RECOIL EFFECTS IN HEAVY-ION TRANSFER REACTIONS*

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Recent heavy-ion experiments involving both single-particle and cluster transfer at energies well above the Coulomb barrier exhibit several unusual and unexpected features.[1-3] The angular distributions are smooth, in striking contrast to the oscillatory distributions characteristic both of transfer reactions initiated by protons and deuterons, and of elastic heavy-ion scattering. Furthermore, the data are in disagreement with the strong oscillatory cross sections predicted by previous theoretical diffraction models for high-energy heavy-ion transfer reactions.[4]

When plotted as a function of the linear-momentum transfer, the experimental angular distributions fall on a curve which appears to be independent of the nature of the projectile and target as well as independent of the incident energy and the angular momentum transferred. Typical results for proton transfer leading to a variety of final states[1-3] are shown in Fig. 1(a). Deuteron[1,3] and alpha[3] transfer reactions are compared with the proton data in Figs. 1(b) and 1(c). All of these data show the same monotonic dependence on q, the linear-momentum transfer, and the slope seems to depend only on the mass of the transferred particle.

The disagreement between diffraction theory and experiment could perhaps reflect a failure of strong-absorption models to represent these reactions adequately. Such a failure is quite unlikely, however, in view of the significant successes that strong-absorption (or cutoff) models have had in explaining experimental data from both elastic and transfer scattering at lower energies,[4,5] as well as the success for reactions induced by lighter projectiles.[6,7] Another possible explanation is that the smooth distributions arise from the presence of a very diffuse surface for these systems. However, a large surface thickness is inconsistent with elastic-scattering information for the same systems.[8]

The purpose of this note is to show that a satisfactory explanation of these phenomena is obtained with a sharp-cutoff diffraction model, if both finite-range effects and recoil effects are included.

Earlier treatments of heavy-ion transfer reactions[4,5] have neglected specific consideration of recoil effects; i.e., terms of order $1/A$

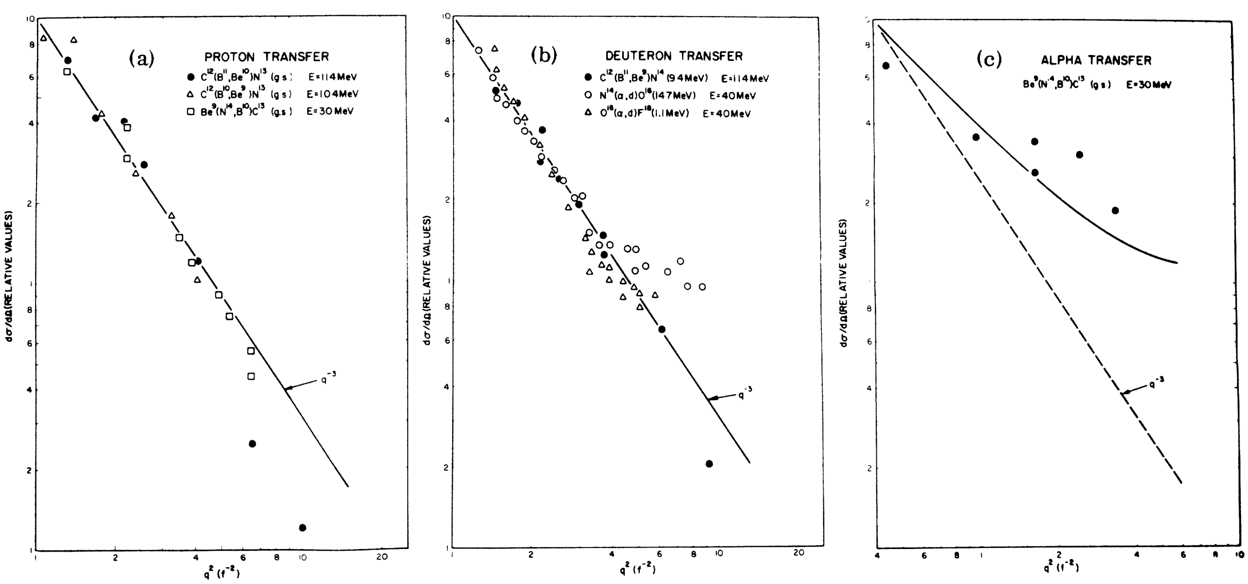

FIG. 1. (a) Differential cross sections for proton transfer in several typical heavy-ion reactions (references 1 and 3). The energy quoted is the laboratory energy in each case. The solid curve shows the q^{-3} dependence obtained from the model in the text. (b) Same as Fig. 1(a) for deuteron transfer (references 1 and 2). (c) Same as Fig. 1(a) for alpha-particle transfer (reference 3). The dashed line shows the q^{-3} dependence, while the solid line is obtained from the more exact expression in Eq. (9).

