gy region 600-1000 MeV by the $I=\frac{1}{2}$ state, it is sufficient to fit the present existing data with one set of parameters R and α correspondingly to the $I=\frac{1}{2}$ scattering amplitude.

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POSSIBLE DISCREPANCY BETWEEN THE REGGE-POLE HYPOTHESIS AND THE DIFFERENCE OF THE π^+p AND π^-p DIFFERENTIAL CROSS SECTIONS*

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(Received 6 April 1965)

High-energy $\pi^\pm p$ elastic scattering data[1] have been recently analyzed[2-4] in terms of the exchange of two Regge poles. The amplitudes for $\pi^\pm p$ elastic scattering, $A^\pm$, are related to A_0 and A_1, the amplitudes for the exchange in the crossed channel of isotopic spin $I=0$ and $I=1$, respectively, by the relation

$$A^\pm = A_0 \mp A_1. \tag{1}$$

The amplitude A_0 in these analyses was assumed to be dominated at high energy by the exchange of the Pomeranchuk pole, P, and Igi's[5] second Pomeranchuk pole, the P'. The amplitude A_1, on the other hand, was ignored because the difference between the π^-p and π^+p cross sections was small compared with the cross sections themselves. Good fits based on these assumptions were obtained for the mean of π^+p and π^-p scattering. The difference, $D(t) \equiv d\sigma^-/dt - d\sigma^+/dt$,[6] which depends on both A_0 and A_1, was not considered. A recent study[7] of high-energy π^-p charge-exchange data[8] indicates that the amplitude A_1 is dominated by the exchange of a single ρ Regge pole. Making use of this result and other experimental information, we shall show that the simple form of A_0, using two poles and no spin dependence, is inconsistent with the t dependence of $D(t)$.

Although $D(t)$ is not well known for all values of t (it is small and often of the same order of magnitude as the experimental errors), one essential feature does emerge.[8] $D(t)$ is positive at $t=0$ and becomes negative as $|t|$ increases, due to the fact that the π^-p diffraction peak is narrower than the π^+p peak.[1] Examination of the fits[1] of the $\pi^\pm p$ differential cross section to the form $\exp(a+bt+ct^2)$ reveals that this crossover occurs at $t=-0.12 \pm 0.10$ $(\mathrm{GeV}/c)^2$ in the energy range 6-18 GeV. Using the relationship $d\sigma^\pm/dt = (16\pi/s^2)|A_0 \mp A_1|^2$, we find that

$$D(t) = (16\pi/s^2)\,\mathrm{Im}A_1\,\mathrm{Im}A_0(1+R_0R_1), \tag{2}$$

where $R_i(E,t) = \mathrm{Re}A_i/\mathrm{Im}A_i$. Since neither $\mathrm{Im}A_0$ nor $\mathrm{Im}A_1$ goes to zero at $t=-0.12$, the condition $D(-0.12)=0$ implies that $1+R_0R_1=0$, and therefore that

$$R_0(t=-0.12) = \frac{-1}{R_1(-0.12)}. \tag{3}$$

Since we have assumed A_1 is dominated by a single ρ Regge pole, R_1 has no energy dependence, and hence the above expression for R_0 holds at all energies. We find that $R_0(-0.12) = -0.85$ by evaluating $R_1(-0.12)$ using the results of the π^-p charge-exchange analysis in which it was found that the data were consistent with the exchange of a single ρ Regge pole. The trajectory $\alpha_\rho(t)$ of this Regge pole was found to be $0.6+(0.4\pm0.4)t$ for $t<0$, giving $R_1(-0.12)=1.18$. R_0 at $t=0$, on the other hand, may be evaluated by making use of the recent measurements[9] of $R_\pm(0) = \mathrm{Re}A^\pm(0)/\mathrm{Im}A^\pm(0)$ obtained by observing the interference of the nuclear and Coulomb parts of the $\pi^\pm p$ scattering. Since $A^\pm = A_0 \mp A_1$ and $\mathrm{Im}A_0 \gg \mathrm{Im}A_1$, we may approximate

