Several theories[8] advanced to explain the apparent CP-invariance violation in K_2^0 decay[1] demand that no time-reversal noninvariance effects appear in $K_{\mu 3}^+$ decay. Cabibbo[9] has advanced a theory in which $K_{\mu 3}^+$ could show an effect, with $\mathrm{Re}\xi = 0$ and $\mathrm{Im}\xi \neq 0$, but no prediction is made for the expected value of $\mathrm{Im}\xi$.

The authors are indebted to Alex Maksymowicz, Adam Bincer, and Robert Sachs for useful discussions of the theoretical aspects of the problem.

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*Work supported by the U. S. Atomic Energy Commission.

†Accepted without review under policy announced in Editorial of 20 July 1964 [Phys. Rev. Letters 13, 79 (1964)].

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EXISTENCE OF LIGHT DOUBLE HYPERNUCLEI*

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(Received 29 April 1965)

With the discovery by Danysz et al.[1] of a double hypernuclear event which is best interpreted as $_{\Lambda\Lambda}\mathrm{Be}^{10}$, the question naturally arises whether there exist other lighter double hypernuclei which are particle-stable. In this note, we report the results of such an investigation using a variational method on the double hypernuclear systems $_{\Lambda\Lambda}Z^A$ with $A = 3$ to 6.

In our calculation, the nucleon-nucleon potential is chosen to fit the two-body low-energy scattering data and the binding energies and rms radii of the triton and the alpha particle.[2] It has the form

$$V_{NN}(r) = \frac{1+P_{NN}^{\sigma}}{2} V_t(r_{NN}) + \frac{1-P_{NN}^{\sigma}}{2} V_s(r_{NN}) + V_c(r_{NN})\epsilon_{NN}, \quad (1)$$

where P_{NN}^{σ} denotes the spin-exchange operator and the last term represents the Coulomb interaction, with ϵ_{NN} equal to 1 if both nucleons are protons and 0 otherwise. The quantities $V_t(r)$ and $V_s(r)$ are the triplet and singlet potentials in the even states and are chosen to be of the following type[3]:

$$\begin{aligned} V_t(r) &= \infty, & r &< r_c, \\ &= -v_{0t}\exp[-\kappa_t(r-r_c)], & r &> r_c; \\ V_s(r) &= \infty, & r &< r_c, \\ &= -v_{0s}\exp[-\kappa_s(r-r_c)], & r &> r_c; \end{aligned} \quad (2)$$

with $r_c = 0.4$ F, $v_{0t} = 475.044$ MeV, $v_{0s} = 235.414$ MeV, $\kappa_t = 2.5214$ F^{-1}, and $\kappa_s = 2.0344$ F^{-1}. The potential in the odd states does not need to be specified, since, in this investigation, the variational function is taken to be symmetric with respect to the space exchange of all the nucleons.

The Λ-nucleon potential is taken from our recent analysis of the S-shell hypernuclei.[4] It is

$$U_{\Lambda N}(r)=\tfrac{1}{2}(1+P_{\Lambda N}{}^{\sigma})U_t(r_{\Lambda N})+\tfrac{1}{2}(1-P_{\Lambda N}{}^{\sigma})U_s(r_{\Lambda N}), \qquad (3)$$

with

$$\begin{aligned} U_t(r)&=\infty, & r<r_c,\\ &=-u_{0t}\exp[-\lambda(r-r_c)], & r>r_c;\\ U_s(r)&=\infty, & r<r_c\\ &=-u_{0s}\exp[-\lambda(r-r_c)], & r>r_c. \end{aligned} \qquad (4)$$

The value of λ is chosen as 5.059 F^{-1}, corresponding to an intrinsic range of 1.5 F. The depths u_{0t} and u_{0s} are determined from the binding energies of the S-shell hypernuclei; their values are 954.1 and 1221.1 MeV, respectively.

