(1964). Most of these theorems require some assumption about minimality, i.e., lack of derivative couplings, which is not necessary for the theorems presented here.

[5]S. Weinberg, Phys. Rev. 112, 1375 (1958).

[6]J. Bernstein, G. Feinberg, and T. D. Lee, to be published.

[7]N. Cabibbo, Phys. Letters 12, 137 (1964).

[8]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[9]For the definition of C for octets see N. Cabibbo, Phys. Rev. Letters 12, 62 (1964); M. Gell-Mann, Phys. Rev. Letters 12, 83 (1964).

[10]Since C- and T-invariance violations are connected (see reference 3), regular and irregular parts of the currents will give rise, respectively, to real and imaginary parts of the F_i and H_i, respectively.

[11]It is in fact equal to $\sin\theta/\sqrt{2}$, see reference 8.

[12]This dependence is fixed by being even under the exchange $a \leftrightarrow b$, as well as under the simultaneous transposition of λ^a, λ^b, and λ^i. The first property is required by CPT, the second by the behavior under C of the irregular octet.

[13]I am grateful to Dr. L. Montanet for a discussion of proton-antiproton annihilation which stimulated this remark.

[14]A test for C conservation in η decays, proposed by R. Friedberg, T. D. Lee, and M. Schwartz, is presently being carried out; I am grateful to P. Franzini for interesting discussions in this respect.

[15]This difficulty is avoided by the authors of reference 2, who assume a small (10^{-2}) violation of C invariance in strong interactions.

GENERALIZATIONS OF THE POINCARÉ GROUP

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I. Introduction.—The recent successful application of the symmetry group SU(6) has opened the floodgates of speculation. The original formulation of SU(6) symmetry[1] suggested that it was incompatible with relativity, and many attempts to formulate a "relativistic version" were made.[2] Except for the suggestion of Wyld[3] and Mahanthappa and Sudarshan,[4] all these generalizations are plagued by difficulties of interpretation. Here we investigate relativistic "generalizations" of internal symmetry groups. Our main conclusion is that an algebra $\mathcal{A}$ that includes that of the Poincaré group must be a semidirect product with $\mathcal{P} = \mathcal{A}/\mathcal{S}$. The assumptions under which this result is derived are (1) relativistic covariance and (2) that the mass spectrum is not continuous. We have studied several choices of $\mathcal{S}$, giving relativistic generalizations of Wigner's supermultiplet theory,[5] as well as Gürsey and Radicati's SU(6) theory.

II. General considerations.—It is our aim to determine every real Lie algebra that satisfies certain conditions that are necessary for a physical interpretation. Let $\mathcal{P}$ be the algebra of the Poincaré group, and let the 10 basic elements of $\mathcal{P}$ be chosen as follows:

$$\mathcal{P} = \{L_{ij}, P_\mu, L_{i0}\},$$

$$i,j = 1,2,3; \quad \mu = 0,1,2,3.$$

Let $\mathcal{P}_{LT}$ be the largest subalgebra of $\mathcal{P}$ that commutes with P_0 and let $\mathcal{P}_L$ be the homogeneous part of $\mathcal{P}_{LT}$. Then the structure of $\mathcal{P}$ is

$$\mathcal{P} = \{\mathcal{P}_{LT}, L_{0i}\}, \tag{2.1}$$

$$\mathcal{P}_{LT} = \mathcal{P}_L \vdash \{P_\mu\}, \tag{2.2}$$

where the symbol $\vdash$ denotes semidirect sum;

$$\mathcal{P}_L = \{L_{ij}\}. \tag{2.3}$$

The semidirect sum will always be written with the invariant subalgebra last.