where A is the ratio of the mass of one of the heavy aggregates to that of the transferred cluster. In addition to neglecting recoil terms, most diffraction treatments also use a zero-range approximation.[4]

The rearrangement process may be written in the form $(a+c)+b \to a+(b+c)$, where the parentheses indicate a bound state, i.e., in the initial state "i," the heavy aggregate b is indicent on the compound system $(a+c)$ with relative momentum $\vec{k}_i$, and in the final state "f," $(b+c)$ and a have relative momentum $\vec{k}_f$. In the distorted-wave Born approximation, the matrix element for the transition $i \to f$ is

$$T_{if} = d\vec{r}\int d\vec{r}'\, \chi_f^{(-)*}\left(\vec{k}_f; \vec{r} - \frac{\vec{r}'}{A_f}\right) u_f^*(\vec{r}') V_{ac}(\vec{r}+\vec{r}') u_i(\vec{r}+\vec{r}') \chi_i^{(+)}\left(\vec{k}_i; \frac{A_i-1}{A_i}\vec{r} - \frac{\vec{r}'}{A_i}\right), \qquad (1)$$

where u_i and u_f are the bound-state wave functions for c in the initial and final states, respectively, and the mass ratios, $A_i = (m_a + m_c)/m_c$, and $A_f = (m_b + m_c)/m_c$ are assumed to be large compared with unity. The vector $\vec{r}$ is the relative coordinate between the centers of mass of the heavy aggregates a and b, while $\vec{r}'$ is the coordinate of c relative to b. The diffraction model for the distorted wave $\chi_i^{(+)}$ or $\chi_f^{(-)}$ is formulated in terms of an undistorted plane wave modulated by a function B_i or B_f which vanishes inside a sphere of radius R_0 as well as in the shadow scattering region.[7]

Our approximation consists of keeping terms in $1/A_i$ and $1/A_f$ in the phase of the distorted waves and neglecting such terms when they appear linearly in B_i and B_f. This procedure is based on the observation that recoil momentum proportional to $1/A$ will in general be quite small compared to k_i or k_f or to the momentum transfer $\vec{q} = [(A_i-1)/A_i]\vec{k}_i - \vec{k}_f$. Consequently, matrix elements proportional to $1/A$ can in general be neglected; however, $1/A$ terms in the phase, although small, can produce significant effects.

With this approximation, B_i and B_f depend only on $\vec{r}$, and the amplitude of Eq. (1) becomes

$$T_{if} = \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} B_i(\vec{r}) B_f^*(\vec{r}) G_{if}(\vec{r}), \qquad (2)$$

where the transfer function is

$$G_{if}(\vec{r}) = \int d\vec{r}'\, u_f(\vec{r}') V_{ac}(\vec{r}+\vec{r}') u_i(\vec{r}+\vec{r}') e^{i\vec{p}\cdot\vec{r}'}, \qquad (3)$$

and includes a recoil factor $e^{i\vec{p}\cdot\vec{r}'}$ with $\vec{p}$ given by

$$\vec{p} = -[\vec{k}_i/A_i + \vec{k}_f/A_f]. \qquad (4)$$

Consider now the simple case in which all spin effects are neglected and (i) the single particle states u_i, u_f may be represented by harmonic oscillator wave functions; (ii) in the initial state, c has zero orbital angular momentum relative to a, and in the final state it has orbital angular momentum L with projection M; and (iii) the finite range potential V_{ac} is a Gaussian.[9] The evaluation of $G_{if}^{LM}(\vec{r})$ by the standard techniques requires the expansion of each of the three functions of the integral, $u_f^{L,M}(\vec{r}')$, $\bar{u}_i(\vec{r}+\vec{r}') \equiv V_{ac}(\vec{r}+\vec{r}')u_i(\vec{r}+\vec{r}')$, and $e^{i\vec{p}\cdot\vec{r}'}$, in spherical harmonics. If the z axis for the integral over $\vec{r}'$ is taken along $\vec{p}$, and the integral over the azimuthal angle is performed, one obtains

$$G_{if}^{LM}(\vec{r}) = \sum_{l'} a_{l'}^{LM}(r) Y_{l'}^{M}(\Omega_{pr}), \qquad (5)$$

with

$$a_{l'}^{LM}(r) \propto \sum_l \int_0^\infty dr' r'^2 u_f^L(r') j_l(pr') \bar{u}_i^{l'}(r,r') \times \int_{-1}^{1} d\mu\, P_l^0(\mu) P_L^M(\mu) P_{l'}^{-M}(\mu). \qquad (6)$$

