lips and Rarita[12] have recently constructed a Regge-pole model which fits the $\pi^{\pm}p$ differential cross-section difference by including a substantial spin-flip contribution. The other alternative is to introduce a cut in the J plane for the amplitude A_0 as suggested by Mandelstam.[13]

I would like to thank Dr. I. Mannelli for pointing out that the πp charge-exchange and elastic scattering data necessitate a strong t dependence of $R_0(t)$ and that this may lead to a contradiction of existing theories. I would like to thank Dr. M. Wahlig for his constant help. I also wish to thank my supervisor Professor F. E. Low for his guidance in this work.

---

*This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-2098.

[1]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 425 (1963).

[2]A. Ahmadzadeh and I. A. Sakmar, Phys. Rev. Letters 11, 439 (1963).

[3]T. D. Binford and B. R. Desai, "High-Energy Elastic Scattering at Low Momentum Transfers" (to be published).

[4]R. J. N. Phillips and W. Rarita, Phys. Rev. Letters 14, 502 (1965).

[5]K. Igi, Phys. Rev. Letters 9, 76 (1962) and Phys. Rev. 130, 820 (1963).

[6]By s and t we denote the usual Mandelstam variables for the square of the center-of-mass energy and the four-momentum transfer, respectively. The energy of the pion in the lab, E, is related to s by the formula $s = 2ME + M^2 + \mu^2$, where M and μ are the nucleon and pion mass, respectively. The $\pi^{\pm}p$ differential cross sections are represented by $d\sigma^{\pm}/dt$. The function D is really a function of both E and t, but since $d\sigma^{\pm}/dt$ shows such little E dependence, we shall assume that D is only a function of t.

[7]R. K. Logan, Phys. Rev. Letters 14, 414 (1965).

[8]I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and F. Sodickson, Phys. Rev. Letters 14, 408 (1965).

[9]S. J. Lindenbaum, Proceedings of the International Conference on High-Energy Physics, Dubna, 1964 (to be published). Note: These ratios were actually measured for $-0.01 \le t \le -0.004$.

[10]The strong t dependence of $R_0(t)$ was first pointed out by I. Mannelli and mentioned in reference 8.

[11]F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters 9, 183 (1962).

[12]R. J. N. Phillips and W. Rarita, Bull. Am. Phys. Soc. 10, 460 (1965). I would like to thank Dr. Phillips for showing me a prepublication version of this work.

[13]S. Mandelstam, Nuovo Cimento 30, 1127, 1148 (1963).

PION-MASS MEASUREMENT BY CRYSTAL DIFFRACTION OF MESONIC X RAYS*

Robert E. Shafer, Kenneth M. Crowe, and David A. Jenkins

Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 5 May 1965)

We report here the recent measurement at the 184-in. cyclotron of the $4F$-$3D$ transitions in π-mesonic calcium and titanium, using a 7.7-m bent-crystal spectrometer,[1,2] and present the preliminary results of a new determination of the charged pion mass.

The experimental arrangement is shown in Figs. 1 and 2. The π^- beam, produced on an internal target, was extracted at 185 MeV/c with a 60% macroscopic duty cycle, and transported to the bent-crystal target. Pions stopping in the target were identified by a six-counter telescope. Typical stopping rates were 800 pions/g sec for the 4-g calcium and 6-g titanium targets. The bent-crystal spectrometer, built on the DuMond geometry (focus at a stationary target),[3] has about a 1×10^{-6} effective fractional solid angle for the mesonic x rays emanating from the target. A fast coincidence between the pion telescope and a 7- by 7- by $\frac{1}{4}$-in. NaI(Tl) crystal behind the spectrometer identified a "real" event, which gated a pulse-height analyzer storing the integrated NaI pulses. The spectrometer was rotated to scan alternately the right and left first-order diffraction peaks.

The data (Fig. 3) were analyzed by a least-squares minimization method, using a known calculated line shape of arbitrary height above an arbitrary flat background. Analysis indicated a counting rate of about two events per hour above a three-per-hour random background. The spectrometer was calibrated by using the $K\alpha_1$ x ray (52.389 ± 0.001 keV)[4] and the nuclear γ ray (84.261 ± 0.003 keV)[5] of Yb^{170}, yielding a quartz (310) d_{18} spacing of 1177.54 ± 0.05 xu.