Let $\mathcal{A}$ be an algebra that contains $\mathcal{P}$ as a subalgebra, and let $\mathcal{A}_{LT}$ be the largest subalgebra of $\mathcal{P}$ that commutes with P_0. Then we shall show that the physical interpretation requires the following structure for $\mathcal{A}$:

$$\mathcal{A} = \{\mathcal{A}_{LT}, L_{0i}\}, \tag{2.4}$$

$$\mathcal{A}_{LT} = \mathcal{A}_L \vdash \{P_\mu\}, \tag{2.5}$$

$$\mathcal{A}_L = \{L_{ij}\} \vdash \mathcal{S}, \tag{2.6}$$

$$\mathcal{A} = \mathcal{P} \vdash \mathcal{S}. \tag{2.7}$$

From (2.4) and (2.5) there follows that $\{P_\mu\}$ is an invariant subalgebra of $\mathcal{A}$. If $\{P_\mu\}$ is an invariant subalgebra of $\mathcal{A}$, and if in addition the mass operator $P_\mu P^\mu$ is an invariant of $\mathcal{A}$, then (2.6) and (2.7) follow. This important result was obtained by Michel.[6] Here we shall assume only that the spectrum of $P_\mu P^\mu$ is not continuous, and not, a priori, that $\{P_\mu\}$ is an invariant subalgebra.

First we show that (2.4) is necessary. Let $\mathfrak{D}$ be a particle-like representation of $\mathfrak{a}$ (i.e., one in which the spectrum of P_0 is bounded below by m, say) and let $\mathcal{H}$ be the Hilbert space in which the operators of $\mathfrak{D}$ act. Let $\mathcal{H}_L$ be the subspace[7] of $\mathcal{H}$ on which P_0 has the eigenvalue m, and let $\mathfrak{D}_L$ be the representation of $\mathfrak{a}_L$ induced in $\mathcal{H}_L$. The basis vectors of $\mathcal{H}_L$ are, for an appropriate choice of $\mathfrak{D}$, the states of a single particle at rest; we may call them $|\alpha\rangle$, $\alpha=1,2,\cdots$, where α stands for discrete quantum numbers like spin, charge, and strangeness. Let a_i, $i=1,2,\cdots$, be a maximal set of basis elements in $\mathfrak{a}$ that are linearly independent modulo $\mathfrak{a}_{LT}$. Let $U(\epsilon)$ be a unitary operator $1+\sum\epsilon_i a_i$, where the ϵ_i are arbitrarily small real numbers. Then $U(\epsilon)|\alpha\rangle=|\alpha,\epsilon\rangle$ is not in $\mathcal{H}_L$. Thus, the elements of $\mathfrak{a}$ that are not in $\mathfrak{a}_{LT}$ may be used, in addition to the label α, as labels to denote those vectors of $\mathcal{H}$ that are close to $\mathcal{H}_L$. Let us decompose this part of $\mathcal{H}$ into a direct sum of subspaces $\mathcal{H}_\alpha$, where $\mathcal{H}_\alpha$ consists of all vectors $|\alpha,\epsilon\rangle$ with fixed α. Then $\mathcal{H}_\alpha$ contains all the states of small velocity of a particle with well-defined internal quantum numbers. Now we come to our main point, namely, the dimension of $\mathcal{H}_\alpha$ must be three. It is at least three, because the three components of momentum are independent of each other. It is not more than three because the principle of relativity requires that, if a particle is found in a well-defined state by an observer at rest relative to it, then its state must likewise be well defined as seen by an observer moving slowly relatively to it. It follows that the three operators L_{0i}, when adjoined to $\mathfrak{a}_{LT}$, complete $\mathfrak{a}$, and we have proved (2.4).

Let $a\in\mathfrak{a}_{LT}$ and consider the commutator

$$[a,L_{0i}]=-C(a)_i^{\ j}L_{0j}+b, \tag{2.8}$$

where $b\in\mathfrak{a}_{LT}$. Calculating the commutator of both sides of (2.8) with P_0 we obtain

$$[a,P_i]=-C(a)_i^{\ j}P_j, \tag{2.9}$$

which proves (2.5).

In (2.9) let $a\in\mathfrak{a}_L$ and let us now introduce the assumption that the spectrum of $P_\mu P^\mu$ is not continuous. Then the matrices $C(a)_i^{\ j}$ are antisymmetric and form a faithful representation of the subalgebra $\{L_{ij}\}$ of $\mathfrak{a}_L$. Therefore, $\mathfrak{a}_L$ must have an invariant subalgebra $\mathcal{S}$, say, such that $\mathcal{S}$ commutes with $\{P_i\}$, and $\{L_{ij}\}$ is the factor algebra $\mathfrak{a}_L/\mathcal{S}$. Thus we have proved (2.6); (2.7) follows immediately by the observation that the matrices $C(a)_i^{\ j}$ in (2.8) and in (2.9) are the same. Note that $\mathcal{S}$ commutes with $\{P_\mu\}$.

