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C-INVARIANCE VIOLATION IN STRONG AND WEAK INTERACTIONS

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One of the most interesting proposals advanced to explain the discovery of a decay mode $K_2^0 \to \pi^+ + \pi^-$ is the suggestion that C invariance is violated directly at the level of strong interactions. Such C-invariance violation could appear at the level of SU(3)-breaking interactions, as originally proposed by Prentki and Veltman,[1] or with a somewhat smaller strength, comparable to that of electromagnetic interactions, as porposed by Lee and Wolfenstein.[2] In this note we wish to consider a more extreme possibility, namely, we will propose that C need not be at all a basic symmetry of strong interactions.

For a limited but important class of phenomena we show that the predictions of C and of T invariance follow (with different degrees of approximation) from the CPT theorem, P conservation,[3] and the approximate isotopic spin [SU(2)] and SU(3) symmetries. That such things could happen has been known for a long time.[4]

The class of phenomena for which theorems are actually stated here includes the coupling of pseudoscalar particles to spin-$\frac{1}{2}$ baryons, and the coupling of vector mesons to conserved currents of spin-$\frac{1}{2}$ baryons and of spinless mesons. Although more extensive theorems can be stated, it is both amusing and interesting to point out some cases where theorems of this kind are not valid, except perhaps under more stringent conditions. These are the coupling of scalar and of axial-vector (1^+) particles to spin-$\frac{1}{2}$ baryons. The interest of this fact in respect to the present theory has been pointed out by Pais, and it is hardly necessary to emphasize its possible implications.

Let us first consider theorems which deal with the coupling of pseudoscalar mesons to spin-$\frac{1}{2}$ particles:

T_1.—The P-conserving coupling of a pseudoscalar neutral particle to a spin-$\frac{1}{2}$ particle on the energy shell is T invariant.

T_2.—If isospin is conserved, the above results are generalized to πN couplings (with N on the energy shell).

T_3.—In the limit of exact SU(3) the above is generalized to the couplings among octet pseudoscalar meson and the baryon octet. It is easy to see that theorems analogous to T_1-T_3 cannot be proved for the case of scalar (0^+) particles interacting with baryons, unless more restrictive assumptions are made.

Let us consider next the matrix elements of a Hermitian conserved vector current among two states of the same particle. Write such a matrix element in the form

$$\langle p'|j_\mu|p\rangle = i\bar{u}(p')[F_1(k^2)\gamma_\mu + F_2(k^2)\frac{1}{2M}\sigma_{\mu\nu}k_\nu + H_1(k^2)\frac{1}{2M}k_\mu]u(p), \tag{1}$$

in the case of a spin-$\frac{1}{2}$ particle, and

$$\langle p'|j_\mu|p\rangle = F_3(k^2)(p+p')_\mu + H_2(k^2)(p-p')_\mu, \tag{2}$$

for a spin-zero particle; F_1, F_2, and F_3 are usually called first-class form factors, H_1 and H_2 second-class form factors.[5] From the fact that j_μ is Hermitian (for $\mu = 0, 1, 2, 3$) and conserved, there follows

T_4.—$H_1(k^2) = H_2(k^2) = 0$; F_1, F_2, and F_3 are real, i.e., the same limitations one would obtain from C and T. Furthermore,

T_5.—If the scalar (or PS) particle in Eq. (2) is self-conjugate under CPT, then also $F_3 = 0$ and the particle is strictly neutral.

These results apply directly to diagonal matrix elements of the electromagnetic current which has all the required properties. The results can be generalized by the further assumption of exact isospin conservation and SU(3) invariance:

T_6.—The matrix elements of the isospin currents among members of a single isospin multiplet with either spin 0 or spin $\frac{1}{2}$ obey all formal requirements of T conservation.

This is evident if one notes that the current j^3 has the properties required for theorem T_4, and all other matrix elements can be obtained from those of j^3.

If we assume that the nonstrange vector mesons (ρ, ω, φ) are coupled to conserved currents (isospin currents, Y and B currents), it follows from the above theorems that nuclear forces should be T conserving to the extent to which nucleon-nucleon interactions are due to the single exchange of π, η, and the said vector mesons.

These results can also be extended to SU(3) by the following theorem:

T_7.—In the limit of exact SU(3) the above results of T_4 apply to matrix elements of the SU(3) currents $j_\mu{}^i$ $(i=1,\cdots,8)$ among octets of baryons and of pseudoscalar mesons.

This theorem covers some nondiagonal matrix elements of the electromagnetic (em) current; in particular we obtain

T_8.—T noninvariance in the $\Sigma^0 \to \Lambda^0 + \gamma$ (real or virtual γ) amplitude is at most of the first order in the SU(3)-breaking interaction.

