

ing change in  $T_C$  was twenty times as large as and of the same sign as Glover's, for induced charge per atom twenty times as great as his. The changes in the reflection spectra will be reported separately.<sup>6</sup>

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P. J. Zachmanidis gave skillful assistance in the measurements.

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### PROPAGATION OF TRANSVERSE ACOUSTIC WAVES IN A SPIN-DENSITY-WAVE METAL\*

Roger C. Alig,† John J. Quinn, and Sergio Rodriguez

Department of Physics, Purdue University, Lafayette, Indiana

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Overhauser<sup>1</sup> has suggested recently that the electronic ground states of sodium, potassium, rubidium, and cesium possess spin-density waves (SDW). In order to explain why the distortion of the Fermi surface in the direction of the SDW is not observed experimentally,<sup>2</sup> he has postulated that the SDW orients itself parallel to the direction of a sufficiently strong magnetic field. One obvious experimental test of Overhauser's hypothesis is the study of the propagation of acoustic waves parallel to the dc magnetic field.<sup>3</sup> The object of this note is to present the results of a calculation of the attenuation and velocity of shear acoustic waves propagating parallel to the magnetic field for a SDW model of a metal, and to compare these results with the predictions of the free-electron model.<sup>4</sup> For example, in Fig. 1 we display the attenuation coefficient  $\gamma$  as a function of the magnetic induction  $B_0$ . The calculation was carried out for a model appropriate to potassium, assuming that the SDW is characterized by an energy gap  $G = 0.62$  eV as required to explain<sup>1</sup> the optical data of Mayer and El Naby.<sup>5</sup>

The position of the absorption edge (called the Kjeldaas edge) in Fig. 1 is determined by the condition

$$\omega_c = qv_m = (\omega/s)v_m, \quad (1)$$

namely that the frequency experienced by at least some electrons moving in the self-consistent electromagnetic field associated with the acoustic wave be equal to the cyclotron resonance frequency  $\omega_c$ . In Eq. (1) we have neglect-

ed  $\omega$ , the frequency of the acoustic wave, as compared to  $\omega_c$ . The symbol  $q = \omega/s$  stands for the wave vector of the sound wave,  $s$  is the velocity of sound, and  $v_m$  the maximum velocity of an electron on the Fermi surface in the direction of  $\vec{B}_0$ . For the free-electron model  $v_m$  is equal to the Fermi velocity  $v_F$ , but for a SDW model<sup>6</sup>  $v_m$  is considerably smaller.

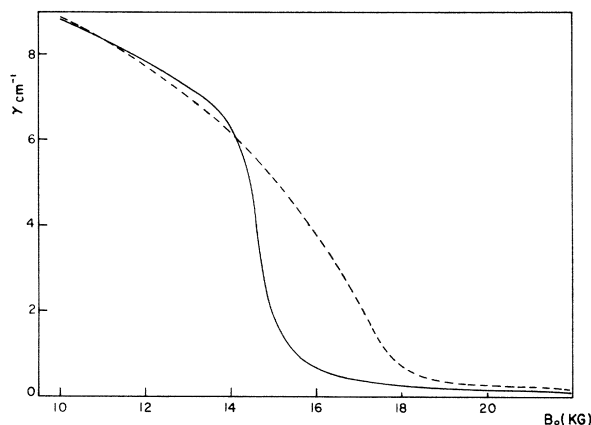


FIG. 1. The attenuation coefficient  $\gamma$  in  $\text{cm}^{-1}$  versus magnetic induction  $B_0$  in kilogauss for a left-circularly polarized sound wave. The solid curve is for the linear SDW model, and the dashed curve for the free-electron model. Both curves are appropriate to an acoustic frequency of 100 megacycles per second and an electron relaxation time of  $1.58 \times 10^{-10}$  sec, giving a value of the parameter  $ql$  of approximately 50. Detailed calculations for the velocity and attenuation of both circular polarizations and for helicons have been carried out. A more complete account of this work will be reported elsewhere.

This results in a shift of the Kjeldaas edge to lower magnetic fields as can be seen in Fig. 1. In Fig. 2 we display the derivative of  $\gamma$  with respect to the magnetic induction  $B_0$  as a function of  $B_0$ . The shape of the curve for the SDW model<sup>7</sup> differs appreciably from that for the free-electron model, indicating that, at least in the alkali metals, this experiment could indeed be used to test for the existence of a SDW.

