MEASUREMENT OF THE QUANTUM-ELECTRODYNAMIC LEVEL SHIFT IN THE $n=3$ STATE OF $(He^4)^+$ †

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Precision measurements of the level shift in hydrogenic systems have provided a fundamental test of the theory of quantum electrodynamics. However, at the present writing experimental results have been reported for relatively few levels.[1-8] In this Letter we report the first observation of microwave resonances between $n=3$ states of He^+ and the results of an initial low-precision measurement of the level shift $S=E(3\,^2S_{1/2})-E(3\,^2P_{1/2})$. It is felt that a new test of the n and Z dependence of the theory will eventually be provided.

The pertinent energy levels with their approximate separations are shown in Fig. 1 which allows a general description of the experimental method. In the absence of radio-frequency radiation, all He^+ ions decaying spontaneously from the $3S$ state do so via the $2P$ levels in $\sim 1\times 10^{-8}$ seconds with the emission of a 1640Å quantum followed by a 303Å quantum. Application of microwave radiation of the proper frequency to induce the electric dipole transition $3\,^2S_{1/2}-3\,^2P_{1/2}$ alters the cascade process markedly. Since most of the ions in the $3\,^2P_{1/2}$ state decay directly to the ground state in $\sim 3\times 10^{-10}$ seconds,[9] a diminution in the amount of 1640Å decay radiation is expected when the microwave transition is induced.

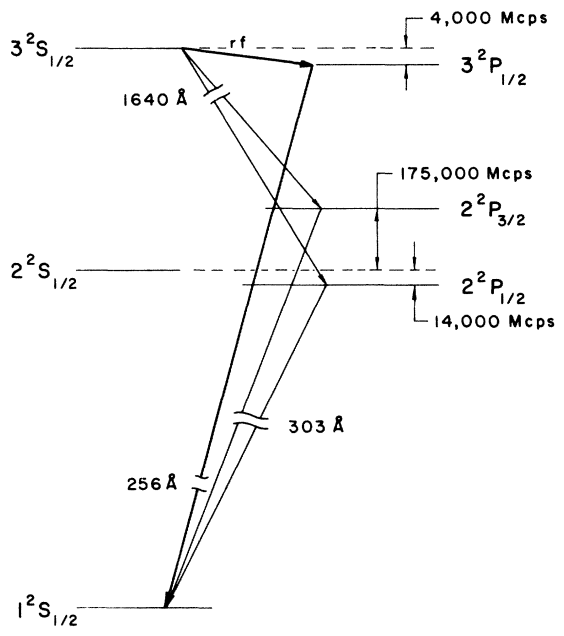

FIG. 1. The decay scheme of the $3\,^2S_{1/2}$ and $3\,^2P_{1/2}$ levels of He^+. Decay of $3\,^2P_{1/2}$ via $2\,^2S_{1/2}$ is not indicated. The fine-structure splittings are not given to scale, and $3\,^2P_{3/2}$, $3\,^2D_{3/2}$, and $3\,^2D_{5/2}$ are not shown.

A schematic diagram of the experimental apparatus is given in Fig. 2. An electron gun is operated in a helium atmosphere of about 10^{-2} Torr. Helium ions in the $3\,^2S_{1/2}$ state are produced by simultaneous ionization and excitation of the atoms during collisions with electrons. An estimate of the cross section for this process made using the sudden approximation[10] yields a value of 5×10^{-20} cm^2 for 200-V incident electrons. The electron beam passes between a pair of parallel rf plates located at one focus of an ellipsoidal light pipe. A photomultiplier sensitive to the 1640Å radiation is located at the second focus. The electron gun is situated between the poles of an electromagnet. The magnetic field permits the scanning of resonances between the Zeeman sublevels of $3\,^2S_{1/2}$ and $3\,^2P_{1/2}$ and also serves to collimate the electron beam. Radiation of wavelength shorter than 1415 Å is prevented from reaching the detector by a sapphire window. A straightforward lock-in detection scheme follows the photomultiplier. The synchronous signal is provided by 100% square-wave amplitude modulation of the microwave field. Recorder tracings of the dc output at fixed microwave frequency are obtained as the magnetic field is swept through resonance. Figure 3 shows a typical resonance curve which is of the expected Lorentzian shape and width.