For the Λ-Λ potential, we use

$$\begin{aligned} W_{\Lambda\Lambda}(r)&=\infty, & r<r_c,\\ &=-w_0\exp[-\mu(r-r_c)], & r>r_c, \end{aligned} \qquad (5)$$

where w_0 is the depth in the $^1S_0$ state. From our analysis of the double hypernucleus $_{\Lambda\Lambda}Be^{10}$,[5] we have found that the value of w_0 necessary to yield the experimental separation energy $B_{\Lambda\Lambda}$ of the two Λ particles is 944 ± 40 MeV when μ is chosen as 5.059 F^{-1}, corresponding to the mechanism of two-pion exchange.

From the depths of the Λ-nucleon and Λ-Λ potentials, it can be concluded that $_{\Lambda\Lambda}H^3$ or $_{\Lambda\Lambda}n^3$ has no particle-stable bound state. This is so since, in these double hypernuclei, the average Λ-nucleon interaction is the same as that in the $I=1$ configuration of $_{\Lambda}H^3$, but the Λ-Λ well-depth parameter (0.732 ± 0.03) is much smaller than the nucleon-nucleon well-depth parameter (0.949) in the $^1S_0$ state. Hence, the nonexistence of an $I=1$ bound state[6] for $_{\Lambda}H^3$ implies also the nonexistence of a bound state for $_{\Lambda\Lambda}H^3$ or $_{\Lambda\Lambda}n^3$.

On the other hand, the fact that the Λ-Λ potential is attractive implies immediately that $_{\Lambda\Lambda}He^6$ is bound.[5,7] With $w_0 = 944 \pm 40$ MeV, a calculation using a three-body model of $\alpha + 2\Lambda$ for $_{\Lambda\Lambda}He^6$ yields the result that the value of $B_{\Lambda\Lambda}$ is 9.33 ± 0.50 MeV.[5] In this investigation, we shall refine the calculation further by treating $_{\Lambda\Lambda}He^6$ as a six-body system. In this way, the effect of distortion of the alpha-particle core can be correctly taken into account.

The variational wave function used for the double hypernuclear systems with $A=4$ to 6 is of the form

$$\Psi=F(r_{12})\left[\prod_{i=1}^{2}\prod_{k=3}^{A}G(r_{ik})\right]\left[\prod_{\substack{i,k=3\\ i<k}}^{A}H(r_{ik})\right]\chi, \qquad (6)$$

where 1, 2 denote the two Λ particles coupled into a $^1S_0$ state and χ represents the appropriate spin function. For the function $F(r)$, we use the following form:

$$\begin{aligned} F(r)&\\ &=u_f(r)/r, & r<d_f,\\ &=A_f r^{n_f}[\exp(-\alpha_f r)+B_f\exp(-\beta_f r)], & r>d_f, \end{aligned} \qquad (7)$$

where $u_f(r)$ is a solution of the equation

$$-\frac{\hbar^2}{2\mu_f}\frac{d^2}{dr^2}u_f(r)+[W_{\Lambda\Lambda}(r)-e_f]u_f(r)=0, \qquad (8)$$

with μ_f being the reduced mass of the two Λ particles. The constants A_f and B_f in Eq. (7) are adjusted such that the function $F(r)$ and its first derivative are continuous at the separation distance d_f. There are a total of five variational parameters in this function, namely, α_f, β_f, d_f, e_f, and n_f. The functions $G(r)$ and $H(r)$ are defined in an analogous manner, except that the reduced mass μ_f and the potential function $W_{\Lambda\Lambda}(r)$ in Eq. (8) are replaced by quantities appropriate to the Λ-nucleon and nucleon-nucleon branches. In the function $G(r)$, the potential function is $\frac{3}{4}U_t(r)+\frac{1}{4}U_s(r)$ for all double hypernuclear systems considered, while in the function $H(r)$, the potential function is $V_t(r)$ for the $I=0$ state of $_{\Lambda\Lambda}H^4$, $V_s(r)$ for the $I=1$ state of $_{\Lambda\Lambda}H^4$, and $\frac{1}{2}[V_t(r)+V_s(r)]$ for the cases of $_{\Lambda\Lambda}H^5$ and $_{\Lambda\Lambda}He^6$. The variational parameters in these functions are α_g, β_g, d_g, e_g, n_g, α_h, β_h, d_h, e_h, and n_h.

