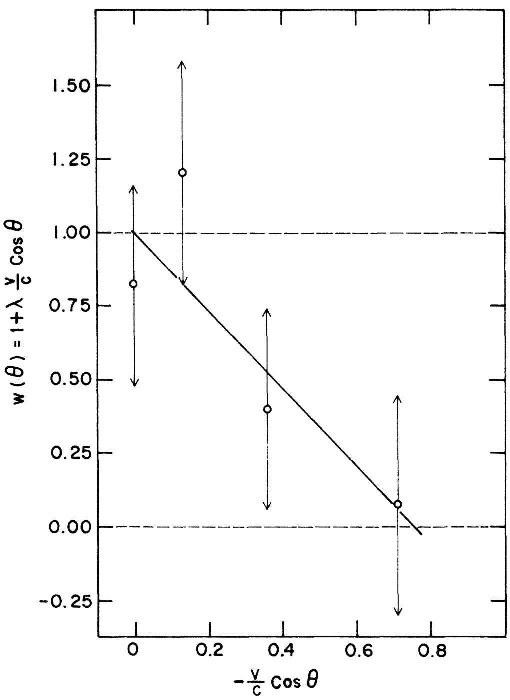

FIG. 2. Beta-neutrino angular correlation in Sb^{122}.

ations of the semi-empirical energy surface, and the shell model of the nucleus. On the other hand, Pursey[9] estimates Λ_V using the Hartree-type single particle model, which gives $\Lambda_V \simeq 2.1$ for our case. On the basis of the conserved current theory, Fujita[10] has derived an expression for Λ_V, giving 2.5 for the Sb^{122} decay. Owing to the smallness of the vector matrix elements and the consequent wide margin of uncertainty in Λ_V, a comparison with the various estimates is ineffective. On the other hand, Λ_A, which is known without much ambiguity, appears to be somewhat larger than the Ahrens-Feenberg value. However, when the fairly wide approximations used in obtaining the theoretical estimate are considered, the present result may not be regarded as a substantial deviation from theory.

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ANALYSIS FOR THE PRODUCTION OF AN ISOSCALAR S-WAVE DI-PION RESONANCE

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The various speculations concerning the existence of di-pion resonances in the region below about 750 MeV are at present very confusing. On the one hand, there are experimental results[1-4] for multi-pion reactions which indicate structure in the two-pion $I=0$ system at various energies between 350 and 750 MeV; however, it is not clear what are the effects of competing resonances, especially the (3,3) pion-nucleon resonance, on these final pion states. On the other hand, various theoretical approaches[5] have also implied the existence of isoscalar S-wave di-pion resonances in roughly the same energy region, but in these cases there arises the question of the accuracy of the theoretical models being used as well as the sensitivity of the results to the parameters of the resonance. In an attempt to overcome these various difficulties, we outline here a fully relativistic and unitarity-preserving, albeit purely phenomenological, model for the one-pion production reaction

$$\pi^- + p \to \pi^- + \pi^+ + n. \quad (1)$$

In particular, our approach incorporates the effects of the (3,3) resonance on the final two-pion system, as well as examining the possibility of a di-pion resonance. The tests of the model for a wide range of data serve to insure its validity and likewise to define its parameters rather precisely.

Our treatment is based on the work of Macfarlane,[6] which deals with three-body final states in a relativistic S-matrix formalism. To obtain an explicit form for the scattering amplitude, we assume that the initial pion-nucleon system is dominated by the ($J^P = \frac{1}{2}^+, I = \frac{1}{2}$) resonance at about 1480 MeV and by the ($J^P = \frac{3}{2}^-$, $I = \frac{1}{2}$) resonance at 1512 MeV for the energy region in which we are interested. To test this assumption, we see in Fig. 1 that the pion-nucleon elastic scattering in the $I = \frac{1}{2}$ state is described quite well in the energy region up to 700 MeV by taking the scattering to be mainly resonant.[7,8] In order to incorporate inelastic effects, we consider that the (3,3) pion-nucleon resonance and possibly a ($J^P = 0^+, I = 0$) di-pion resonance dominate in the final state of Reaction (1). Since we expect the production process to proceed via S waves near threshold, we see that the 1480-MeV pion-nucleon resonance will lead only to the di-pion resonance plus nucleon production, while the 1512-MeV resonance contributes only to the (3,3) resonance plus pion production, as far as the total cross-section is concerned. It can be shown by the methods of reference 6 that these channels with different total angular momentum also do not interfere for the differential cross sections calculated below. We assume that Reaction (1) proceeds through a total $I = \frac{1}{2}$ state; consequently, the $\pi^- n$ contribution to the (3,3) resonant will be dominant, and we neglect the resonant contribution from the $\pi^+ n$ final system.