The results shown in Figs. 1 and 2 are obtained by solving the equation of motion of the positive ions within the metal in the presence of the magnetic field  $\vec{B}_0$ . The model of a metal and the equation of motion of the ions have been discussed in detail elsewhere.<sup>8</sup> We have considered a transverse acoustic wave of given wavelength and have calculated the frequency as a function of  $B_0$ . We obtain the following cubic equation<sup>8</sup> for the angular frequency  $\omega$ :

$$(\omega^2 - s_0^2 q^2 \pm \Omega_c \omega)(\omega G_{\pm} - i\beta_0 \omega_0) = (im\omega/M\tau)(\omega - i\beta_0 \omega_0)(1 - G_{\pm}). \quad (2)$$

The components of the magnetoconductivity tensor appropriate to the propagation of circularly polarized waves are given by

$$\sigma_{\pm} = \sigma_{xx} \mp i\sigma_{xy} = \frac{e^2 m \tau}{4\pi^2 \hbar^2} \int dk_z \frac{v_{\perp}^2}{1 + i(\omega\tau \mp \omega_c \tau - qv_z \tau)}. \quad (3)$$

In this equation  $v_{\perp}$  and  $v_z$  are the components of the velocity of an electron on the Fermi surface perpendicular and parallel to  $\vec{B}_0$ , respectively.

In the present problem the only significant difference between the free-electron model and a SDW model of a metal is the difference in the constant-energy surfaces in  $\vec{k}$  space. In-

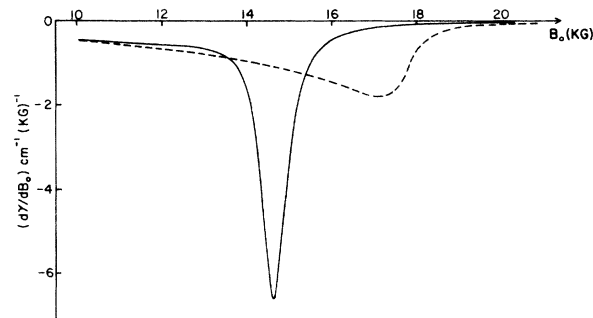


FIG. 2. The derivative of  $\gamma$  with respect to  $B_0$  vs  $B_0$  for the same experimental conditions as in Fig. 1. The ordinate is measured in  $\text{cm}^{-1} \text{KG}^{-1}$  and the abscissa in kilogauss.

stead of the spherical energy surfaces of the free-electron model, given by  $\epsilon = \hbar^2 k^2 / 2m$ , the energy surfaces appropriate to a metal with a linear SDW of wave vector  $\vec{Q}$  oriented parallel to the  $z$  direction are given approximately<sup>1</sup> by

$$\epsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m} + \mu \left( \frac{Q}{2} - |k_z| \right) - \left[ \mu^2 \left( \frac{Q}{2} - |k_z| \right)^2 + \frac{G^2}{4} \right]^{1/2}. \quad (4)$$

In this equation  $\mu = \hbar^2 Q / 2m$ ,  $|k_z|$  denotes magnitude of  $k_z$ , and  $G$  is the energy gap due to the SDW. We have evaluated numerically the real and imaginary parts of  $G_{\pm} = \sigma_0^{-1} \sigma_{\pm}$  as functions of  $B_0$  for a number of values of the wave vector  $q$  and the relaxation time  $\tau$ . We have then solved Eq. (2) numerically. The three roots of Eq. (2) correspond to right- and left-circularly polarized transverse acoustic waves and the helicon wave. The velocities and attenuation coefficients for each of these waves have been determined from the real and imaginary parts, respectively, of the appropriate roots of Eq. (2).

In the numerical calculations we have chosen parameters (such as electron density, sound velocity, etc.) appropriate to potassium, and an effective mass equal to the free-electron mass. For a spherical Fermi surface (but not necessarily one corresponding to a parabolic energy band) the position of the Kjeldaas edge depends only on the radius of curvature  $k_F = (3\pi^2 n)^{1/3}$ , where  $n$  is the electron concentration. If the metal possesses a SDW ground state, we expect the Kjeldaas edge to be shifted to lower magnetic fields.

If an experiment is performed with linearly polarized shear waves, then its interpretation requires consideration of the following facts. First, the difference in the velocities of the right- and left-circularly polarized components gives rise to a rotation of the plane of polarization of the acoustic wave. Second, the difference in the attenuation coefficients of these two components results in an elliptically polarized wave.

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†National Aeronautics and Space Administration trainee.

