due to a small amount of contamination coming through the traps.

[15]If it were produced in a reaction involving the emission of a negative pion or from the 1% isotope C^{13}, it could contain seven neutrons. However, its cross section from carbon would probably be two orders of magnitude lower than from heavier targets, while in fact, the yields from C, O, and F were comparable.

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SPECTROSCOPIC FACTORS AND l VALUES FROM DEUTERON STRIPPING

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It is the purpose of this note to report how l values and spectroscopic factors may apparently be accurately obtained from deuteron stripping results by a quite elementary computation which contains no adjustable or ambiguous parameters. This work represents a preliminary investigation of a new method for stripping recently proposed by Butler[1] and Tanifuji.[2]

We consider a (d,p) stripping reaction in which the incident and outgoing wave vectors are $\vec{k}_d$ and $\vec{k}_p$, respectively, and where the spins of the initial and final nuclei are J_i and J_f, respectively. The theoretical cross section, in terms of optical-model wave functions, is[3,4]

$$\frac{d\sigma}{d\Omega}=\frac{\frac{1}{2}m_p m_d}{(2\pi\hbar^2)^2}\frac{k_p}{k_d}\frac{(2J_f+1)}{(2J_i+1)}\sum_{lm}\frac{1}{2l+1}S(l,J_i,J_f)|M|^2, \quad (1)$$

where the matrix element M is given as

$$M(\vec{k}_p,\vec{k}_d)=\langle\psi_d^{+}(\vec{k}_d,\vec{r}_p,\vec{r}_n)|V_{np} \times|F_l^{m}(\vec{r}_n)\psi_p^{-}(\vec{k}_p,\vec{r}_p)\rangle\rangle. \quad (2)$$

Here $\vec{r}_p$ and $\vec{r}_n$ are the neutron and proton coordinates, respectively. The wave function ψ_d^+ describes elastically scattered deuterons with outgoing spherical waves, ψ_p^- describes elastically scattered protons with incoming spherical waves, and F_l^m is the wave function of the final bound neutron with orbital angular momentum l and projection m, normalized to unity. The normal neutron-proton interaction is represented by V_{np}. In the form of Eq. (1) all other nuclear coordinates have been integrated out so that ψ_p^- may be considered to be a known optical-model wave function. The factor $S(l,J_i,J_f)$ is the so-called spectroscopic factor which is a real positive number.

Strictly Eq. (1) may be considered to be a distorted-wave Born approximation (DWBA) cross section. However, in the usual evaluation of the matrix element M associated with DWBA calculations, the interaction V_{np} is taken to be of zero range, and in the wave function ψ_d^+ all internal distortion or polarization of the deuteron is neglected; in these calculations ψ_d^+ describes the center-of-mass motion of the deuteron as an optical-model wave function, but leaves the internal motion unpolarized, a procedure which is very difficult to justify.[3]

Extensive exploration of this DWBA approach over the past decade has left its status still somewhat obscure.[4] There are always ambiguities in the deuteron optical parameters, and until recently it was considered usually necessary to choose optical parameters different from those required to fit elastic scattering.[5] However, a satisfactory theory should, with no adjustable parameters, be able to fit data such that l values can be determined unambiguously and spectroscopic factors given accurately.[6] A recent DWBA study of (d,p) reactions on Ca^{40}, involving careful selection of deuteron optical parameters from among those consistent with elastic-scattering data, has yielded satisfactory results. Yet there still remains a large number of cross sections reported in the literature which would appear to require optical parameters not in agreement with elastic-scattering data.