Let s_A, $A=1,2,\cdots$, be a basis in the algebra $\mathcal{S}$;

$$[s_A,L_{\mu\nu}]=C_{A,\mu\nu}^{\ \ \ \ B}s_B, \tag{2.10}$$

where the matrices $C_{A,\mu\nu}^{\ \ \ \ B}$ form a real, finite-dimensional representation of the Lorentz algebra $\{L_{\mu\nu}\}$. Such a representation is a direct sum of tensor representations. The index A may be replaced by an aggregate of indices $(\lambda_1\cdots\lambda_n,a)$ where all except the last one are four-vector indices, such that (2.10) takes the form of a set of equations

$$\begin{aligned}[s_{\lambda_1\cdots\lambda_n,a},L_{\mu\nu}]&=i(g_{\lambda_1\mu}\delta_\nu^{\ \lambda_1'}-g_{\lambda_1\nu}\delta_\mu^{\ \lambda_1'})s_{\lambda_1'\cdots\lambda_n,a}\\&+\cdots+i(g_{\lambda_n\mu}\delta_\nu^{\ \lambda_n'}\\&-g_{\lambda_n\nu}\delta_\mu^{\ \lambda_n'})s_{\lambda_1\cdots\lambda_n',a},\end{aligned} \tag{2.11}$$

with $n=0.1,\cdots$. The range of the index a will, in general, depend on n.

The structure constants of $\mathcal{S}$ itself, defined by

$$\begin{aligned}[s_{\lambda_1\cdots\lambda_l,a},s_{\rho_1\cdots\rho_m,b}]&=C_{\lambda_1\cdots\lambda_l,a,\rho_1\cdots\rho_m,b}^{\qquad\qquad\sigma_1\cdots\sigma_n,c}\\&\times s_{\sigma_1\cdots\sigma_n,c},\end{aligned} \tag{2.12}$$

must of course satisfy the usual conditions that make $\mathcal{S}$ a Lie algebra. In addition, (2.11) and (2.12) are consistent if and only if (2.12) is Lorentz covariant.

In general $\mathcal{S}$ will include elements with no vector indices. These commute with $\mathcal{P}$ and form the algebra $\mathcal{S}_0$ of the internal symmetry group.

III. Examples without internal symmetries. – To construct the smallest $\mathfrak{a}$ that is not simply a direct sum of $\mathcal{S}_0$ and $\mathcal{P}$, let some of the elements of $\mathcal{S}$ be labelled by a single vector index, i.e., $s_\mu \in \mathcal{S}$, $\mu = 0, 1, 2, 3$. Then the commutator $[s_\mu, s_\nu] \equiv s_{\mu\nu}$ is an antisymmetric tensor. If $s_{\mu\nu} = 0$ then we may take $\mathcal{S}$ to consist exclusively of $\{s_\mu\}$, thus $\mathcal{S}_0 = 0$. In this example $\mathcal{A}_L = \{L_{ij}\} \vdash \{s_\mu\}$ is isomorphic to the direct sum $U_1 \oplus E_3$, where E_3 is the three-dimensional Euclidean group. If $s_{\mu\nu} \neq 0$, then it cannot be expressed linearly and covariantly in terms of the s_μ; hence $s_{\mu\nu}$ would be independent elements of $\mathcal{S}$.[8]

For another example, suppose that $s_{\mu\nu} = -s_{\nu\mu} \in \mathcal{S}$. Then it is possible to write covariant commutation relations, for example

$$[s_{\mu\nu}, s_{\lambda\rho}] = -i(g_{\mu\lambda}s_{\nu\rho} - g_{\mu\rho}s_{\nu\lambda} - g_{\nu\lambda}s_{\mu\rho} + g_{\nu\rho}s_{\mu\lambda}). \quad (3.1)$$

