matched by either curve, to within the accuracy of the cosmic normalization; however, the steep slope of the data, particularly as shown in the spectrum of Metzger et al., strongly suggests that the blackbody radiation is present and has steepened the cosmic electron spectrum. The solid curve is then the relevant one.

We have seen that this steepening of the x-ray spectrum follows naturally from the assumptions that the universal blackbody radiation is present and that the fast electrons are intergalactic. Whether this steepening could occur for radiation originating in the galactic halo, under the very different conditions prevailing there, is uncertain. This question, together with further details of the research reported here, will be discussed in forthcoming papers.

P. Morrison provided valuable advice at the inception of this work and suggested at an early date the possible importance of cosmic blackbody radiation to the problem. I have enjoyed informative conversations with G. B. Field and R. J. Gould.

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SELF-FOCUSING OF OPTICAL BEAMS

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The possibility has been noted of the self-trapping of optical-frequency electromagnetic beams due to a nonlinear increase in index of refraction, and various physical mechanisms which could give rise to the nonlinearity have been discussed.[1-3] Trapped modes have been found.[3-5]

In this note we study a situation not examined previously: the situation where the self-focusing effect due to the nonlinear index increase is not compensated for by diffraction. In this case there will be a build-up in intensity of part of the beam as a function of distance in the direction of propagation. We will define a self-focusing length, and show that this definition is reasonable by numerically solving the nonlinear wave equation. This self-focusing length is the distance in which the intensity of the self-focused region tends to become anomalously large. Other optical nonlinearities are likely to limit the focusing process in intense beams and may stabilize the intense region into filaments.

The importance of the effect in stimulated Raman emission as well as in other nonlinear effects should be considerable.[3] In a self-focused region the Raman gain should be anomalously large. The anomalous Raman gain has been the subject of some controversy and a number of communications have appeared.[6] One of the important observed features is that the anomalous gain in liquids occurs only after the beam has traveled some distance through the liquid. It is proposed here that this distance is the self-focusing length described in the present note and that the calculation report-

ed here will help shed some light on the anomalous-gain controversy. Recent experimental results seem to confirm this.[7]

We take as our starting equation Eq. (3) of Ref. 3,

$$\nabla^2\vec{E}-\frac{\epsilon_0}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}-\frac{\epsilon_2}{c^2}\frac{\partial^2(E^2\vec{E})}{\partial t^2}=0, \tag{1}$$

where ϵ_0 and ϵ_2 are real, and $\epsilon_2E^2\ll 1$. We assume a linearly polarized wave of frequency ω and propagating along the z axis, so that

$$\vec{E}=\frac{\hat{e}}{2}(E'e^{i(kz-\omega t)}+\text{c.c.}), \tag{2}$$

where $k=\epsilon_0^{1/2}\omega/c$, the factor $\exp(ikz-i\omega t)$ represents the propagating part of the wave, and E' is the slowly varying part. Substituting (2) into (1) and neglecting the third-harmonic term, we obtain

$$2ik\frac{\partial E'}{\partial z}+\nabla^2E'+\frac{\epsilon_2'k^2}{\epsilon_0}|E'|^2E'=0, \tag{3}$$

where $\epsilon_2'=3\epsilon_2/4$. Note, $\delta n=n_2'E'^2=\epsilon_2'E'^2/2n_0$ is the dc change in index as defined by Maker and Terhune.[8] We drop the term in the second z derivative of E', assuming it to be small:

$$2ik\frac{\partial E'}{\partial z}+(\nabla_x^2+\nabla_y^2)E'+\frac{\epsilon_2'k^2}{\epsilon_0}|E'|^2E'=0. \tag{4}$$

This is a familiar approximation in optics and has been used by Vainshtein[9] in solving the resonator problem. It is valid when the distance characteristic of the change of E' in the z direction is much larger than a wavelength. The equation is of a parabolic type and can be solved with an open boundary. Transversality is assumed to have a negligible effect.

We note that (4) has a simple solution when there is no transverse variation of the beam:

$$E'=E_0'\exp(in_2'k|E_0|^2z/n_0); \tag{5}$$

in other words, the nonlinearity introduces a phase change proportional to the intensity. The phase velocity decreases with increasing intensity. Thus the equiphase surfaces are depressed where the beam is most intense. From Huygen's principle, therefore, the rays should move toward the region of highest intensity and the intensity of the center should increase. We use this solution to obtain a characteristic focusing distance. We assume that the transverse second derivative of E' in (4) depends on axial distance through the transverse variation of the intensity in the exponent of (5). This in turn gives rise to an appreciable change in field along the axis when

$$z=z_{\text{focus}}\equiv\tfrac{1}{2}a(n_0/n_2')^{1/2}/E_m', \tag{6}$$

where a is a characteristic transverse radius of curvature of the input intensity and E_m' is the peak field value. The focusing distance is defined assuming an equiphase input beam. It is not expected that the solution for the beam center will be strictly correct for distances within a few wavelengths of the focusing distance because of the neglect of the second derivative in (4).