From our past experience with this type of trial function, we know that it is not necessary to vary all of these parameters.[8] Thus, in this calculation, we have set $d_f = d_g = 1.0$ F and $d_h = 1.2$ F for all cases considered, and $n_f = n_g = -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}$ and $n_h = -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}$ for the cases with $A = 4, 5, 6$, respectively.

For the evaluation of the various expectation values, we have used a Monte-Carlo method. As this method has already been discussed in detail previously,[9] we shall not go into it here.

The results of this calculation are given in Table I, where E_A denotes the energy of the double hypernucleus $_{\Lambda\Lambda}Z^A$ and $\langle r_{NN}^2\rangle^{1/2}$ denotes the rms distance of separation between the nucleons. The quantity $B_{\Lambda\Lambda}$ is the separation energy of the two Λ particles; it is given by

$$B_{\Lambda\Lambda} = -(E_A - E_c - E_{\Lambda\Lambda}), \tag{9}$$

where E_c is the energy of the nuclear core and $E_{\Lambda\Lambda}$ is the energy of the Λ-Λ system in its lowest state. With the nucleon-nucleon potential used here, E_c is equal to -2.25, -7.78 ± 0.05, and -29.75 ± 0.18 MeV when the core nucleus is a deuteron, triton, and alpha particle, respectively.[2,4] As for $E_{\Lambda\Lambda}$, it is equal to zero when w_0 is less than 1290 MeV ($s_{\Lambda\Lambda} = 1$); for w_0 equal to 1300, 1330, and 1500 MeV, $E_{\Lambda\Lambda}$ is equal to -0.01, -0.11, and -3.37 MeV, respectively.

From the results given in Table I, we draw the following conclusions.

(A) $_{\Lambda\Lambda}H^4$.—(1) $I = 0$ configuration. If this double hypernucleus is to be stable against decaying into $_{\Lambda}H^3 + \Lambda$, $B_{\Lambda\Lambda}$ has to be greater than or equal to 0.31 ± 0.15 MeV which is the binding energy B_Λ of a Λ particle in the hypertriton.[10] From the dependence of $B_{\Lambda\Lambda}$ on w_0 shown in Table I, one sees that unless w_0 is greater than about 1200 MeV, $B_{\Lambda\Lambda}$ will be smaller than this value. Since the value of w_0 as determined from the analysis of $_{\Lambda\Lambda}Be^{10}$ is only 944 ± 40 MeV, we can safely conclude that it is very unlikely for $_{\Lambda\Lambda}H^4$ to have a particle-stable $I = 0$ state.

(2) $I = 1$ configuration. We note that $B_{\Lambda\Lambda}$ is only 0.47 MeV for w_0 equal to 1500 MeV, which is even smaller than the value of $B_{\Lambda\Lambda}$ in the $I = 0$ case with w_0 equal to 1300 MeV. This can be explained by observing that the large separation between the nucleons is unfavorable for binding by a short-range Λ-nucleon interaction. Since, from the above discussion, we have

Table I. Binding energies $B_{\Lambda\Lambda}$ for the double hypernuclei with $A = 4$-6.[a]