T_9.—The matrix element of the em current among an η^0 and a π^0 is of the first order in the symmetry-breaking interaction.

The search for T-invariance violations in correlations of the decay $\Sigma^0 \to \Lambda^0 + e^+ + e^-$, and for the C-invariance violating decay mode $\eta^0 \to \pi^0 + e^+ + e^-$ have been recently proposed[6]; theorems T_7 and T_8 suggest that these effects could be smaller than expected on the basis of simple estimates. It is again easy to see that theorems T_4 to T_7 cannot be simply extended to axial-vector currents.

Effects on leptonic weak interactions.—In a preceding paper[7] the author has suggested a possible way of introducing CP nonconservation in weak interactions. This scheme assumed that the weak currents were built in the usual way[8] from a single octet $J_\mu{}^i$ of mixed C and P properties:

$$J_\mu{}^i = j_\mu{}^{i,R} + j_\mu{}^{i,I} + g_\mu{}^{i,R} + g_\mu{}^{i,I}.$$

The j's are vector, the g's axial-vector currents; the superscripts R and I stand for regular and irregular C behavior.[9] In that paper C conservation in strong interactions was tacitly assumed, so that the em current was naturally assigned to the regular octet $j_\mu{}^{i,R}$.

In the present theory in which we do not assume any a priori C behavior for the strong interactions, the following assumption is more natural:

(A) The electromagnetic current and the weak vector currents belong to the same octet, i.e., to $j_\mu{}^{i,R} + j_\mu{}^{i,I}$.

With respect to the properties of the amplitudes for weak processes of the form $B \to B' + l + \nu$ or $P \to P' + l + \nu$, where P represents a pseudoscalar meson, the weaker assumption of reference 7 implies that in the limit of exact SU(3) the irregular currents contribute only to the second-class form factors $H_1(k^2)$ and $H_2(k^2)$ [see Eqs. (1) and (2)] and the "axial magnetism" term.

The new assumption, which ties the em and vector weak currents, has stronger implications:

T_{10}.—Under the above assumption it follows from T_6 and T_7 that there is no T-invariance violation in weak vector processes in the limit of exact symmetry, i.e., $H_1(k^2)$, $H_2(k^2)$, and the ratio $\mathrm{Im}[F_i(k^2)]/\mathrm{Re}[F_i(k^2)]$ $(i=1,2,3)$ are at least of the first order in the symmetry-breaking interactions[10] for $\Delta S = \pm 1$ decays and for nondiagonal $\Delta S = 0$ transitions like $\Sigma^\pm \to \Lambda^0 + e^\pm + \nu$. For $\Delta S = 0$ diagonal transitions (beta decay or $\pi^+ \to \pi^0 + e^+ + \nu$) these quantities are at least of order $(137)^{-1}$ (from theorem T_6).

Since the terms in H_1 and H_2 give contributions proportional to the lepton mass, these conclusions are only of practical interest in the case of muon emission. A process which was suggested for experimental study in reference 7 is the decay $K^+ \to \pi^0 + \mu^+ + \nu_\mu$. For this process we are able—under assumption (A)—to reach an even stronger conclusion than the one suggested by T_{10}, namely,

T_{11}.—Under assumption (A), time-reversal invariance violations in $K_{\mu 3}$ are at least of the second order in the SU(3)-breaking interaction.

To prove this theorem we first note that the time-reversal noninvariant correltion $\vec{\sigma}(\mu) \cdot [\vec{p}(\mu) \times \vec{p}(\pi)]$ is proportional to

$$\mathrm{Im}(F_3 H_2{}^*) = -\mathrm{Re}(F_3)\,\mathrm{Im}(H_2) + \mathrm{Im}(F_3)\,\mathrm{Re}(H_2).$$

In this expression only $\mathrm{Re}(F_3)$ is different from zero in the SU(3) limit.[11] Thus we have only to prove that $\mathrm{Im}(H_2)$ vanishes up to the second order in the symmetry breaking. Consider the general matrix element of $j_\mu{}^{i,I}$ among two pseudoscalar mesons M_a and M_b:

$$\langle M_b | j_\mu^{i,I} | M_a \rangle = i[\mathrm{Im}(F_3)(p^a + p^b)_\mu + \mathrm{Im}(H_2)(p^a - p^b)_\mu].$$