The product of $a_{l'}^{LM}(r)$, which decreases rapidly with increasing r, and $B_i B_f^*$ limits the radial integral over r to the surface region. To illustrate the effect of $G_{if}^{L,M}$ in Eq. (2), we assume for the next few paragraphs that the surface region may be represented approximately by a delta function $\delta(r-R_0)$. The angular dependence in the boundary conditions $B_i B_f^*$ limits the integral over the solid angle in $\vec{r}$ space to an annular region which can be approximated by the ring-locus model.[6,7] If the z axis in $\vec{r}$ space is chosen in the $\vec{p}$ direction, the general adiabatic conditions[10] require that the ring locus of annulus lie in a plane perpendicular to $\vec{p}$, or at $\theta=\pi/2$, where θ is the polar angle. Thus, as in the Blair-Dar theory, the spherical harmonic $Y_{l'}{}^{M}(\Omega_{pr})$ of Eq. (5) is evaluated at $\theta=\pi/2$, and the angular integral depends only on the azimuthal angle. The expression obtained finally for the amplitude is

$$T_{if}{}^{LM} \propto \sum_{l'} J_M(qR_0) P_{l'}{}^{M}(\pi/2) a_{l'}{}^{LM}(R_0), \qquad (7)$$

where J_M is the cylindrical Bessel function of order M.

The amplitude of Eq. (7) includes the usual Blair-type phase rules as special cases. First, it is evident that in the zero-range approximation, $V_{ac}(\vec{r}+\vec{r}\,')=\delta(\vec{r}+\vec{r}\,')$, the spherical harmonic $Y_L{}^M(\Omega_{r'})$ appearing in $u_f{}^{LM}(\vec{r}\,')$ of Eq. (3) becomes $Y_L{}^M(\Omega_r)$; and that the $\vec{r}$ integration immediately yields the usual phase rules for the cross section, since $L+M$ must be even on the ring. Secondly, even with finite-range potentials, if the recoil terms proportional to $1/A$ are neglected, the recoil momentum $\vec{p}$ vanishes. Then the radial integral in Eq. (6) is zero unless $l=0$ [due to the term $j_l(0)$], and consequently, the angular integral in Eq. (6) yields $\delta_{L,l'}$. Eq. (7) becomes

$$T_{if}{}^{LM} \propto J_M(qR_0) P_L{}^{M}(\pi/2), \qquad (8)$$

which, as in the zero-range theory, also obtains the usual phase rules. Thus, in either the zero-range approximation or the approximation that neglects terms $\sim 1/A$ in the phase, the model predicts strong oscillations with L-dependent phases, in striking disagreement with the experimental data.

However, for the case described by the conditions (i), (ii), and (iii), we have calculated the coefficients $a_{l'}{}^{L,M}(r)$ in Eq. (6) by numerical integration without neglecting the recoil momentum. This calculation yields nonvanishing coefficients $a_{l'}{}^{L,M}(r)$ for all l'. The important thing to note is that the restriction that $(l'+M)$ be even, due to the factor $P_{l'}{}^{M}(\pi/2)$, does not require that $L+M$ be even. In fact, in the cases considered, all substates, $-L \leq M \leq L$, make nearly equal contributions to the cross section, when $1/p$ is of the order of the range a of the bound-state wave functions. As a result the oscillatory nature of the angular distribution

$$d\sigma_L(\theta)/d\Omega = \sum_M |T_{if}{}^{L,M}|^2$$

disappears.

For $L>1$ and q large, the transfer function $G_{if}{}^{L,M}$ of Eq. (2) can be evaluated in closed form, and the resulting angular distribution is

$$\frac{d\sigma_L}{d\Omega} \xrightarrow[q\gg 1/a]{} q^{-3}\exp(-p^2a^2/6), \qquad (9)$$

in agreement with the results of the numerical computation. The details of this evaluation are somewhat involved and will appear in a separate publication. For $pa \lesssim 1$ the exponent in Eq. (9) is very small, and the variation of σ with q is independent of L, R_0, and any other nuclear parameters.[11] These predictions are in qualitative agreement with the available data which are compared with the q^{-3} prediction of the theory in Figs. 1(a) and 1(b). The difference in slope for alpha-particle transfer [Fig. 1(c)] is also predicted by our model and is due to the larger value of p in Eq. (9).

