These differences may mean the following: (1) Few of the transmitted particles radiate in our wavelength range. This, probable for charge stages 3+ or greater, is implausible for neutrals. (2) The lifetimes of many excited states are too short for adequate deflection prior to decay. (3) Our foils do not produce excitation equilibrium.

Kay[6] used a fast beam source to infer charge stages responsible for nitrogen spectral lines. He assumed knowledge of the charge stage for one observed spectral line, but an incorrect choice would vitiate the other assignments. He assumed that the relative population of excited states for any degree of ionization is independent of particle velocity. This lacks experimental and theoretical justification. Finally, he assumed that an observed line can be ascribed to but one charge stage. The separations demonstrated herein show that this assumption is untenable.

Unlike earlier work, our experiment gives a direct and definitive measurement of the ionization charge stages for observed spectral lines. One limitation of this method is that states with lifetimes shorter than $\sim 10^{-9}$ sec may decay too soon to allow charge identification. This could cause one to miss some charge stages in lines with mixed charge components. Another limitation is that the low light intensity of the source prescribes the use of fast spectrographs with a consequent sacrifice in resolution. At low resolution, separation of lines within a given charge stage may often be inadequate to permit unique assignments of spectral lines. However, in some instances for which charge separation is very difficult to obtain by conventional methods, e.g., for Ne III–Ne VII,[7] the present method should complement and simplify the higher resolution observations. No basic experimental difficulties (except possibly that of source intensity) should arise with the present method as the incident beam energy is increased to excite higher charge stages.

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STRUCTURE AND OSCILLATOR STRENGTHS OF AUTOIONIZING LEVELS IN HELIUM*

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Recent experiments[1-3] have stimulated theoretical study of the autoionizing levels in helium. By now the positions of the lowest lying S and P levels have been well established through the work of Burke and McVicar[4] (hereafter called I), who used a close-coupling approximation, and O'Malley and Geltman[5] (hereafter called II), who performed a variational calculation. However, neither of these techniques gives detailed information on the structure of these levels. Of particular interest is the suggestion by Cooper, Fano, and Prats[6] that these levels should be labeled $(n,n')\pm$ denoting the state $(nsn'p \pm n'snp)$, rather than by a single configuration. This suggestion qualitatively explains the experimental data on the ^1P levels by predicting an alternating series of strong and weak lines.

We have calculated eigenvalues and eigenvectors for several low-lying ^1P and ^3P autoionizing levels in helium. The structure of these levels is thus explicitly obtained. Quantitative verification is given to the plus-minus classification scheme for the ^1P states, and the ^3P are classified similarly. In addition, oscilla-

Table I. Energies and oscillator strengths of the P states.

Level designation	ENERGIES (volts above ground state) This paper	Burke and McVicar[a]	O'Malley and Geltman[b]	Madden and Codling[c,d]	OSCILLATOR STRENGTHS This paper	Burke and McVicar[a]	Madden and Codling[c,d]
$^1P(2,2)+$	60.35	60.28	60.19	60.14	0.495×10^{-2}	0.506×10^{-2}	0.53×10^{-2}
$^1P(2,3)-$	62.79	62.77	62.78	62.76	0.820×10^{-4}	0.206×10^{-4}	
$^1P(2,3)+$	63.71	63.69	63.72	63.66	0.809×10^{-3}	0.773×10^{-3}	0.4×10^{-3}
$^1P(2\text{-}4)-$	64.15	64.13	64.15	64.14	0.397×10^{-4}	0.86×10^{-5}	
$^1P(3d)$	64.17	64.17	64.19		0.47×10^{-7}	0.26×10^{-7}	
$^1P(2,4)+$	64.54	64.48	64.62		0.557×10^{-3}	0.311×10^{-3}	
$^3P(2,2)+$	58.41	58.35	58.30				
$^3P(2,3)+$	63.18	63.14	63.14				
$^3P(2,3)-$	63.29	63.28	63.93				
$^3P(3d)$	64.12	64.12	64.32				
$^3P(2,4)+$	64.28	64.26	64.49				
$^3P(2,4)-$	64.34	64.33					

[a]Reference 4. [b]Reference 5. [c]Reference 1. [d]Denotes experimental values.

tor strengths for the transitions from the ground state to the ^1P levels have been computed using a six-parameter Hylleraas function for the lower state.[7]

The method used was configuration interaction with a basis set of hydrogenic orbitals having $Z=2$. 10 configurations were employed ($2s2p$; $2snp$; $2pns$; $2pnd$; $n=3,4,5$). With such a basis, which does not include the $1s$ orbital, we may use the full Hamiltonian as the energy operator and the results will be upper bounds on the eigenvalues of H_{QQ}, an operator defined in II. It is shown there that the eigenenergies for H_{QQ} lie very close to the observed positions of the autoionizing levels, the differences in position being due to interaction with the background continuum.

The results are presented in Tables I and II. The energies listed are the approximate eigenvalues of H_{QQ}. Thus, the shift due to interaction with the continuum computed in I, which is very small, has been subtracted off the results of I and the results of Madden and Codling[1] (hereafter called III). It is seen that the energies computed here differ by less than 0.1% from the more elaborate calculation in I, and that the values for the higher lying states lie lower than those computed variationally in II.