In the present approach no such difficulties arise. On the basis of a sudden approximation, in which the reaction time is considered short compared to the natural deuteron period, the wave function ψ_d^+ assumes a simple form.[1,2] It appears as a momentum-space convolution integral between a free-deuteron wave function and optical-model wave functions for neutron and proton interacting separately with the nucleus. The resulting expression for the matrix element M, without recourse to zero-range approximation and involving only nucleon optical wave functions, is[1,2]

$$M=\frac{\hbar^2N}{2\pi^2m}\int d\vec{k}_p{}'g(\vec{k}_d,\vec{k}_p{}')\langle\psi_p{}^+(\vec{k}_p{}',\vec{r}_p)|\psi_p{}^-(\vec{k}_p,\vec{r}_p)\rangle \times\langle\psi_n{}^+(\vec{Q}',\vec{r}_n)|F_l{}^m(\vec{r}_n)\rangle, \quad (3)$$

where m is the nucleon mass, $\vec{Q}'=\vec{k}_d-(M_i/M_f)\vec{k}_p{}'$ and $N^{-2}=1/2\gamma-\frac{1}{2}\rho_t$ with $\hbar^2\gamma^2/m$ the deuteron binding energy, and ρ_t the effective range of the neutron-proton interaction. The factor g is defined in Ref. 1; it is very close to unity for all regions of contribution to the integral.

The quantity $\langle\psi_p{}^+|\psi_p{}^-\rangle$ is determinable[7]; it is

$$\langle\psi_p{}^+(\vec{k}_p{}',\vec{r}_p)|\psi_p{}^-(\vec{k}_p,\vec{r}_p)\rangle =(2\pi)^3\left[\delta(\vec{k}_p-\vec{k}_p{}')-\frac{i}{\pi}\delta(k_p{}^2-k_p{}'^2)f(\vec{k}_p,\vec{k}_p{}')\right], \quad (4)$$

where f is the proton elastic-scattering amplitude so normalized that $|f|^2$ is directly the differential cross section.

The first term in Eq. (4) by itself yields a cross section sharply peaked at some forward angle and rapidly becomes negligibly small beyond the peak. The second term is strongly coupled to the proton elastic scattering; in many cases it is completely swamped by the first term in the vicinity of the forward peak, but rapidly takes over and dominates at larger angles.[8] For reactions involving heavy nuclei, the second term can be of importance also at small angles of scattering, tending to eliminate the appearance of a characteristic forward stripping structure; for light and medium-heavy nuclei this seldom, if ever, occurs.

Insertion of Eq. (4) into the matrix element (3) yields a final cross section which may be coded for computing. For an initial investigation, we chose reactions for which the first term of Eq. (4) does completely dominate at the main structural peak. Reliable criteria for this may readily be found.[9]

We then computed a cross section yielded by the first term of Eq. (4), assuming a square well of radius r_0 rather than the usual Saxon-Wood potential for determining the neutron optical and bound-state wave functions. The disadvantages of a square well are well recognized; it is, however, clearly sufficient for the aim of the present work.

An optical square-well potential of depth V_0+iW is chosen for determining ψ_n, and a real square well of depth U_0, with the same radius, is chosen for determining F_n. The element $\langle\psi_n|F_n\rangle$ may then be represented analytically.[10] It is

$$\langle\psi_n|F_n\rangle=\delta_{m0}\frac{[4\pi(2l+1)]^{1/2}r_0{}^2}{Q^2+\kappa^2}\left\{\frac{\Delta V-iW}{(E_Q+E_B+\Delta V)-iW}\right\} \times A_l e^{i\delta_l}\cos\delta_l[W(j_l(Qr_0),h_l(i\kappa r_0)) -\tan\delta_l W(n_l(Qr_0),h_l(i\kappa r_0))], \quad (5)$$

where $E_Q=\hbar^2Q^2/2m$, $E_B=\hbar^2\kappa^2/2m$ is the neutron binding energy, $\delta_l(\vec{Q})$ is the complex phase shift for the lth partial wave of ψ_n, h_l is the Hankel function of the first kind, and j_l and n_l are the spherical Bessel functions of the first and second kinds, respectively. Moreover $\Delta V=|V_0|-|U_0|$ and W denotes a Wronskian with derivatives taken with respect to r_0. We have also written $F_n=R_l(r_n)Y_{lm}(\theta_n\varphi_n)$, and A_l is defined in terms of R_l by the equation $R_l(r_0)=A_lh_l(i\kappa r_0)$. This expression for the cross section reduces to the familiar Butler-Born result[3,11] if we put $\delta_l=0$ and replace the factor inside the curly brackets by unity.

