tion that determines the radius of the fast-proton plasma. Figure 2 is an example of the correlation of these losses with magnetic-probe signals (dB_Z/dt) near the proton-cyclotron frequency.

The magnetic probe signals are dominated by a microinstability having two distinct modes. One, termed the gyrofrequency mode, generates signals at frequencies (≈15 Mc/sec) appropriate for proton-cyclotron rotation near the equilibrium orbit. The radio-frequency signals from the second mode, termed the diminished frequency mode, are lower by 1-2 Mc/sec. Most of the losses are due to the gyrofrequency mode.

We have not been able to make unique mode assignments for the microinstabilities. The negative mass,[10] drift cyclotron ($k_{11} \neq 0$),[11] maser ($k_{11} \neq 0$),[12] and Harris[13] ($k_{11} \neq 0$) instabilities are the theoretically recognized possibilities.

A complete report of the microinstability limitations of this plasma is in preparation.

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†Deceased.

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PHASE COHERENCE AND STABILITY OF HELIUM II IN NARROW CHANNELS*

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There has been considerable interest recently in exploring analogies between superconductivity and superfluidity. In particular, Richards and Anderson[1] have been able to observe the analog of the ac Josephson effect at a small orifice connecting two baths of helium II. The purpose of this communication is to study the flow of superfluid through narrow channels, narrow in the sense that the normal fluid is effectively clamped by its viscosity.

Consider first the situation illustrated in Fig. 1 where the flow of the saturated film is considered as one example of a narrow channel. Beliaev[2] has shown that the order parameter for

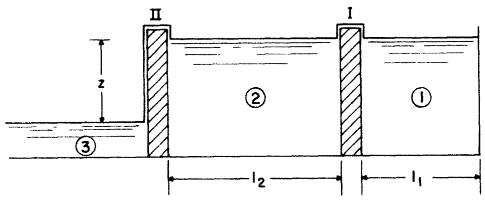

FIG. 1. Sketch of a three-bath experiment with helium II. The analog of the dc Josephson effect occurs at film junction I, and the ac effect occurs at II.

the background fluid contains a phase factor $\exp(-i\mu t/\hbar)$, where μ is the chemical potential. Thus the superfluid behaves, by analogy with quantum electrodynamics, like an external field with frequency $\omega = \mu/\hbar$. We can appreciate this identification of μ by supposing that baths 2 and 3 are in equilibrium, with bath 2 at a temperature ΔT higher than bath 3. The condition for equilibrium is $\Delta\mu = 0$, which leads at once to the relation for the thermomechanical pressure

$$\Delta p/\Delta T = \rho S \tag{1}$$

(to first order in the differences), S being the entropy per gram. Thus the levels will be separated by a distance z given by $\rho g z = \rho S \Delta T$. The generalization of (1) to include the effects of rotation and internal motion of the normal and superfluid components will be considered elsewhere.

Now let us suppose that under isothermal conditions we arrange a steady flow from bath 1 to bath 2 to bath 3. The levels in baths 1 and 2 will coincide exactly (otherwise superfluid would transfer back from bath 2 to bath 1), and the velocity of superflow through film junction I would be

$$v_1 = (l_1/l_2)v_c \quad (l_1 < l_2), \tag{2}$$

where the flow through junction II is at the critical velocity v_c. The three-bath arrangement is analogous to a constant-current source in electricity, and the subcritical flow through I is not in internal equilibrium. An experimental realization of this situation was devised by Daunt and Mendelssohn: the ingenious double-beaker experiment.[3] The situation just described is analogous to the dc Josephson effect—a current flow without potential drop.