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<sup>3</sup>Another possible test is the study of helicon propagation near the absorption edge. This problem has been considered by A. W. Overhauser and S. Rodriguez, to be published, and by J. C. McGroddy, J. L. Stanford, and E. A. Stern, to be published. See J. C. McGroddy and J. L. Stanford, Bull. Am. Phys. Soc. **10**, 384 (1965).

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<sup>6</sup>If we assume an effective mass equal to the free-electron mass in potassium, then  $v_F = 0.864 \times 10^8$  cm/

sec, and for the SDW model used here  $v_m = 0.714 \times 10^8$  cm/sec.

<sup>7</sup>In this work we have considered a linear SDW only. For a spiral SDW, we expect the attenuation coefficient to behave like a combination of the results for the free-electron and linear SDW models.

<sup>8</sup>J. J. Quinn and S. Rodriguez, Phys. Rev. **133**, A1589 (1964). A derivation of Eq. (2) is given in this reference. The symbols used in the present paper are the same as in this reference: Namely,  $s_0$  is the velocity of low-frequency transverse acoustic waves in the absence of a magnetic field (for potassium we take  $s_0 = 1.74 \times 10^5$  cm/sec appropriate to shear waves propagating in a [100] direction);  $\Omega_C$  is the cyclotron frequency of the positive ions;  $q$  is the wave vector of the sound wave;  $\omega_0 = s_0 q$ ;  $\beta_0 = (c^2 q^2 / 4\pi\omega_0\sigma_0)$ ;  $\sigma_0$  is the dc electrical conductivity;  $\sigma_0 G_{\pm} = (\sigma_{xx} \mp i\sigma_{xy})$ , where  $\sigma_{xx}$  and  $\sigma_{xy}$  are components of the magnetoconductivity tensor along Cartesian axes perpendicular to  $B_0$ ;  $M$  is the ionic mass; and  $\tau$  the electron relaxation time.

## BETA-NEUTRINO ANGULAR CORRELATION IN $Sb^{122*}$ †

J. C. Palathingal‡

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

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From a comprehensive analysis of the various experiments performed on the  $2^- - 2^+$  first-forbidden nonunique beta transition from  $Sb^{122}$  to the first excited state of  $Te^{122}$ , Pipkin, Sanderson, and Weyhmann<sup>1</sup> have arrived at possible solutions for the six nuclear matrix elements involved in this decay. The results are summarized in eight sets of different values for the six matrix elements, and a ninth set corresponding to the modified  $B_{ij}$  approximation.<sup>2</sup>

An experimental determination of the beta-neutrino angular correlation for this decay has now made it possible to pick out one set of matrix elements from those presented by Pipkin, Sanderson, and Weyhmann. When the nuclear Coulomb effect is taken into account, the beta-neutrino angular correlation in first-forbidden decay can be approximated by the simple allowed form  $1 + (v/c) \cos\theta$  in a manner analogous to the  $\xi$  approximation.<sup>3-5</sup> An expression for  $\lambda$  is derived, given by

$$\lambda = \frac{C_A^2 |i f \gamma_5 - \xi f \vec{\sigma} \cdot \vec{r}|^2 - \frac{1}{3} |C_V i f \vec{\alpha} + \xi (C_A i f \vec{\sigma} \times \vec{r} - C_V f \vec{r})|^2}{C_A^2 |f \gamma_5 - \xi f \vec{\sigma} \cdot \vec{r}|^2 + |C_V i f \vec{\alpha} + \xi (C_A i f \vec{\sigma} \times \vec{r} - C_V f \vec{r})|^2}, \quad (1)$$

where  $\xi = \alpha Z / 2R \approx 12$  is equal to half the potential energy of an electron at the nuclear surface.

The experimental procedure consists of observing the variation of the resonance fluorescence scattering cross section for  $\gamma$  rays from a gaseous source of  $Sb^{122}$  (in the form of antimony trihydride) in coincidence with beta rays of selected energy, as a function of beta energy. The beta rays were selected along a direction approximately at  $180^\circ$  relative to the direc-

tion of the gamma rays (Fig. 1). The Doppler shift condition of the resonance fluorescence effect requires the neutrinos to propagate within a narrow cone surrounding the beta-gamma axis, subtending an angle  $\theta$  given by

$$E_\gamma = cp_\beta + cp_\nu \cos\theta, \quad (2)$$

where  $E_\gamma$  is the gamma energy,  $p_\beta$  and  $p_\nu$  are