The depth U_0 is chosen to fit the observed binding energy. For the neutron optical potential we take one potential which gives a good fit to neutron scattering at all energies E_Q

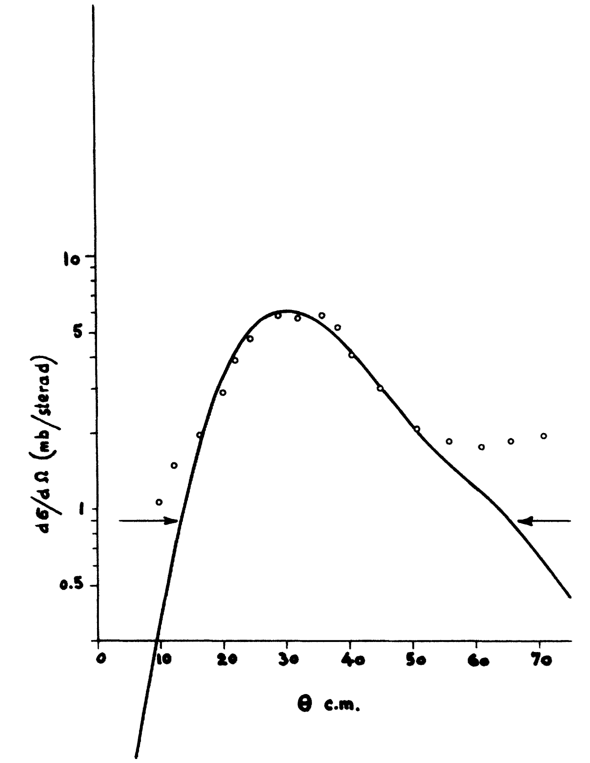

FIG. 1. Experimental points[4] and theoretical curve for $Ca^{40}(d,p)Ca^{41}$(g.s.) ($E_d = 12$ MeV; $Q = 6.14$ MeV; $l = 3$; $S = 1$). The arrows indicate roughly the point at which the second term in Eq. (4) becomes important.[9]

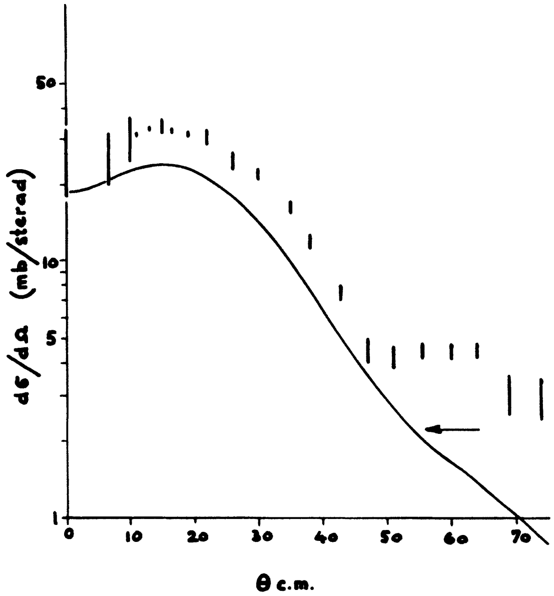

FIG. 2. Experimental points[14] and theoretical curve for $O^{16}(d,p)O^{17}$(g.s.) ($E_d = 15$ MeV; $Q = 1.92$ MeV; $l = 2$; $S = 1$). The error bars represent relative errors; the over-all experimental cross section is ±25%.

of relevance and on all target nuclei[12,13]:

$$V_0(\vec{Q}) = -(52.6 - 0.6E_Q), \quad E_Q < 21;$$

$$= -(40 - 10.8 \ln\{E_Q/21\}), \quad E_Q > 21;$$

$$W(\vec{Q}) = -(2.5 + 0.3E_Q), \quad E_Q < 21;$$

$$= -(8.8 + 2.25 \ln\{E_Q/21\}, \quad E_Q > 21;$$

$$r_0 = (1.33A^{1/3} + 0.30)\mathrm{F}. \qquad (6)$$

(V_0, W, and E_Q are expressed in MeV units.) Notice that, unlike Butler-Born theory, the nuclear radius is not treated as an adjustable parameter.