This algebra is of order six[9]; it is isomorphic to $\mathrm{SL}(2, c)$. It has a two-dimensional representation

$$s_{ij} = \tfrac{1}{2}\sigma_k, \quad s_{0i} = \frac{1}{2i}\sigma_i. \quad (3.2)$$

The algebra $\mathcal{A}_L$ is $\{L_{ij}\} \vdash \{s_{\mu\nu}\}$. From the commutation relations (2.11) and (3.1) it follows that $\{s_{\mu\nu}\}$ commutes with $\{L_{ij} - s_{ij}\}$; therefore $\mathcal{A}_L$ is isomorphic to $\mathrm{SL}(2, c) \oplus \mathrm{SU}(2)$, where the second term is $\{L_{ij} - s_{ij}\}$ and not $\{L_{ij}\}$. The unitary irreducible representations of $\mathcal{A}_L$ are given by a pair of unitary irreducible representations of the invariant subalgebras; let us consider briefly those representations of $\mathcal{A}_L$ that are obtained by choosing the trivial representation for the second one.

The unitary irreducible representations of $\mathrm{SU}(2, c)$ were given by Naimark.[10] They may be reduced according to its compact subalgebra, which is isomorphic to SU(2), and are then found to contain an infinite sum of irreducible representations. Each irreducible representation of the SU(2) subalgebra with "spin" larger than some minimum value occurs precisely once. These representations may be associated with the rotational levels of nuclei for fixed isotopic spin.[11]

It is important to realize that, in the type of representation just considered, the operators L_{ij} and s_{ij} are equal only in the rest system, i.e., on $\mathcal{H}_L$. Because the commutation relations between L_{ij} and s_{ij} with accelerations and with momenta are entirely different, this equality does not hold in other reference systems. In fact, on states with momentum $\vec{p}$,

$$L_{ij} = L_{ij}^{(0)} - i\left(p_i \frac{\partial}{\partial p_j} - p_j \frac{\partial}{\partial p_i}\right),$$

where $L_{ij}^{(0)}$ are the spin operators in the rest system, while

$$s_{\mu\nu} = \Lambda_\mu{}^\lambda \Lambda_\nu{}^\rho s_{\lambda\rho}^{(0)},$$

where $\Lambda_\mu{}^\lambda$ is the 4 by 4 matrix of the Lorentz transformation that transforms p_μ to rest. We have $L_{ij}^{(0)} = s_{ij}^{(0)}$ but $L_{ij} \neq s_{ij}$ for states with $\vec{p} \neq 0$.

IV. Example with isotopic spin. – Let $\mathcal{S}_0 = \{I_a\}$, where I_a, $a = 1, 2, 3$, are the isotopic spin operators, and let us construct a relativistic generalization of Wigner's supermultiplet theory. Both $\mathcal{S}_0$ and the algebra $\{s_{\mu\nu}\}$ considered in the preceding section have two-dimensional representations, given by (3.2) and by $I_a = \frac{1}{2}\tau_a$. In these representations we may calculate anticommutators as well as commutators; it is particularly important that those of $\{s_{\mu\nu}\}$ may be expressed covariantly:

$$[s_{\mu\nu}, s_{\lambda\rho}]_+ = \tfrac{1}{2}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + \tfrac{1}{2}\epsilon_{\mu\nu\lambda\rho}. \quad (4.1)$$

Wigner's supermultiplet theory[5] is based on the algebra SU(4) given by the matrices

$$I_a = \tfrac{1}{2}(1 \otimes \tau_a), \quad s_{ij} = \tfrac{1}{2}(\sigma_k \otimes 1),$$
$$s_{ij,a} = \tfrac{1}{2}(\sigma_k \otimes \tau_a). \quad (4.2)$$

The relativistic theory is constructed in the same way, and gives the algebra $\mathcal{S}$:

$$I_a = \tfrac{1}{2}(1 \otimes \tau_a), \quad I_a' = \frac{1}{2i}(1 \otimes \tau_a),$$
$$s_{ij} = \tfrac{1}{2}(\sigma_k \otimes 1), \quad s_{0i} = \frac{1}{2i}(\sigma_i \otimes 1),$$
$$s_{ij,a} = \tfrac{1}{2}(\sigma_k \otimes \tau_a), \quad s_{0i,a} = \frac{1}{2i}(\sigma_i \otimes \tau_a). \quad (4.3)$$