What properties can we ascribe to a film junction? In the absence of a potential difference we may suppose the film to act like a two-di-

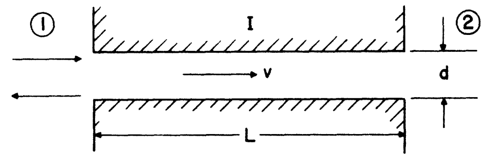

FIG. 2. Sketch of a narrow channel with supercurrent at velocity v.

mensional slit of thickness d ($\approx 3\times10^{-6}$ cm), as shown in Fig. 2. Feynman[4] has shown that the wave function describing the flow of superfluid from left to right will be $\psi = \Psi \exp(i\vec{k}\cdot\vec{r}) = \Psi \exp(i\varphi)$, and the current density will be

$$J = (\hbar/m)\Psi\Psi^*\nabla\varphi = (\hbar/m)\rho_S\nabla\varphi, \tag{3}$$

where the phase acts as a velocity potential for the flow. Potential flow can occur in the channel because the wave function for the superfluid approaches zero on the walls of the slit in a "healing length" of the order of a few angstroms[5]; thus the superfluid slips at the walls because $\psi \to 0$, and not because $v \to 0$. Neglecting end effects (of order d),[6] the current density in I and phase difference between 1 and 2 are related by

$$J = \rho_S v = \rho_S(\hbar/m)(\varphi_1 - \varphi_2)/L. \tag{4}$$

(We shall observe shortly that there is an upper limit on v.) Thus the film junction has the property that the phase is coherent throughout. If the film is ruptured and subsequently reunited, the phases will again be locked by emission of phonons to eliminate the accompanying density fluctuation.[7] In contrast to the Josephson junction, there is no region where superfluidity does not occur, and hence the phase remains a real number throughout. The same considerations apply to channels or filter materials until the dimension d approaches the order of the healing length where presumably superfluid "tunnelling" could occur and $J \propto \cos\Delta\varphi$ as in Josephson tunnelling.[6,8] Such a material would exhibit a reduction in the λ point.[9]

Finally, let us consider the flow through junction II when the chemical potentials of the baths are different. Under isothermal conditions $\mu_2 = \mu_3 + mgz$, and the coupling through the film induces a beat frequency between baths, given by

$$\omega = \Delta\mu/\hbar = mgz/\hbar. \tag{5}$$

Richards and Anderson[1] interpret this beat frequency as a phase slippage between baths which produces a quantized vortex line every time the phase changes by 2π. Thus vortices should be produced as suggested in Fig. 3 at a rate

$$\nu = gz/2\pi(\hbar/m), \tag{6}$$

which for $z = 1$ cm corresponds to 980 kc/sec. These vortices should slow down principally by collisions with rotons, radiating second sound at frequency ν as they do. This is the analog of the ac Josephson effect. Since the condition (6) refers only to average frequency, Richards and Anderson[1] have observed the effect by synchronization with a quartz oscillator.

We can show that the ac Josephson effect has a familiar consequence. If the energy of a vortex line is ϵ erg per cm, a film of width W will produce vortices consuming $W\epsilon\nu$ erg/sec. This energy must be supplied by the potential energy gain of the descending superfluid, $\rho_S v_c dWgz$. Estimating ϵ as[4]

$$\epsilon = \rho_S \pi(\hbar/m)^2 \ln d/a, \tag{7}$$

where a is ≈ 1 Å, we equate the two power estimates to find

$$J_c = \rho_S v_c = (\rho_S/2d)(\hbar/m)\ln(d/a), \tag{8}$$

giving a value of v_c independent of z and T. Relation (8) gives $v_c d = 4.52\times10^{-4}$ cm^2/sec, which is to be compared with the data of Snyder and Donnelly[10] who give $v_c d = 9.65\times10^{-5}$ cm^2/sec. The difference probably lies in the estimate of ϵ, which should be corrected for the effects of the wall. Equation (8) gives a physically reasonable explanation for the critical transfer rate and coincides with Feynman's expression.[4] The independence of $\Delta\mu$ in (8) means that (fourth) sound cannot be transmitted through such a channel or filter material when a steady chemical potential difference prevails: The effect will be to frequency-modulate the rate of vortex formation.