Cross sections so obtained from the first term of Eq. (4), both as regards absolute magnitude and angular distribution, are displayed in Figs. 1 and 2. In each case the spectroscopic factor S was taken to be unity. The arrows on the curves label the regions where we expect the second term in (4) to assume significance.[9] The reactions $O^{16}(d,p)O^{17}$ and $Ca^{40}(d,p)Ca^{41}$ were chosen because the initial nuclei are doubly magic, and the assignment $S = 1$ should be reliable. The experimental[4,14] absolute magnitudes are subject to over-all systematic errors of ±25% for O^{16} and ±10% for Ca^{40}. Equally good fits have been obtained [with the same potential (6)] for the peaks in $Ca^{40}(d,p)Ca^{41}$ at other deuteron energies and for excited states, and also for $C^{12}(d,p)C^{13}$(g.s.) ($S = 0.5$),[15,16] $K^{39}(d,p)K^{40}$(g.s.) ($S = 0.45$),[17] and the set[18] of reactions $Kr^{86}(d,p)Kr^{87}$(g.s.) ($S = 0.4$), $Sr^{88}(d,p)Sr^{89}$(g.s.) ($S = 0.5$), and $Zr^{90}(d,p)Zr^{91}$(g.s.) ($S = 0.6$). The spectroscopic factors in brackets are those which achieve exact agreement between experimental peak magnitudes and our theory; they are consistent with shell-model theory.

On the basis of these results a code is being prepared for the Sydney University KDF 9 computer for calculating complete cross sections using both terms of Eq. (4) and standard Saxon-Wood potentials throughout. In any case, however, the simple square-well cross section presented here, which involves no adjustable parameters and whose coding for a computer is trivial, would seem to provide a satisfactory basis for analyzing many experimental results.

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[8]The Coulomb singularity in f at zero scattering angle causes no trouble. Under the sudden approximation of Ref. 1 it may be seen that a realistic approach is to provide a Coulomb cutoff at say 1-2 nuclear radii from the nuclear surface. Such a cutoff plays no role in the final result.

[9]For $l=0$, for example, one can estimate that the second term of Eq. (4) becomes significant only at those angles for which

$$\frac{d\sigma_1(\theta)}{d\Omega} \gtrsim \left[k_p^2 \frac{d\sigma_1(\theta)}{d\Omega}\frac{(\Delta\theta)^4}{4}\right]\frac{d\sigma_p(\theta)}{d\Omega}.$$

Here $\Delta\theta$ is the half-width of the forward stripping peak yielded by the first term, $d\sigma_1/d\Omega$ is the cross section yielded by the first term, and $d\sigma_p/d\Omega$ is the proton elastic-scattering cross section. Similar estimates are available for $l \neq 0$.

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ANGULAR MOMENTUM EFFECTS IN REACTIONS WITH HEAVY IONS*

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Rearrangement collisions between complex nuclei differ from direct nuclear reactions with simple nuclear projectiles, at least in three aspects: (1) Coulomb effects are stronger. (2) The dimensions of the projectile are not small in comparison with the dimensions of the target nucleus. (3) The transferred particle is usually not in a state of zero orbital angular momentum in the incident particle.

These effects may lead to unusual experimental cross sections. Recently, heavy-ion experiments involving both single-particle and cluster transfer at energies well above the Coulomb barrier actually exhibited several unusual and unexpected features.[1-3] In some cases the angular distribution is smooth, in other cases it is oscillatory.

Dodd and Greider suggested[4] that the smooth behavior is due to recoil effects; however, recent measurements by Bock et al.[3] exhibit a clear oscillatory behavior in many cases, in contrast with Dodd and Greider's predictions. The purpose of this note is to provide a simple explanation for this behavior and to relate the oscillatory angular distribution in a simple way to the angular momenta involved in the reactions.

Our approach is based on the smooth-cutoff diffraction model (SCDM) for direct nuclear reactions.[5] According to this model the tran-