The theory of the Zeeman effect for the $n=3$ state of the hydrogen atom has been given in detail by Lamb and Sanders.[11] Their results may be taken over directly with an appropriate adjustment of parameters. The Zeeman levels of importance in the present experiment are shown in Fig. 4, labeled in the notation of earlier workers. The levels were computed using the theoretical values $3\,^2P_{3/2}-3\,^2P_{1/2}=52\,020$ Mc/sec and $S=4183$ Mc/sec. The theoretical value of S was obtained from a systematic treatment of the hydrogenic level-shift prob-

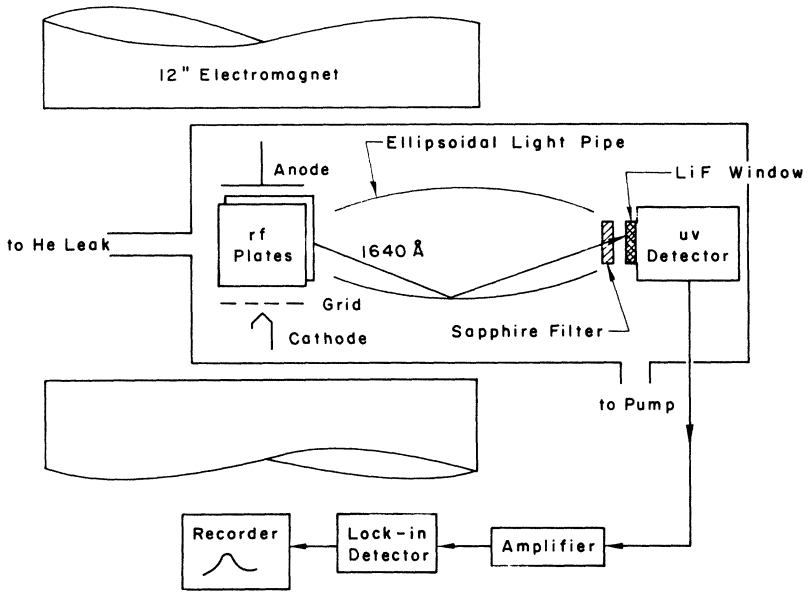

FIG. 2. Schematic diagram of the experimental apparatus employed in the present work.

lem given by Erickson.[12]

A preliminary measurement of S has been obtained from eight runs of the transition αf at 1465 G and six runs of βe at 6100 G. For the purpose of this exploratory work, the resonances have been treated as perfectly symmetrical and their centers located graphically.

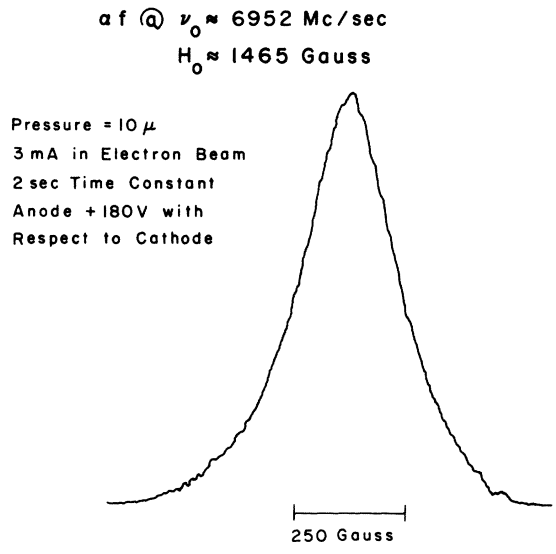

FIG. 3. A typical resonance curve indicating some of the experimental conditions. In obtaining experimental data, resonances were taken in pairs so as to negate errors associated with the direction of the magnetic field scan.

No attempt has been made to make the many corrections[1-8] to the raw data which are re-