The measured transition energies and the

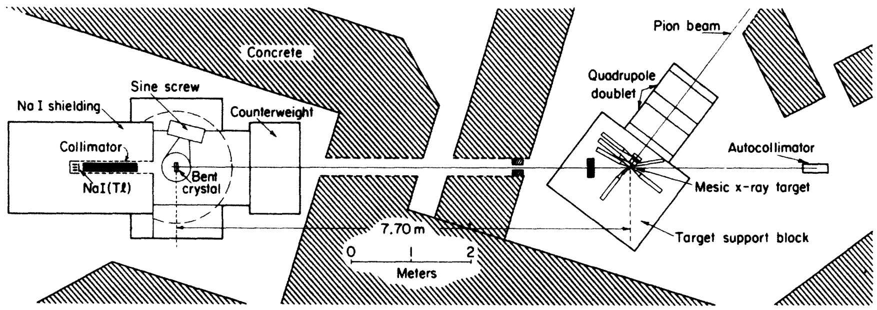

FIG. 1. Experimental arrangement. The autocollimator line of sight was held to ±0.002 in. throughout the experiment. Considerable shielding was required around the NaI detector because of cyclotron-generated neutrons.

results of calculations of these transitions are shown in Table I. As pions have no spin, the Klein-Gordon equation is used. The vacuum polarization[6,7] has been checked to 1 % in the hydrogen atom[8] and 3 % in muonic phosphorus.[9] The strong-interaction shift is calculated by scaling the measured shift in the $3D$-$2P$ π-mesonic aluminum transition[2] to the present cases by first-order perturbation theory using an optical model to represent the interaction. Some studies have been made on the Z, $\vec{T}$, and effective-mass dependence in the nuclear interaction in π-mesonic atoms,[10] but as the interaction is not well understood and the experimental data are scarce, an error of ±100% is being assigned.

The quantum-electrodynamic corrections not included here can be estimated from the energy-level calculation for the $3D$-$2P$ muonic phosphorus transition.[11] The largest remaining correction is expected to be the fourth-order vacuum polarization (~+2 eV). Wave-function corrections to the second order are about +1 eV.[12] The finite-nuclear-size effect on the Coulomb potential is less than 0.1 eV. We use here an all-inclusive (3 ± 3)-eV correction to include these effects.

The weighted average of the two measurements in Table I is 139.580 ± 0.015 MeV,[13] in agreement with the presently accepted value of 139.60 ± 0.05 MeV. The latter value was derived by combining the muon mass[9] with the measured muon momentum from pion decay at rest,[14] assuming that the muon neutrino has zero mass. By comparing these measurements, we find

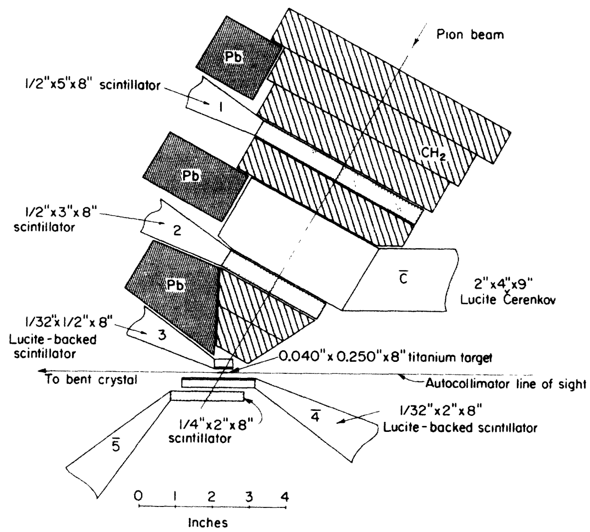

FIG. 2. Counter telescope. Logic $12\bar{C}\bar{3}\bar{4}\bar{5}$ identified a stopping pion.

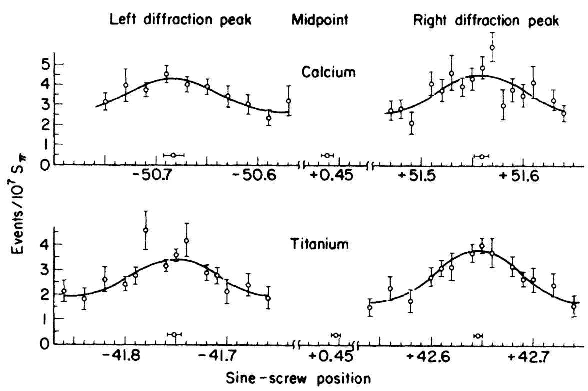

FIG. 3. The right and left first-order diffraction peaks for Ca and Ti, plotted as events per 10^7 stopped pions versus sine-screw turns. The smooth curve represents the best fit as described in the text, yielding (for all curves combined) $\chi^2/N-1=31/41$. Horizontal flags represent the calculated means and their standard deviations. The two midpoints differ by 0.009 ± 0.007 turn (20% probability). The standard deviation of the spectrometer resolution is 60 eV for Ca and 90 eV for Ti.

Table I. Calculations of the $4F$-$3D$ transition energies using $m_\pi c^2 = 139.580$ MeV.