$$R_\pm = \frac{\mathrm{Re}A_0 \mp \mathrm{Re}A_1}{\mathrm{Im}A_0 \mp \mathrm{Im}A_1} \text{ by } \frac{\mathrm{Re}A_0}{\mathrm{Im}A_0} \mp \frac{\mathrm{Re}A_1}{\mathrm{Im}A_0} \pm \frac{\mathrm{Re}A_0\,\mathrm{Im}A_1}{(\mathrm{Im}A_0)^2},$$

and hence $R_0 \approx \frac{1}{2}(R_+ + R_-)$. Foley et al.[1] found that at $E=10$ GeV, $R_+(0) = -0.33 \pm 0.13$ and $R_-(0) = -0.23 \pm 0.18$; thus we may take $R_0(10,0) = -0.28 \pm 0.11$. We shall show that the strong t dependence[10] of $R_0(t)$ indicated by these two measurements at $t=0$ and $t=-0.12$ is inconsistent with the hypothesis that A_0 is dominated by P and P' exchange.

The amplitude, A_0, for the exchange of the

P and the P' is given by

$$A_0=\{-\cot[\tfrac{1}{2}\pi\alpha_P(t)]+i\}B_P(t)\exp[\alpha_P(t)\ln E]$$

$$+\{-\cot[\tfrac{1}{2}\pi\alpha_{P'}(t)]+i\}B_{P'}(t)\exp[\alpha_{P'}(t)\ln E], \quad (4)$$

where $\alpha_P, \alpha_{P'}$ and $B_P, B_{P'}$ are the trajectory and residue functions for the two Regge poles, respectively. Assuming for the sake of simplicity that $\alpha_P(t)$ and $\alpha_{P'}(t)$ are linear in t, we have

$$\alpha_P(t)=1.0+mt \text{ and } \alpha_{P'}(t)=0.5+m't, \quad (5)$$

where m and m' are constants. The condition $\alpha_P(0)=1.0$ is required to obtain constant cross sections at infinite energy, whereas the condition that $\alpha_{P'}(0)=0.5$ is required to explain the energy dependence of the total pp, $\bar{p}p$,[11] and πp[5] cross sections. Using Eqs. (4) and (5), we find that the ratio of the real to the imaginary part of A_0 is given by

$$R_0(E,t)=-\frac{B_P(t)\cot[\frac{1}{2}\pi(1+mt)]E^{1+mt}+B_P(t)\cot[\frac{1}{2}\pi(0.5+m't)]E^{0.5+m't}}{B_P(t)E^{1+mt}+B_{P'}(t)E^{0.5+m't}}. \quad (6)$$

Applying the condition that $R_0(10,0)=-0.28$ to this equation, we obtain the relationship that $B_{P'}(0)/B_P(0)=1.26$, in agreement with Igi's original estimates for $B_P(0)$ and $B_{P'}(0)$. Substituting this relation into Eq. (6) and setting $B_{P'}(t)/B_P(t)=[B_{P'}(0)/B_P(0)]X(t)$, we find that

$$R_0(10,t)=-\frac{\cot[\frac{1}{2}\pi(1+mt)]+0.4X(t)10^{(m'-m)t}\cot[\frac{1}{2}\pi(0.5+m't)]}{1+0.4X(t)10^{(m'-m)t}}. \quad (7)$$

Analysis of Eq. (7) reveals that we must take both m and m' as large as possible to satisfy the condition that $R_0(10,-0.12)=-0.85$. The nonshrinkage of the πp diffraction peak, on the other hand, requires that m and $m'\leq 0.5$ (GeV)$^{-2}$. Setting $m=m'=0.5$, we find that we must take $X(-0.12)=6.6$ to satisfy the condition that $R_0 \times(-0.12)=-0.85$. This value of X is much too large, however, to fit the π^+p and π^-p differential cross sections in terms of the exchange of the P, P', and ρ poles. A recent two-pole analysis,[3] for which the best fit to the data was obtained with $m=m'=0.41$ and

$$X(t)=\left(\frac{1-1.4t}{1-2.8t}\right)^{2.5},$$

indicates that one must take $X(t)<1$. We confirmed this by unsuccessfully trying to fit the $\pi^{\pm}p$ data with $m=m'=0.5$ and $X(t)=1$. In fact, our χ^2 was so large that $X(t)=1$ is a safe upper limit on the value of $X(t)$ for which one can obtain a fit to the $\pi^{\pm}p$ differential cross sections.