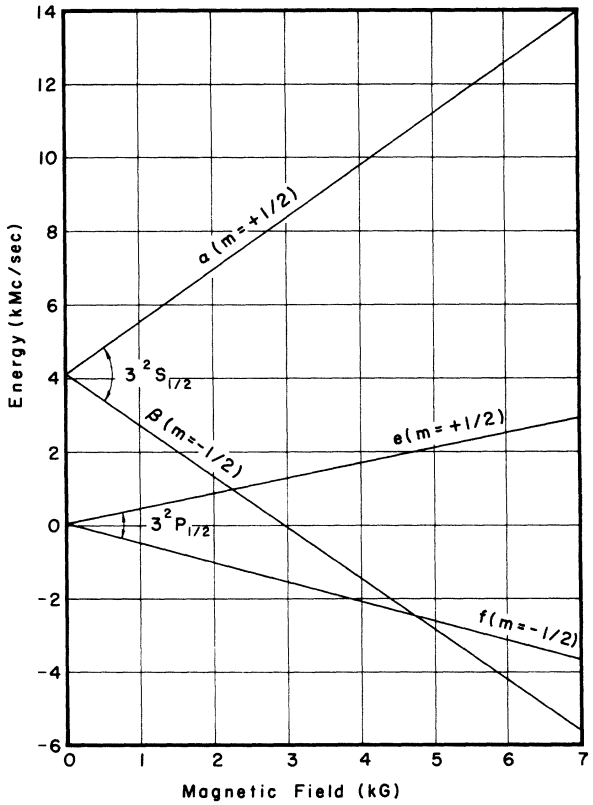

FIG. 4. Zeeman splitting of the $3\,^2S_{1/2}$ and $3\,^2P_{1/2}$ levels of He^+. The $3\,^2S_{1/2}$ state has been raised 4183 Mc/sec relative to the $3\,^2P_{1/2}$ state.

quired in a precision measurement. The results are for αf,

$$\$ = 4198 \pm 20 \text{ Mc/sec};$$

for βe,

$$\$ = 4167 \pm 20 \text{ Mc/sec}.$$

The quoted uncertainties are twice the statistical fluctuations in the data and are thought to allow for the neglected corrections. Clearly no large discrepancy, i.e., greater than 1%, with theory has been found. The quality of the data indicates that a much higher precision measurement should be possible. This will be attempted in the near future utilizing computer techniques to perform a detailed treatment of the resonance line shape, and it is believed that a sensitive test of quantum electrodynamics will be provided.

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ELECTRON-COUPLED PROTON DEUTERON COUPLING CONSTANT IN HD†

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The coupling of nuclear spins by second-order perturbation mechanisms was first proposed and discussed by Ramsey and Purcell[1] in explanation of experimental results obtained by Hahn and Maxwell.[2] Ramsey[3] showed that the dominant contribution to the scalar coupling constant, J, defined in terms of the coupling energy by

$$E = hJ\vec{I}_H \cdot \vec{I}_D + h\vec{I}_H \cdot \mathcal{J} \cdot \vec{I}_D$$

arises from the second-order perturbation theory energy of the Fermi contact Hamiltonian. We have calculated the first term in a perturbation-theory expansion for the Fermi contact contribution, J_F, to the indirect scalar coupling between the nuclear spins in hydrogen deuteride.

In the presence of the dominant spin interactions, the total Born-Oppenheimer Hamiltonian for the HD molecule can be written as

$$H = H_0 + \lambda H_{100} + \vec{I}_H \cdot \vec{H}_{010} + \vec{I}_D \cdot \vec{H}_{001}, \quad (1)$$

where

$$H_0 = -\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2) - \frac{e^2}{r_{1H}} - \frac{e^2}{r_{2D}}, \quad (2a)$$

$$\lambda H_{100} = -\frac{e^2}{r_{2H}} - \frac{e^2}{r_{1D}} + \frac{e^2}{r_{12}} + \frac{e^2}{R}, \quad (2b)$$

$$\vec{I}_H \cdot \vec{H}_{010} = (16\pi\beta\hbar\gamma_H/3) \times [\delta(\vec{r}_{1H})\vec{S}_1 + \delta(\vec{r}_{2H})\vec{S}_2] \cdot \vec{I}_H, \quad (2c)$$

$$\vec{I}_D \cdot \vec{H}_{001} = (16\pi\beta\hbar\gamma_D/3) \times [\delta(\vec{r}_{1D})\vec{S}_1 + \delta(\vec{r}_{2D})\vec{S}_2] \cdot \vec{I}_D, \quad (2d)$$

and where $r_{kN} \equiv r_k - R_N$, S_k and I_N are the electron and nuclear spins, respectively, and λ is a dummy parameter. The nonsymmetrical separation of the purely electronic Hamiltonian into H_0 and λH_{100} has been discussed elsewhere[4] where it has been shown that other properties of the hydrogen molecule to zeroth order in λ are in good agreement with experiment. The rapid convergence of such an electronic unsymmetrical perturbation theory—which only exists when the exact wave function is separable in space and spin coordinates—has recently been demonstrated by Bailey[5] for H_2^+.

Using straightforward coupled perturbation theory, J_F is given by the energies linear in