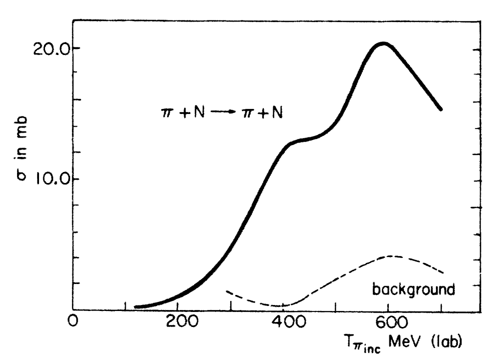

FIG. 1. The solid curve represents $\frac{2}{3}$ of the pion-nucleon elastic cross section in the $I = \frac{1}{2}$ state, as calculated by the resonance-dominated model described in the text. The background is obtained by subtracting the calculated values from the experimental results of reference 7.

We can now construct a unitary scattering matrix which is essentially a product of terms of the Breit-Wigner type with proper threshold behavior.[6,9] Thus, for example, the amplitude for Reaction (1) may be written as the sum of two terms (i.e., for the total angular momentum $J = \frac{1}{2}$ and $\frac{3}{2}$) of the form

$$\langle \zeta_3 j_2 \eta_2 | T_J(\epsilon_2, E) | \eta_{\pi N} \rangle = \langle \eta_2 | t_{j2}(\epsilon_2) | \eta_2 \rangle \langle \zeta_3 | M_J(E_1 \epsilon_2) | \eta_{\pi N} \rangle, \quad (2)$$

with $\langle \eta_2 | t_{j2}(\epsilon_2) | \eta_2 \rangle$ being the elastic rescattering amplitude for the two final resonant particles, and $\langle \zeta_3 | M_J(E_1 \epsilon_2) | \eta_{\pi N} \rangle$ the production amplitude through an initial pion-nucleon resonance. The total angular momentum and invariant energy for the whole system are denoted by J and E, respectively, while j_2 and ϵ_2 are the corresponding quantities for the final two-particle resonance. The symbol $\eta_{\pi N}$ contains essentially the spin and orbital subquantum numbers of the initial pion-nucleon system, and η_2 represents the same quantities for the final two-particle resonance. Finally, ζ_3 gives the spin and orbital quantum numbers associated with the coupling of the final two-particle resonant system with the third final particle.[6] Isotopic spin can be trivially incorporated as well. Our model, of course, picks out the assumed dominant quantum numbers. The important problem concerning recoupling of final particles, in order to determine the effects of final two particle resonances on one another, can also be solved in this scheme. We may then calculate the total inelastic cross section for Reaction (1) to check that our assumptions still apply.[10] The results are shown in Fig. 2; the agreement for a wide energy range shows that a rather sensitive test of our model is satisfied. The assumption of pure $I = \frac{1}{2}$ production breaks down above about 600 MeV, where $I = \frac{3}{2}$

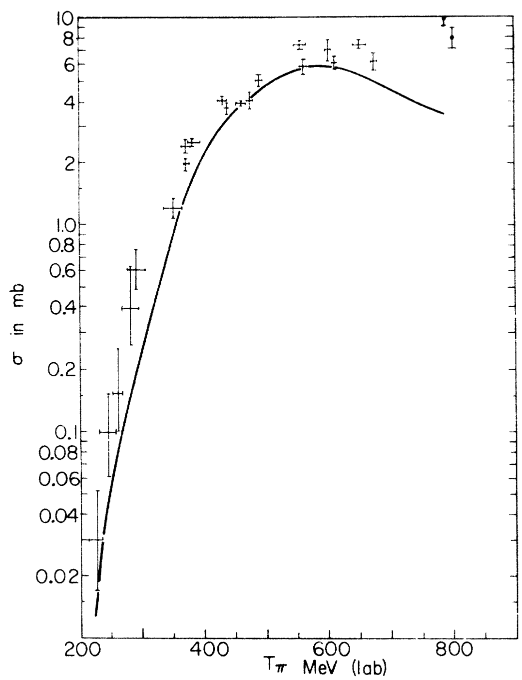

FIG. 2. The calculated inelastic cross section for the process $\pi^- + p \to \pi^- + \pi^+ + n$ as given by the solid curve, compared with the experimental values summarized in reference 8 and reproduced here.

inelastic scattering becomes sizable,[7] and the calculated values begin to fall off from the experimental ones in this region. Finally, the $\pi^+\pi^-$ and π^-n invariant mass distributions can be computed, and with masses and widths (being here the S-matrix pole positions in the energy plane) chosen as $M(3,3) = 1240$ MeV, $\Gamma(3,3) = 120$ MeV, $M(2\pi) = 490$ MeV, $\Gamma(2\pi) = 110$ MeV, the results are given in Figs. 3 and 4. The agreement with experiment is generally quite good;[11] the apparent shifting of the $\pi^+\pi^-$ peak with increasing incident energy may be attributed to the effect of a phase space factor which forces the curves to zero at the upper kinematical limit, and also slightly to the effect of the (3,3) resonance. The decrease in size of the peaks for the $\pi^+\pi^-$ system as the incident energy becomes larger is due to the increasing competition of the (3,3) resonance, proceeding through the $J = \frac{3}{2}$ channel. We note that there are some residual effects of the di-pion resonance in our model above an incident energy of about 600 MeV, while the experimental results seem to imply that the two-pion system may still tend to peak towards its upper kinematical limit until a phase space distribution is reached near