Using the data compiled in Ref. 8, we estimate that $n_2'\approx(0.2\text{-}1.5)\times10^{-11}$ esu for CS_2. For a 1-MW beam of 2-mm diameter, we therefore have a focusing length of 40-100 cm in CS_2. It is likely that the n_2' at the ruby-laser frequency in many liquids is due to a near two-photon resonance with vibronic states.

Equating z_{focus} to the diffraction length we obtain an approximate threshold power for cylindrical beam trapping (equating a to the radius of beam),

$$P_{cr}=(1.22\lambda)^2c/512n_2'. \tag{7}$$

Since $n_2'=n_2/2$, this power is $\frac{1}{4}$ of the threshold power found in Ref. 3. The net focusing distance after correcting for diffraction is

$$z_{\text{net}}=\frac{a}{2}(n_0n_2')^{1/2}(E_m'-E_{cr})^{-1}, \tag{8}$$

where $E_{cr}=1.22\lambda/8a(n_2'n_0)^{1/2}$.

To verify these considerations, Eq. (4) is solved numerically for a cylindrically symmetrical beam. To facilitate this (4) is rewritten in the following dimensionless form:

$$i\frac{\partial E^*}{\partial z^*}+\frac{\partial^2E^*}{\partial r^{*2}}+\frac{1}{r^*}\frac{\partial E^*}{\partial r^*}+|E^*|^2E^*=0, \tag{9}$$

where $r^*=r/a$, $z^*=z/2ka^2$, and $E^*=(\epsilon_2/\epsilon_0)^{1/2}kaE'$. The starred notation for the dimensionless quantities is similar to that of Ref. 3; it does not indicate complex conjugation. In this dimensionless form $z_{\text{focus}}^*=(2\sqrt{2}E_m^*)^{-1}$ and $z_{\text{net}}^*=[(2\sqrt{2}E_m^*)(1-1.35/E_m^*)]^{-1}$.

We have solved (9) by a finite-difference or mesh technique discussed by Harmuth[10] and modified to include the nonlinearity. An equiphase Gaussian intensity profile was assumed for the input beam. Examples of the numerical results obtained on an SDS 930 computer

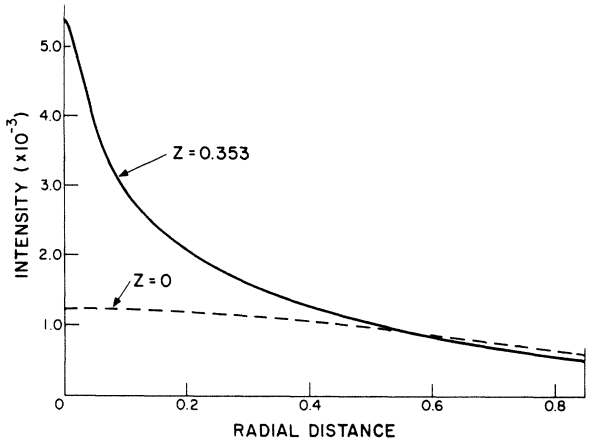

FIG. 1. Calculated intensity of beam I^* versus radial distance r^* for $z^* = 0.353/E_m^*$. For comparison the Gaussian initial profile ($z^* = 0$) is shown dashed.

are given in Figs. 1 and 2. E_m^* was chosen large enough to give a fairly small focusing distance. It was found that the beam power remained independent of z^* to at least the fifth decimal place. The width at half-maximum appears to be decreasing roughly as the inverse of the central intensity.

We note that the curve given in Fig. 2 is

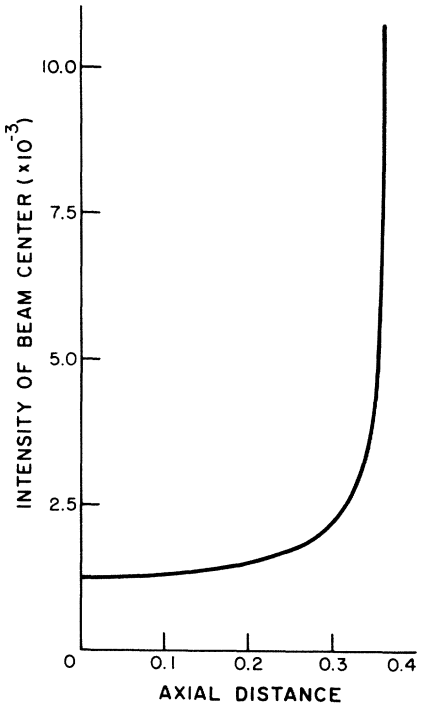

FIG. 2. Calculated intensity of beam center I_m^* vs z^* (in units of $1/E_m^*$).

given to a high accuracy by

$$I_m^*(z^*)/I_m^*(0) = z_0^*(z_0^{*2} - z^{*2})^{-1/2}, \tag{10}$$

where $I_m = |E_m|^2$. We find that $z_0^* = 0.366/E_m^*(0)$, while from (6) and (8) $z_{\text{focus}}^* = 0.353/E_m^*(0)$ and $z_{\text{net}} = 0.368/E_m^*(0)$, respectively.