Double hypernucleus	I	w_0 (MeV)	α_f (F^{-1})	β_f (F^{-1})	e_f (MeV)	α_g (F^{-1})	β_g (F^{-1})	e_g (MeV)	α_h (F^{-1})	β_h (F^{-1})	e_h (MeV)	E_A (MeV)	$B_{\Lambda\Lambda}$ (MeV)	$\langle r_{NN}^2\rangle^{1/2}$ (F)
$_{\Lambda\Lambda}H^4$	0	1270	0.26	6.0	−10.0	0.070	5.0	−11.0	0.31	6.0	−12.0	−2.90 ± 0.08	0.65 ± 0.08	3.06 ± 0.03
		1300	0.30	7.0	−12.0	0.075	6.0	−15.0	0.32	7.0	−14.0	−3.33 ± 0.09	1.07 ± 0.09	2.95 ± 0.03
		1330	0.32	8.0	−15.0	0.080	7.0	−18.0	0.33	7.0	−19.0	−3.80 ± 0.10	1.44 ± 0.10	2.85 ± 0.03
	1	1500	0.60	4.0	−2.0	0.074	6.0	−10.0	0.15	1.7	10.0	−3.84 ± 0.25	0.47 ± 0.25	4.22 ± 0.04
$_{\Lambda\Lambda}H^5$	$\frac{1}{2}$	944	0.12	3.0	−15.0	0.10	5.0	−5.0	0.235	3.5	−15.0	−10.81 ± 0.36	3.03 ± 0.36	2.49 ± 0.02
		1020	0.16	3.0	−25.0	0.105	5.5	−20.0	0.235	3.5	−15.0	−11.37 ± 0.33	3.59 ± 0.33	2.47 ± 0.02
$_{\Lambda\Lambda}He^6$	0	944	0.18	2.5	−5.0	0.13	5.0	−7.0	0.22	6.0	−25.0	−39.43 ± 0.39	9.68 ± 0.43	2.22 ± 0.03

[a]Number of estimates used in the Monte-Carlo calculation are 100 000 for $_{\Lambda\Lambda}H^4$ and $_{\Lambda\Lambda}He^6$, and 30 000 for $_{\Lambda\Lambda}H^5$.

come to the conclusion that $_{\Lambda\Lambda}H^4$ is unlikely to have a bound $I=0$ state, it follows quite clearly that it will also not have a bound $I=1$ state.

(B) $_{\Lambda\Lambda}H^5$.—As in the case of $_{\Lambda\Lambda}H^4$, the coupling of the two Λ particles into a $^1S_0$ state causes the depth of the spin-averaged Λ-nucleon potential in $_{\Lambda\Lambda}H^5$ (1020.8 MeV) to be much smaller than that in the hypernucleus $_\Lambda H^4$ (1087.6 MeV).[4] In fact, using the interpolation formula given in reference 4, B_Λ of the Λ particle in $_\Lambda H^4$ would be reduced to 0.67 MeV, if the depth were only 1020.8 MeV.

The value of $B_{\Lambda\Lambda}$ for $w_0 = 944 \pm 40$ MeV is 3.03 ± 0.46 MeV, which is greater than the value of B_Λ (2.18 MeV) in $_\Lambda H^4$.[4] This indicates that $_{\Lambda\Lambda}H^5$ is not only stable against decaying into $H^3 + 2\Lambda$, but also stable against decaying into $_\Lambda H^4 + \Lambda$.

(C) $_{\Lambda\Lambda}He^6$.—With $w_0 = 944 \pm 40$ BeV, $B_{\Lambda\Lambda}$ is equal to 9.68 ± 0.60 MeV, which is about the same (within the statistical uncertainty) as the value of 9.33 ± 0.50 MeV obtained using a three-body model of $\alpha + 2\Lambda$ for $_{\Lambda\Lambda}He^6$. This indicates that the effect of distortion of the alpha-particle core is not important in the determination of the value of $B_{\Lambda\Lambda}$, which is, of course, a consequence of the fact that the compressibility of the alpha particle is small.

Also, we note that the value of $\langle r_{NN}{}^2\rangle^{1/2}$ computed here (2.22 ± 0.03 F) is not different from that computed in $_\Lambda He^5$ (2.20 ± 0.03 F),[4] when the statistical uncertainty is taken into consideration, which is another evidence that the effect of distortion of the alpha-particle core is quite insignificant.