Because (3.1) and (4.1) are covariant we have covariant commutation relations for $\mathcal{S}$, given by (3.1) and

$$[I_a, I_b] = iI_c, \quad [I_a, I_b'] = iI_c', \quad [I_a', I_b'] = -iI_c, \tag{4.4}$$

$$[I_a, s_{\mu\nu}] = [I_a', s_{\mu\nu}] = 0; \tag{4.5}$$

$$[I_a, s_{\mu\nu,b}] = is_{\mu\nu,c}, \quad [I_a', s_{\mu\nu,b}] = -i\tilde{s}_{\mu\nu,c}, \tag{4.6}$$

$$[s_{\mu\nu}, s_{\lambda\rho,a}] = -i(g_{\mu\lambda}s_{\nu\rho,a} - g_{\mu\rho}s_{\nu\lambda,a} - g_{\nu\lambda}s_{\mu\rho,a} + g_{\nu\rho}s_{\mu\lambda,a}), \tag{4.7}$$

$$\begin{aligned}[s_{\mu\nu,a}, s_{\lambda\rho,b}] &= i(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})I_c - i\epsilon_{\mu\nu\lambda\rho}I_c' \\ &\quad - i\delta_{ab}(g_{\mu\lambda}s_{\nu\rho} - g_{\mu\rho}s_{\nu\lambda} - g_{\nu\lambda}s_{\mu\rho} + g_{\nu\rho}s_{\mu\lambda}),\end{aligned} \tag{4.8}$$

where $\tilde{s}_{\mu\nu,c} = g_{\mu\lambda}g_{\nu\rho}\epsilon^{\lambda\rho\sigma\tau}s_{\sigma\tau,c}$, and i,j,k and a,b,c are cyclic permutations of 1, 2, 3. These are the commutation relations of SU(4, c).

The algebra $\mathcal{A}_L$ is $\{L_{ij}\} \vdash$ SL(4, c). As in the previous example $\{L_{ij} - s_{ij}\}$ commutes with SL(4, c), so $\mathcal{A}_L$ is isomorphic to SL(4, c)⊕SU(2). Again we may take the trivial representation for SU(2), and construct unitary irreducible representations of SL(4, c). These are sums of unitary, irreducible representations of SU(4); Thus one representation of SL(4, c) is an infinite set of Wigner supermultiplets. We repeat the warning that it is only on $\mathcal{H}_L$ that $L_{ij} = s_{ij}$.

V. Example with unitary symmetry. – Let $\mathcal{S}_0 = \{\lambda_a\}$ where λ_a, $a = 1, \cdots, 8$, are the unitary symmetry operators, and let us construct a relativistic generalization of Gürsey and Radicati's supermultiplet theory.[1] In (4.3) replace $\frac{1}{2}\tau_a$, $a = 1, 2, 3$, by λ_a, $a = 1, \cdots, 8$, taking for the latter one of the three-dimensional representations. Then the matrices in the left-hand column satisfy the commutation relations of SU(6) and those in the right-hand column complete this to $\mathcal{S}$, which is isomorphic to SL(6, c). The commutation relations are given by (3.1), (4.7), and

$$[\lambda_a, \lambda_b] = if_{ab}{}^c\lambda_c, \quad [\lambda_a, \lambda_b'] = if_{ab}{}^c\lambda_c', \quad [\lambda_a', \lambda_b'] = -if_{ab}{}^c\lambda_c,$$

$$[\lambda_a, s_{\mu\nu}] = [\lambda_a', s_{\mu\nu}] = 0,$$

$$[\lambda_a, s_{\mu\nu,b}] = if_{ab}{}^c s_{\mu\nu,c}, \quad [\lambda_a', s_{\mu\nu,b}] = -if_{ab}{}^c\tilde{s}_{\mu\nu,c},$$