To the first order in the symmetry-breaking interaction, contributions to $\mathrm{Im}(H_2)$ have the following dependence[12] on (a, b, i):

$$\mathrm{Im}(H_2)|_{(a,b,i)} = \alpha\,\mathrm{Sp}(\{\lambda^a, \lambda^b\}\{\lambda^i, \lambda^8\}) + \beta\,\mathrm{Sp}(\lambda^a\lambda^b)\,\mathrm{Sp}(\lambda^i\lambda^8) + \gamma[\mathrm{Sp}(\lambda^a\lambda^i)\,\mathrm{Sp}(\lambda^b\lambda^8) + \mathrm{Sp}(\lambda^a\lambda^8)\,\mathrm{Sp}(\lambda^b\lambda^i)];$$

however, if we consider $j_\mu^{i,I}$ to be the irregular part of the electromagnetic current, $j_\mu^{3,I} + (1/\sqrt{3})j_\mu^{8,I}$, all its diagonal matrix elements (i.e., when $M_a = M_b = \pi^\pm, \pi^0, \eta, K^\pm, K^0$) should vanish (see theorem T_4), from which we derive $\alpha = \beta = \gamma = 0$, which completes the proof of T_{11}.

The last theorem again puts the emphasis on the usefulness of an accurate measurement of T-invariance violation in $K\mu_3$ decays; the finding of considerable violation (larger than 1 or 2%) would certainly exclude our assumption (A).

The theorems T_1 to T_8 show that in a certain class of "simple" strong-interaction phenomena, violations of C and T symmetries can be strongly limited by parity, Hermiticity, and electric-current conservation, combined with isospin or SU(3) symmetry. Considering the technique of proving these theorems, it is clear that the proofs go through because in all instances we are considering either diagonal matrix elements of Hermitian operators (as in T_2, T_4, and T_5) or matrix elements which are equivalent to these through some symmetry operation [SU(2) or SU(3)]. It is quite possible that these theorems can be generalized by making use of SU(6) symmetry, and that they can be extended to cover the case of matrix elements among states in the same SU(6) multiplet, like N and N^*, pseudoscalar and vector mesons.

In conclusion we have pointed out that a theory in which there actually is no C invariance is in many "simple" cases equivalent to that proposed by Prentki and Veltman, in which C-invariance violations appear only at the first order of symmetry breaking. Our brief discussion on the technique of proving the theorems T_1 to T_8 suggests that larger C-invariance breakings might occur (within this theory) if one goes away from the simple cases, namely, if one studies "complicated" processes, where more than three particles are involved in an essential manner.

By essential I mean that the main contribution to the process should not come from peripheral graphs which involve three-particle vertices. Among processes which seem promising according to these rules are proton-antiproton annihilations[13] and the decay $\eta \to \pi^+ + \pi^- + \pi^0$.[14] Another class of possibilities is suggested by the lack of theorems equivalent to T_1-T_7 in the case of vertices involving 0^+ and 1^+ particles.

In the field of weak interaction the theory presented here—integrated by assumption (A)—reproduces the results of reference 6, but gives stronger limitations on the amount of C-invariance violation in vector currents. The smallness of the rate of $K_2^0 \to \pi^+ + \pi^-$ is again a difficulty to be explained.[15]

I am deeply indebted to J. Prentki and M. Veltman for long discussions on their proposal, which stimulated many of the above ideas, to Dr. A. Pais for encouragement and a clarifying discussion, and to Dr. P. Franzini, Dr. T. D. Lee, and Dr. G. Feinberg for private communications on their work on this subject.

I owe special gratitude to Professor A. Pais for the warm hospitality I found at the Rockefeller Institute where this work was done.

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[3]P conservation is assumed here as in references 1 and 2, in view of the strong limits existing on electric dipole moments, especially of the neutron. The TCP theorem then establishes the equivalence among C- and T-invariance violations. Although we speak sometimes of the one, sometimes of the other, this equivalence should be kept in mind, and will be sometimes used in the following.

[4]It is well known that C and P can follow from CP and other conditions like Hermiticity, current conservation, SU(3) and isotopic-spin invariance: G. Morpurgo and B. Touschek, unpublished; G. Feinberg, Phys. Rev. 108, 878 (1957); S. Gupta, Can. J. Phys. 35, 1309 (1957); V. Soloviev, Zh. Eksperim. i Teor. Fiz. 33, 537, 796 (1957) [translation: Soviet Phys.—JETP 6, 419, 613 (1958)]; G. Feinberg and F. Gürsey, Phys. Rev. 114, 1153 (1957); J. Sakurai, Phys. Rev. 113, 1679 (1959); A. Pais, Phys. Rev. Letters 12, 432

(1964). Most of these theorems require some assumption about minimality, i.e., lack of derivative couplings, which is not necessary for the theorems presented here.