The quantity δ defined as

$$\delta = B_{\Lambda\Lambda} - 2\bar{B}_\Lambda, \tag{10}$$

where $\bar{B}_\Lambda$ is the binding energy of the Λ particle in the hypernucleus $_\Lambda Z^{A-1}$ using the spin-averaged Λ-nucleon potential appropriate to the double hypernucleus $_{\Lambda\Lambda}Z^A$, is equal to 1.69 ± 0.46, 3.48 ± 0.60, and 4.5 ± 0.5 MeV for the double hypernuclei $_{\Lambda\Lambda}H^5$, $_{\Lambda\Lambda}He^6$, and $_{\Lambda\Lambda}Be^{10}$, respectively. As for the $I=0$ configuration of $_{\Lambda\Lambda}H^4$, δ is rather small, being only about 1.3 MeV even for w_0 as large as 1300 MeV. This shows that δ depends sensitively on A for the light double hypernuclei considered here, but begins to show a tendency toward saturation at A equal to 10. This latter observation leads us to believe that, for heavy double hypernuclei, the value of $B_{\Lambda\Lambda}$ is around 60 MeV, since recent experimental results indicate that the binding energy of a Λ particle in a heavy hypernucleus is about 25 MeV.[11]

In conclusion, we believe that the double hypernucleus $_{\Lambda\Lambda}H^4$ is not likely to be bound, and the lightest double hypernucleus which has a particle-stable bound state is $_{\Lambda\Lambda}H^5$ or $_{\Lambda\Lambda}He^5$.[12]

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*Work supported by the U. S. Atomic Energy Commission.

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PRODUCTION OF ANTIPROTONS VIA NUCLEAR MOTION*

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(Received 4 May 1965)

Since the very first observation of meson production at an accelerator,[1] it has been recognized that "Fermi motion" provides large assistance in making extra energy available in the c.m. system. Present experimental techniques permit the observation of reactions having exceedingly small partial cross sections; these techniques could be applied to the question of just how much energy this complex nuclear effect could provide. For example, a 30-BeV proton collision in which the entire laboratory recoiled coherently would yield 30 BeV in the c.m. system, equivalent to the collision of a 450-BeV proton with a nucleon at rest. If a copper nucleus could take up the recoil, the available energy would be ~20 BeV (200-BeV accelerator). One can then ask whether the probability for similar events is large enough to be detected, in view of the great sensitivity of present techniques. Since no conservation laws are violated, it is safe to say that the question must be answered experimentally. The existence of such (very improbable) correlations is a problem which may perhaps be of interest to nuclear-structure theorists.

Although an extensive literature exists on nuclear internal motions, there is no "well-known" prescription for describing these results in a useful way from the viewpoint of a high-energy collision. We have, therefore, made a series of measurements on the reaction

$$p + \mathrm{Cu} \rightarrow \bar{p} + \text{residue}, \tag{1}$$

using protons of energies near and below threshold for antiproton production via the free-particle reaction

$$p + \binom{n}{p} \rightarrow p + p + \binom{n}{p} + \bar{p}. \tag{2}$$

For collisions below the threshold of the above reaction, some complex nuclear effect is necessary for $\bar{p}$ production. The observed $\bar{p}$ yield will then be a measure of the relative probability of such effects. More precisely, the observation of $\bar{p}$ at a bombarding energy T represents the existence of an interaction in which more than $4m_p$ is available in the c.m. system. Although the detailed mechanism for providing this energy is not known (it could be provided by intermediate pions, or by the coherent recoil of clusters), the main objective of this work is to find the distribution function

$$P_T(W_0)dW_0, \tag{3}$$

i.e., the probability of observing a c.m. total energy W_0 when the bombarding energy in the laboratory system is T. We assume that dynamics enters simply via a threshold function $(W - 4m_p)^n$, although a careful investigation of $\bar{p}$ production by free protons above threshold would