The discrepancy between experiment and the two-pole model for A_0 is established by demonstrating that the constraints placed on the parameters of the P and P' poles by the measurements $D(-0.12)=0$, $R_0(0)=-0.28$, and $R_1(-0.12)=1.18$ make a P, P' fit of the differential cross section impossible. Each of these measurements contains, of course, a certain amount of uncertainty due to experimental errors. We have established the bound within which these values may vary, however, without changing our results and found that the experimental error lies well within them. For example, if $D(t)$ goes to zero anywhere in the range $-0.80\leq t\leq 0$, then $X(t)$ is greater than unity, the upper limit on X for which one may obtain a decent fit to the data. Similarly, we find our results are not sensitive to the uncertainty in the value of $R_0(0)$, since $X(t)\geq 1$ for $R_0(0)\geq -0.67$. The t dependence of $R_0(t)$ is not as strong as before if we take $\alpha_\rho(t)=0.6$, the largest possible value of $\alpha_\rho(t)$ consistent with the π^-p charge-exchange data. We still find that $X(t)\geq 1$ if D goes to zero anywhere in the range $-0.40\leq t\leq 0$. Thus we see that if the amplitude A_1 is dominated by the exchange of a ρ Regge pole, the two-pole model for A_0 is inconsistent with experiment. If one introduces a third Pomeranchuk pole P'' for A_0, the above difficulties still persist.

There are two possible ways in which this difficulty may be overcome. One is to consider the contribution of the spin-flip amplitude, which has been ignored up to this point. In fact, Phillips and Rarita[12] have recently constructed a Regge-pole model which fits the $\pi^{\pm}p$ differential cross-section difference by including a substantial spin-flip contribution. The other alternative is to introduce a cut in the J plane for the amplitude A_0 as suggested by Mandelstam.[13]

I would like to thank Dr. I. Mannelli for pointing out that the πp charge-exchange and elastic scattering data necessitate a strong t dependence of $R_0(t)$ and that this may lead to a contradiction of existing theories. I would like to thank Dr. M. Wahlig for his constant help. I also wish to thank my supervisor Professor F. E. Low for his guidance in this work.

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*This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)-2098.

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PION-MASS MEASUREMENT BY CRYSTAL DIFFRACTION OF MESONIC X RAYS*

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(Received 5 May 1965)

We report here the recent measurement at the 184-in. cyclotron of the $4F$-$3D$ transitions in π-mesonic calcium and titanium, using a 7.7-m bent-crystal spectrometer,[1,2] and present the preliminary results of a new determination of the charged pion mass.

The experimental arrangement is shown in Figs. 1 and 2. The π^- beam, produced on an internal target, was extracted at 185 MeV/c with a 60% macroscopic duty cycle, and transported to the bent-crystal target. Pions stopping in the target were identified by a six-counter telescope. Typical stopping rates were 800 pions/g sec for the 4-g calcium and 6-g titanium targets. The bent-crystal spectrometer, built on the DuMond geometry (focus at a stationary target),[3] has about a 1×10^{-6} effective fractional solid angle for the mesonic x rays emanating from the target. A fast coincidence between the pion telescope and a 7- by 7- by $\frac{1}{4}$-in. NaI(Tl) crystal behind the spectrometer identified a "real" event, which gated a pulse-height analyzer storing the integrated NaI pulses. The spectrometer was rotated to scan alternately the right and left first-order diffraction peaks.

The data (Fig. 3) were analyzed by a least-squares minimization method, using a known calculated line shape of arbitrary height above an arbitrary flat background. Analysis indicated a counting rate of about two events per hour above a three-per-hour random background. The spectrometer was calibrated by using the $K\alpha_1$ x ray (52.389 ± 0.001 keV)[4] and the nuclear γ ray (84.261 ± 0.003 keV)[5] of Yb^{170}, yielding a quartz (310) d_{18} spacing of 1177.54 ± 0.05 xu.

The measured transition energies and the