[5]S. Weinberg, Phys. Rev. 112, 1375 (1958).

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[8]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[9]For the definition of C for octets see N. Cabibbo, Phys. Rev. Letters 12, 62 (1964); M. Gell-Mann, Phys. Rev. Letters 12, 83 (1964).

[10]Since C- and T-invariance violations are connected (see reference 3), regular and irregular parts of the currents will give rise, respectively, to real and imaginary parts of the F_i and H_i, respectively.

[11]It is in fact equal to $\sin\theta/\sqrt{2}$, see reference 8.

[12]This dependence is fixed by being even under the exchange $a \leftrightarrow b$, as well as under the simultaneous transposition of λ^a, λ^b, and λ^i. The first property is required by CPT, the second by the behavior under C of the irregular octet.

[13]I am grateful to Dr. L. Montanet for a discussion of proton-antiproton annihilation which stimulated this remark.

[14]A test for C conservation in η decays, proposed by R. Friedberg, T. D. Lee, and M. Schwartz, is presently being carried out; I am grateful to P. Franzini for interesting discussions in this respect.

[15]This difficulty is avoided by the authors of reference 2, who assume a small (10^{-2}) violation of C invariance in strong interactions.

GENERALIZATIONS OF THE POINCARÉ GROUP

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I. Introduction.—The recent successful application of the symmetry group SU(6) has opened the floodgates of speculation. The original formulation of SU(6) symmetry[1] suggested that it was incompatible with relativity, and many attempts to formulate a "relativistic version" were made.[2] Except for the suggestion of Wyld[3] and Mahanthappa and Sudarshan,[4] all these generalizations are plagued by difficulties of interpretation. Here we investigate relativistic "generalizations" of internal symmetry groups. Our main conclusion is that an algebra $\mathfrak{A}$ that includes that of the Poincaré group must be a semidirect product with $\mathcal{P} = \mathfrak{A}/\mathcal{S}$. The assumptions under which this result is derived are (1) relativistic covariance and (2) that the mass spectrum is not continuous. We have studied several choices of $\mathcal{S}$, giving relativistic generalizations of Wigner's supermultiplet theory,[5] as well as Gürsey and Radicati's SU(6) theory.

II. General considerations.—It is our aim to determine every real Lie algebra that satisfies certain conditions that are necessary for a physical interpretation. Let $\mathcal{P}$ be the algebra of the Poincaré group, and let the 10 basic elements of $\mathcal{P}$ be chosen as follows:

$$\mathcal{P} = \{L_{ij}, P_\mu, L_{i0}\},$$

$$i, j = 1, 2, 3; \quad \mu = 0, 1, 2, 3.$$

Let $\mathcal{P}_{LT}$ be the largest subalgebra of $\mathcal{P}$ that commutes with P_0 and let $\mathcal{P}_L$ be the homogeneous part of $\mathcal{P}_{LT}$. Then the structure of $\mathcal{P}$ is

$$\mathcal{P} = \{\mathcal{P}_{LT}, L_{0i}\}, \tag{2.1}$$

$$\mathcal{P}_{LT} = \mathcal{P}_L \vdash \{P_\mu\}, \tag{2.2}$$

where the symbol $\vdash$ denotes semidirect sum;

$$\mathcal{P}_L = \{L_{ij}\}. \tag{2.3}$$

The semidirect sum will always be written with the invariant subalgebra last.

Let $\mathfrak{A}$ be an algebra that contains $\mathcal{P}$ as a subalgebra, and let $\mathfrak{A}_{LT}$ be the largest subalgebra of $\mathcal{P}$ that commutes with P_0. Then we shall show that the physical interpretation requires the following structure for $\mathfrak{A}$:

$$\mathfrak{A} = \{\mathfrak{A}_{LT}, L_{0i}\}, \tag{2.4}$$

$$\mathfrak{A}_{LT} = \mathfrak{A}_L \vdash \{P_\mu\}, \tag{2.5}$$

$$\mathfrak{A}_L = \{L_{ij}\} \vdash \mathcal{S}, \tag{2.6}$$

$$\mathfrak{A} = \mathcal{P} \vdash \mathcal{S}. \tag{2.7}$$

From (2.4) and (2.5) there follows that $\{P_\mu\}$ is an invariant subalgebra of $\mathfrak{A}$. If $\{P_\mu\}$ is an invariant subalgebra of $\mathfrak{A}$, and if in addition the mass operator $P_\mu P^\mu$ is an invariant of $\mathfrak{A}$,