Effect	Calcium	Titanium
Klein-Gordon equation[a] (keV)	72.388 ± 0.001	87.622 ± 0.001
Reduced mass (keV)	−0.270	−0.273
Vacuum polarization (second order) (keV)	+0.228	+0.298
Orbital-electron screening (keV)	−0.001 ± 0.001	−0.001 ± 0.001
Strong-interaction shift (keV)	+0.002 ± 0.002	+0.004 ± 0.004
Other quantum electrodynamics (keV)	+0.003 ± 0.003	+0.003 ± 0.003
Transition energy (calc.) (keV)	72.350 ± 0.004	87.653 ± 0.005
Transition energy (meas.) (keV)	72.352 ± 0.009	87.652 ± 0.009
Natural linewidth (eV)	~6	~10
Calculated π^- mass (MeV)	139.584 ± 0.020	139.578 ± 0.017

[a] $\alpha^2 = 5.32492 \times 10^{-5} \pm 9$ ppm [Phys. Today 17, No. 2, 49 (1964)].

the most probable (real) value of the muon neutrino mass to be zero, with upper limits of 2.1 MeV (68% confidence level) and 2.8 MeV (90% confidence level).

The authors are indebted to Professor Robert Karplus and Professor Emilio Segrè for their helpful discussions on the energy-level calculations, and to Professor Felix Boehm for his suggestions on the calibration of the spectrometer. The authors are also grateful to Dr. A. Astbury, Dr. J. P. Deutsch, and Dr. R. E. Taylor for their contributions to earlier phases of the mesonic x-ray program.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]The spectrometer and experiment will be described in detail by R. E. Shafer, thesis (to be published).

[2]A. Astbury, J. P. Deutsch, K. M. Crowe, R. E. Shafer, and R. E. Taylor, Compt. Rend. Congr. Intern. Phys. Nucl. 2, 225 (1964).

[3]J. W. M. DuMond, Ann. Rev. Nucl. Sci. 8, 163 (1958), gives a general review of bent-crystal spectrometry.

[4]J. A. Bearden, U. S. Atomic Energy Commission Report No. NYO-10586, 1964 (unpublished). This author quotes deviations in probable errors.

[5]E. N. Hatch and F. Boehm, Z. Physik 155, 609 (1959), quote 84.26 ± 0.01 keV; I. Marklund and B. Lindstrom, Nucl. Phys. 40, 329 (1963), quote 84.260 ± 0.004 keV. The calibration line of the latter has been adjusted to Bearden's value (reference 4). F. Boehm (private communication) has obtained 84.261 ± 0.003 keV.

[6]S. Koslov, Nevis Cyclotron Laboratory Report No. 19, 1956 (unpublished).

[7]A. B. Mickelwait, thesis, Carnegie Institute of Technology, 1954 (unpublished); A. B. Mickelwait and H. C. Corben, Phys. Rev. 96, 1145 (1954).

[8]R. P. Feynman, in Solvay Institute, 12th Physics Conference Proceedings (Interscience, New York, 1961), p. 61.

[9]G. Feinberg and L. M. Lederman, Ann. Rev. Nucl. Sci. 13, 431 (1963).

[10]M. Ericson, Compt. Rend. 257, 3831 (1963).

[11]A. Petermann and Y. Yamaguchi, Phys. Rev. Letters 2, 359 (1959).

[12]R. Glauber, W. Rarita, and P. Schwed, Phys. Rev. 120, 609 (1960).

[13]All errors are quoted here in standard deviations. When a reference has used another deviation, it has been changed assuming a normal error curve.

[14]W. H. Barkas, W. Birnbaum, and F. M. Smith, Phys. Rev. 101, 778 (1956), quote deviations in probable errors.

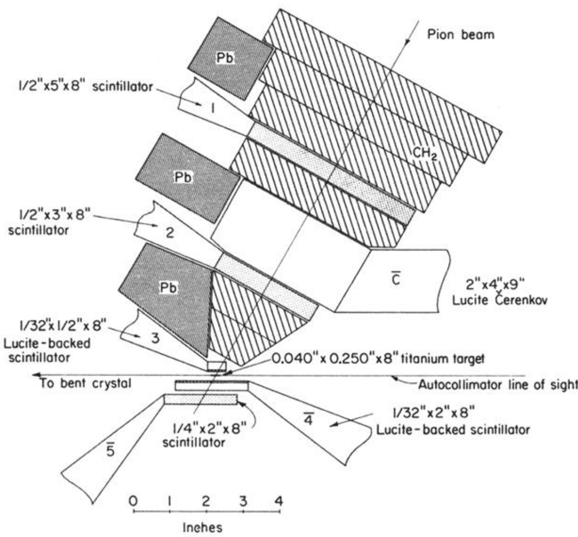

FIG. 2. Counter telescope. Logic $12\bar{C}3\overline{45}$ identified a stopping pion.