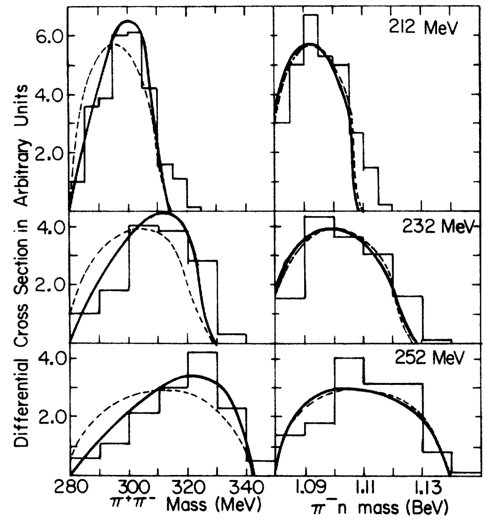

FIG. 3. The solid curves are the calculated results for the $\pi^-\pi^+$ and π^-n effective mass distributions with a scalar di-pion resonance of mass 490 MeV and width 110 MeV. The dashed curves represent phase space and the beam energies are also specified. The experimental results are taken from reference 3.

780 MeV. The discrepancy can probably be explained quite satisfactorily again by our neglect of the $I = \frac{3}{2}$ inelastic scattering, which undoubtedly makes a non-negligible contribution

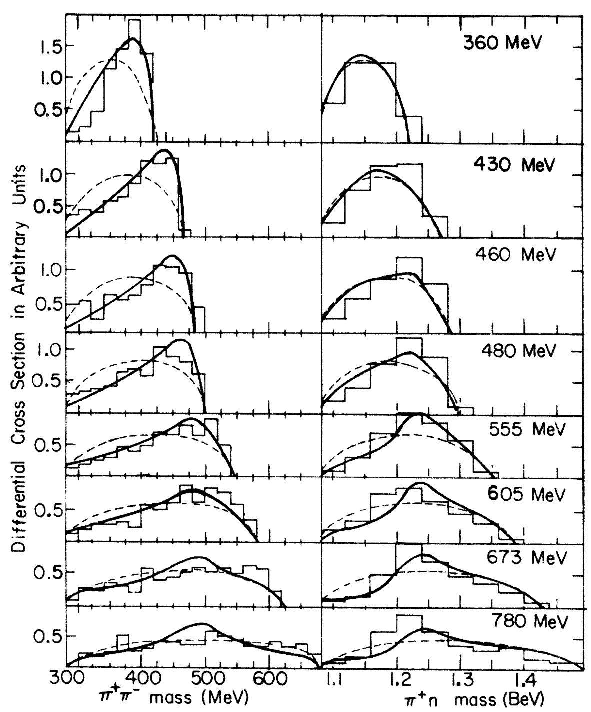

FIG. 4. Same as Fig. 3. The experimental values are from reference 2.

to Reaction (1) above 600 MeV; the effect of this additional contribution will result in a $\pi^+\pi^-$ distribution closer to phase space. On the other hand, the possible influence of higher di-pion resonances, especially a possible scalar resonance[4] proceeding through the $J=\frac{1}{2}$ resonant channel with its broad width,[12] can cause peaking at high invariant-mass energies until the particular production channel becomes no longer important.

The determination of the quantum numbers of the di-pion resonance cannot be precisely obtained by the results calculated here. We point out, however, that successful analyses[13] of pion production have been made without invoking di-pion resonances in cases for which the final pions cannot be in a $I=0$ state. For Reaction (1) such models do not lead to general agreement with experiment; thus, it is reasonable to expect that the resonance presented here is an iso-scalar. The determination of the spin and parity is somewhat more doubtful, but we note that the mass of the di-pion resonance given here tends to imply that the threshold of its production with a nucleon is intimately connected with the 1480-MeV pion-nucleon resonance if the di-pion resonance is in an S state. Since the 1480-MeV "peak" may really be due to a virtual bound state similar to a scattering-length effect,[14] there is good reason for expecting this type of connection.

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[10]Of course, Fig. 1 already includes inelastic effects, since the elastic cross section was for consistency calculated with a unitary scattering amplitude. Similarly in Fig. 2 (as in Fig. 1) the width and position of the di-pion resonance will also affect the calculated curves, and thus the problem is not done in quite as step-wise a manner as outlined in this Letter, but rather a good overall fit is obtained for all the data.

[11]Other parameters for the di-pion resonance do not yield such good agreement generally, while neglect of the resonance altogether gives marked disagreement.

[12]We note that the 490-MeV di-pion resonance presented here has a non-negligible influence on the normalized $\pi^+\pi^-$ mass distribution below the region expected simply from the value of its width. See Fig. 3. However, the actual cross section is small in this region.

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