Let us search further for an interpretation of (8). The deBroglie wavelength of the superfluid advancing through the channel is $\lambda = 2\pi(\hbar/m)v^{-1}$. Low velocities correspond to long wavelengths in the channel. As v increases, λ becomes shorter until a critical minimum wavelength is reached, given by (8),

$$\lambda_c = 4\pi d/\ln(d/a), \tag{9}$$

which is of the order of d. One can now see

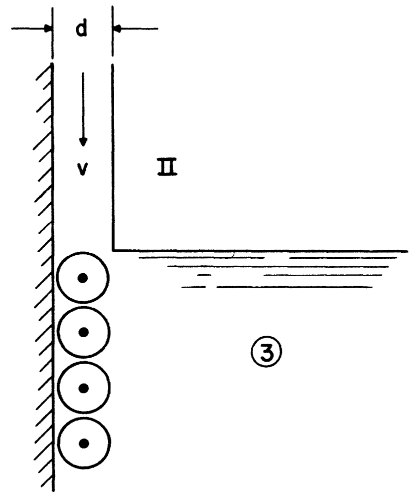

FIG. 3. An interpretation of the production of quantized vortices where film II meets bath 3. The vortices may subsequently fill the film.

the origin of the critical velocity: When the deBroglie wavelength becomes of order d, disturbances are able to cause sections of fluid of different phase to curl together forming vortices of cross section $\sim d^2$. These vortex "rolls" may feed back from the exit to fill the entire length of the channel. Such characteristic vortex formations in classical hydrodynamics have been extensively studied by the author.[11] They involve a transition from two- to three-dimensional flow. We have here, then, an instance of quantum hydrodynamic instability occurring when the dimensionless parameter D reaches a critical value given by

$$D_c = v_c d/(\hbar/m) = \tfrac{1}{2}\ln d/a. \tag{10}$$

Such a quantum Reynolds number was introduced by the author earlier.[12] Presumably the state of marginal stability can be sought through a perturbation analysis of the nonlinear self-consistent field equation.[5] This should yield a better value of D_c than the energy argument given above.

With wider channels the situation is complicated by flow of both normal and superfluid components. Chandrasekhar and Donnelly[13] have shown how, in Couette flow, instabilities may arise in the superfluid and at higher speeds in the normal fluid, and qualitative support for their calculations was obtained by Donnelly.[14] Both these instabilities are preceded by the quantum hydrodynamic instability which introduces quantized vortices into the superfluid to establish the initial stationary shear flow.

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FLOW VISUALIZATION IN He II: DIRECT OBSERVATION OF HELMHOLTZ FLOW*

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It has recently been suggested[1] that in the flow of He II about obstacles, such as Rayleigh disks and airfoils, perfect potential flow does not invariably obtain, and flow separation of the type discovered by Helmholtz[2] commonly occurs. In this Letter we report experiments verifying this suggestion by means of direct observation of the motion of particles of frozen H-D gas about obstacles placed in a bath of rotating He II.

One of the most unambiguous methods of studying the nature of hydrodynamic flow is by introducing into the flow tracer particles which follow trajectories known as path lines. In steady-state flow, path lines are identical with streamlines. Dyes and gas bubbles are typically used in classical studies, but these techniques are impractical in He II. An alternative approach is the use as tracers of small particles of density equal to that of He II. Such particles are unaffected by gravity and remain suspended for long periods. An excellent material for this purpose is frozen H-D gas, which was originally used by Chopra and Brown[3] in studies of acoustic streaming.

In our experiments He II was brought into rotation inside a horizontally mounted cylinder (length 2.9 cm, diameter 2.3 cm) of wire mesh which was driven from outside the cryostat. One end of the cylinder was closed with an optically blackened cover and the other with a glass plate, thereby permitting viewing along the cylinder axis. H-D gas could be introduced above the He level. Upon freezing, particles of 20-100 μ diameter were found, some of which