A further prediction of the model in qualitative agreement with recent experiments is that of the inhibition of population of low-angular-momentum states in the final nucleus.[1,3] Such an effect is to be expected classically since the transferred particle c has a fraction $1/A_i$ of the linear momentum in the entrance channel. When c is captured by b near the nuclear sur-

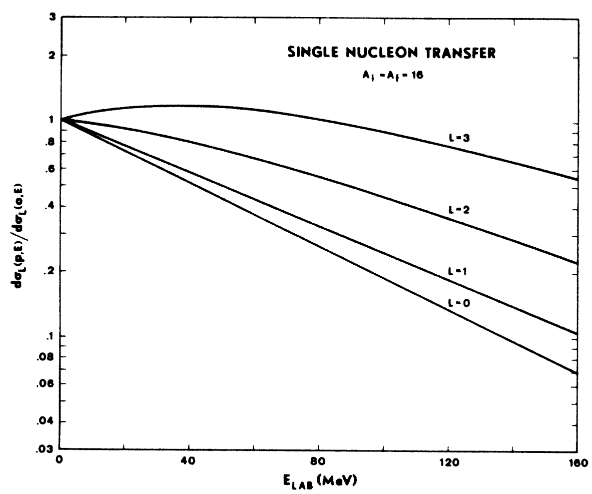

FIG. 2. Ratios of the cross sections predicted by the model with recoil to those obtained by neglecting recoil ($p=0$) are plotted as functions of the incident energy for a typical case of single-nucleon transfer. The cross section for small L transfers are inhibited at high energies.

face R_b, an angular momentum $L_c = k_i R_b / A_i$ is given the final nucleus. Consequently, one would expect to find states of angular-momentum transfer $\approx L_c$ populated with high probability. Figure 2 shows the ratio of the predicted cross section with recoil to that with no recoil ($p = 0$) for a typical case.

It should be emphasized that our model for scattering waves, $\chi_i^{(+)}$ and $\chi_f^{(-)}$, represents an extreme case. The one-parameter ring-locus model obtains the maximum diffraction oscillations in the cross section, and more complicated models for $\chi_1^{(+)}$ and $\chi_f^{(-)}$ are expected to produce smoother distributions. In addition, the introduction of a diffuse surface in the absorption region will cause the angular distribution to fall off faster with q than the sharp-cutoff model.[12] Consequently, the q^{-3} dependence also represents an extreme case for the average dependence of σ on q. It is apparent that the predictions of the model are not particularly sensitive to the scattering wave functions,[9] and that the principal effect of recoil and finite-range potentials is to modify the transfer function, $G_{if}^{LM}(\vec{r})$ in Eqs. (5) and (6).

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[9]The results shown in Figs. 1(a)-1(c) were calculated with the ranges of the initial and final bound-state wave functions equal both to each other and to the potential range, and were taken to be $r_0 A^{1/3}$ where $r_0 = 1.5$ F. The angular distribution falls off like q^{-3} for all reasonable values of the range parameters. In the cases considered, the independence of the monotonic angular distributions on the angular momentum leads to the generalization that all transitions $L' \to L$ will have the same monotonic behavior. The experiments cited represent mixtures of several values of transferred angular momentum, each of which should give the same monotonic dependence on q.

[10]Generalized adiabatic conditions for rearrangement scattering are taken to mean $\vec{k}_i = \vec{k}_f$ and $A_i = A_f$, so that $\vec{p} \cdot \vec{q} = 0$.

[11]This insensitivity of the angular distribution to any nuclear parameters (except perhaps the nuclear surface thickness) may be of some consequence in the proposed uses of high-energy heavy-ion accelerators. If our predictions are borne out further, then it appears that transfer reactions at high energy provide little identifiable nuclear-structure information.

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PRODUCTION OF VERY HEAVY ELEMENTS IN THERMONUCLEAR EXPLOSIONS—TEST BARBEL*

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The possibility of production of very heavy elements by multiple neutron capture in man-made thermonuclear systems was first demonstrated in the Mike explosion, in which exposure of U^{238} led to creation of nuclides as heavy as mass 255.[1] Unfortunately, the disposition of fuel and target material in the device greatly complicated attempts[2-4] to interpret the mass-abundance data in terms of fundamental nuclear parameters (i.e., a single-valued neutron flux and an appropriate set of cross sections).

Two recent low-yield underground test ex-