We defer discussion of the oscillator strengths until the structure of the levels, given in Table II, has been explained. In Table II we list the amplitudes of the configurations making up the state vector. As examples, we have chosen the second and third lowest lying levels designated $(2,3)-$, $(2,3)+$, respectively, for the ^1P states, and $(2,3)+$, $(2,3)-$, respectively, for the ^3P states. On the right-hand side of the

Table II. Eigenvectors of selected P states.

Configuration	$^1P(2,3)-$ (62.79)	$^1P(2,3)+$ (63.71)	$^3P(2,3)+$ (63.18)	$^3P(2,3)-$ (63.29)	Configuration	$^1P(2,3)-$ (62.79)	$^1P(2,3)+$ (63.71)	$^3P(2,3)+$ (63.18)	$^3P(2,3)-$ (63.29)
$2s\,2p$	0.006	0.183	−0.163	−0.028					
$2s\,3p$	−0.647	0.328	−0.519	−0.482	$(2,3)+$	−0.020	0.523	−0.754	0.075
$3s\,2p$	0.619	0.412	−0.548	0.588	$(2,3)-$	−0.896	−0.060	0.020	−0.756
$2s\,4p$	0.264	−0.388	0.422	0.252	$(2,4)+$	0.017	−0.695	0.593	−0.061
$4s\,2p$	−0.240	−0.594	0.417	−0.338	$(2,4)-$	0.357	0.146	0.004	0.417
$2s\,5p$	0.048	0.033	0.049	0.015	$(2,5)+$	0.005	0.064	0.067	−0.006
$5s\,2p$	−0.041	0.057	0.047	−0.023	$(2,5)-$	0.063	−0.018	0.002	0.027
$2p\,3d$	−0.220	0.147	−0.124	−0.395					
$2p\,4d$	0.130	−0.397	0.178	0.295					
$2p\,5d$	0.026	0.030	0.021	0.027					

table the plus-minus amplitudes are given, e.g.,

$$A_{(2,3)\pm} = \frac{A_{2s3p} \pm A_{3s2p}}{\sqrt{2}},$$

where $A_{nsn'p}$ is the amplitude of the configuration $nsn'p$. The energies are given in parentheses below the level designations. By studying the table the plus-minus character of the levels becomes apparent, the best example being $^1P(2,3)-$. For the other examples, the eigenstate has sizable contributions from several shells due to the lack of screening in the basis set. The state labeled $^1P(3d)$ is composed primarily of d configurations and shows no plus-minus structure. Its oscillator strength is so weak, however, that it has not been observed experimentally.

Note that $^3P(2,3)+$ lies below $^3P(2,3)-$ in contrast to the ^1P case. This can be made plausible by extending slightly the development of reference 6. Using the notation of that paper the ^3P function is written

$$\psi(\vec{r}_1, \vec{r}_2) = F(r_1, r_2\theta_{12})Y_{1m}(\theta_1, \varphi_1) - F(r_2, r_1\theta_{12})Y_{1m}(\theta_2, \varphi_2), \quad (1)$$

where

$$F(r_1, r_2, \theta_{12}) = \sum_{l=0}^{\infty} F_l(r_1, r_2)P_l(\cos\theta_{12}). \quad (2)$$

It was found in reference 6 that F_0 is almost symmetric in r_1, r_2 near $r_1 = r_2$ for a plus state, while it is almost antisymmetric in the same region for a minus state. Thus, setting the angular arguments equal in Eq. (1), and subtracting $F_0(r_2, r_1)$ from $F_0(r_1, r_2)$ (instead of adding), we see that the result is a function which is practically zero in the region around $r_1 = r_2$ for a plus state and nonzero in this region for a minus state. Hence the plus state, having a smaller amount of repulsive potential energy, should lie deeper.

The oscillator strengths given in Table I are not directly susceptible to measurement, but Fano[8] has given a formula which makes comparison possible. The formula is

$$f = (df/d\epsilon)_0 \frac{\pi}{2}(q^2-1)\Gamma, \quad (3)$$

where $(df/d\epsilon)_0$ is the continuum differential oscillator strength at the position of the autoionizing level, in the absence of that level. The symbol q is a line-shape parameter, and Γ is the level width. Now $(df/d\epsilon)_0$ is not an experimental quantity either, but must be obtained by extrapolation or by calculation. I lists $(df/d\epsilon)_0$, q, and Γ for the ^1P levels, and thus values for f may be computed. These quantities were calculated with dipole-length matrix elements using a 20-parameter ground-state function. III gives measured q and Γ values for the two lowest lying plus levels, although there is a large uncertainty in their values for the higher level. These numbers have been combined with $(df/d\epsilon)_0$ given by I to yield "experimental" oscillator strengths.

Note that the plus-level oscillator strengths are about an order of magnitude larger than their minus-level counterparts, as predicted by reference 6. The agreement between I and the present work is satisfactory, except for the minus levels. We expect that, due to cancellations, these numbers will be less accurate than for the plus levels, but the discrepancy seems too large in view of the fact that the f values for the d state, which exhibits almost perfect cancellation, are only a factor of two apart. This matter needs more investigation. Our results agree (within experimental error) with the data given in III for the $(2,2)+$ level, but not so for the $(2,3)+$ level. We believe that the discrepancy is due to uncertainty in the experimental quantities.

At present we are extending these calculations to include the interaction with the continuum explicitly.

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