$$\begin{aligned}[s_{\mu\nu,a}, s_{\lambda\rho,b}] &= \frac{i}{4}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})f_{ab}{}^c\lambda_c - \frac{i}{4}\epsilon_{\mu\nu\lambda\rho}f_{ab}{}^c\lambda_c' \\ &\quad - \frac{i}{2}d_{ab}{}^c(g_{\mu\lambda}s_{\nu\rho,c} - g_{\mu\rho}s_{\nu\lambda,c} - g_{\nu\lambda}s_{\mu\rho,c} + g_{\nu\rho}s_{\mu\lambda,c}) \\ &\quad - i\tfrac{2}{3}\delta_{ab}(g_{\mu\lambda}s_{\nu\rho} - g_{\mu\rho}s_{\nu\lambda} - g_{\nu\lambda}s_{\mu\rho} + g_{\nu\rho}s_{\mu\lambda}).\end{aligned}$$

The algebra $\mathcal{A}_L$ is $\{L_{ij}\} \vdash$ SL(6, c) and is isomorphic to SL(6, c)⊕SU(2) where the SU(2) generators are $L_{ij} - s_{ij}$ as in the other example. Taking for SU(2) the trivial representation, we may represent elementary particles by unitary irreducible representations of SL(6, c), which are infinite sums of representations of SU(6). The full relativistic group is given by (2.7) where $\mathcal{S}$ is isomorphic to SL(6, c). The translations commute with $\mathcal{S}$ and the commutation relations between $\mathcal{S}$ and the generators of the homogeneous Lorentz group are given by (2.11). The symmetry is not "intrinsically broken"; if mass splittings are ignored the theory contains an exact symmetry including both the Poincaré group and the internal quantum numbers.

[1] F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).

[2] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) A248, 146 (1965); T. Fulton and J. Wess, to be published; W. Rühl, to be published. K. Bardacki, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 13, 698 (1964); 14, 48 (1965).

[3] H. W. Wyld, to be published.

[4] K. T. Mahanthappa and E. C. G. Sudarshan, to be published.

[5] E. P. Wigner, Phys. Rev. 51, 105 (1937); P. Franzini and L. A. Radicati, Phys. Letters 6, 322 (1963).

[6]L. Michel, private communication. See L. Michel, Phys. Rev. 137, B405 (1965); L. Michel and B. Sakita, to be published.

[7]H_2 is not a proper subspace. A more appropriate terminology would use the concept of the rigged Hilbert space, but for our purposes these trimmings are not essential. For a summary of the theory of rigged Hilbert spaces see A. Böhm, to be published.

[8]If for the $s_{\mu\nu}$ we take the commutation relations (3.1) below, then s_μ and $s_{\mu\nu}$ span the algebra of the 4+1 de Sitter group. In a separate publication we show that this case fits the hydrogen atom.

[9]We may attempt to reduce this to three by taking $s_{\mu\nu}$ to be self-dual, but because of the indefiniteness of the metric we must introduce an imaginary factor: $s_{\mu\nu}=is_{\mu\nu}$. Then $is_{\mu\nu}$ are new elements of the real Lie algebra, and we still have six elements.

[10]M. A. Naimark, Les Representations Lineaires du Groupe de Lorentz (Dunod, Paris, 1962). This book discusses SL(2,C). For SL(n,C) see M. A. Naimark, Mat. Sb. 35, 317 (1954), and 37, 121 (1955) [translated in American Mathematical Society Translations (American Mathematical Society, Providence, Rhode Island, 1958), Ser. 2, Vol. 9, pp. 155, 195].

[11]A. O. Barut and A. Böhm, "Dynamical Groups and Mass Formulae" (to be published).

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PION PRODUCTION IN HIGH-ENERGY MUON-NUCLEON COLLISIONS. P. L. Jain and M. J. McNulty [Phys. Rev. Letters 14, 611 (1965)].

Equation (2) should be

$$L(\epsilon,q^2)=L_0\left(\frac{\Lambda^2}{q^2+\Lambda^2}\right)^2$$

$$=\left(\frac{4\pi}{\epsilon}\right)\sigma_{h\nu}\ \text{with}\ \Lambda^2=\infty,$$

$$=\left(\frac{4\pi}{\epsilon}\right)\sigma_{h\nu}\left(\frac{\Lambda^2}{q^2+\Lambda^2}\right)\ \text{with finite}\ \Lambda.$$