Talanov[11] has recently obtained a result equivalent to (4), of the form

$$k\frac{\partial E_0^2}{\partial z} = -\vec{\nabla}_\perp \cdot (E_0^2 \vec{\nabla}_\perp \varphi) \tag{11a}$$

and

$$2k\frac{\partial \varphi}{\partial z} + (\vec{\nabla}_\perp \varphi)^2 = \frac{\epsilon_2'}{\epsilon_0} k^2 E_0^2 + (\nabla_\perp^2 E_0)/E_0, \tag{11b}$$

where $E' = E_0 \exp(i\varphi)$, and $\perp$ indicates transverse derivatives. For present purposes we use these equations to derive equations for the growth of the beam center ($r = 0$). Neglecting the phase change due to diffraction (i.e., $z_{\text{net}} \approx z_{\text{focus}}$) the equations for the beam center can be written as

$$k\frac{\partial I_m}{\partial z} = -I_m \nabla_\perp^2 \varphi, \text{ and } \varphi = \frac{\epsilon_2' k}{2\epsilon_0}\int_0^z I_m dz. \tag{12}$$

If we make the approximations $\varphi \approx \epsilon_2' k^2 I_m z/2\epsilon_0$ and $\nabla_\perp^2 I_m \approx -4I_m^2/a^2 I_m(0)$ (i.e., the radius of curvature squared is assumed inversely proportional to the intensity, a reasonable assumption based on power conservation), then

$$\frac{\partial I_m}{\partial z} = \frac{I_m^3}{I_m^2(0)} \frac{z}{z_{\text{focus}}^2}, \tag{13}$$

which in dimensionless form has the solution (10) when $z_0^* = z_{\text{focus}}$.

The above considerations are for a smooth beam (i.e., by this we mean a beam whose transverse radius of curvature near the maximum is of the order of transverse radius). Beams whose curvature is greater than the curvature for a smooth beam will exhibit focusing in distances shorter than one would expect from a smooth beam of the same radius. Multimode effects discussed by Bloembergen and Shen[6] should also reduce the focusing distance.

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BEAM DETERIORATION AND STIMULATED RAMAN EFFECT*

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The most important fundamental discrepancy between theory and experiments in the stimulated Raman effect is that the observed Raman gain is one to two orders of magnitude larger than the theoretical value.[1] The latter is given by[2]

$$g = (2\pi\omega_S^2/k_{Sz}c^2)|\chi_S''||E_l|^2,$$

where ω_S is the Stokes frequency, k_S the Stokes wave vector, E_l the laser field amplitude, and χ_S'' the resonant Raman susceptibility whose magnitude can be obtained from the spontaneous Raman-scattering data. It was suggested that the observed anomalous gain might be the result of the multimode structure (or hot filaments) of the laser (pumping) beam,[3] but McClung, Wagner, and Weiner, using a nearly single-mode laser beam in the experiments, still found the presence of such an anomalous gain.[1] This, however, does not eliminate the possibility of deterioration of the laser beam into multimodes as the beam interacts with the medium. In this paper, experimental evidence is presented to suggest that scattering mechanisms in a medium can produce inhomogeneities or filamentary structure in an initially homogeneous beam. We believe that these hot filaments are responsible for the many anomalous effects previously observed.

A laser beam, Q switched by cryptocyanine solution and limited in cross section by an aperture in the cavity, was used to generate Stokes radiation in a 20-cm toluene cell (cell A). The laser intensity was varied by a Polaroid prism outside the laser cavity. Another cell (cell B) of variable length, filled with water, benzene, acetophenone, or nitrobenzene, was inserted between the laser and the toluene cell. The threshold of the stimulated Raman scattering was then measured as a function of the length of cell B. The results are shown in Fig. 1. The curves clearly show that the medium in cell B can distort the laser beam in such a way as to help significantly the Raman generation in toluene. Here, the Raman threshold of toluene first increases and then decreases sharply as the length of cell B is increased.

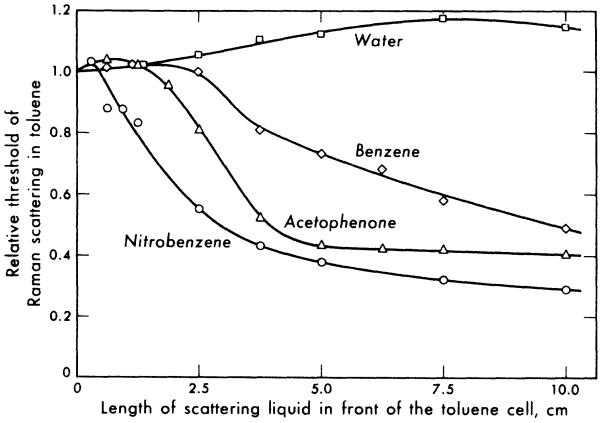

FIG. 1. Raman threshold in toluene versus the cell length of a scattering cell in front of the toluene cell. The scattering cell was filled with water, benzene, acetophenone, or nitrobenzene.