PHONON-PUMPED MAGNON INSTABILITY IN AN ANTIFERROMAGNET*

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In this Letter, the theoretical first-order instability thresholds for phonon-pumped magnons in an unflopped antiferromagnet are given for the case of a longitudinal phonon pump[1] propagating parallel to the dc magnetic field. It appears that the critical values of elastic strain required for phonon parallel pumping can be extremely small for low-anisotropy antiferromagnets in which the magnon losses are reasonable.

Instability thresholds for photon-parallel-pumped magnons in a two-sublattice ferrimagnet have already been given.[2-4] The most efficient first-order process occurs when the pump frequency is the sum of two distinct magnon frequencies—one belonging to each of the two spin-wave branches—and requires that the sublattice g factors be unequal; the magnon frequencies and thresholds for such "exchange-pumped" magnons are particularly low in the vicinity of a spin-compensation point. In an antiferromagnet, spin compensation occurs for all temperatures less than the Néel temperature, and it is therefore particularly disappointing to contemplate the disappearance of "exchange pumping" and the consequent dramatic increase in threshold due to the equal g factors. One is quite naturally led to search for some means of restoring the coupling; what is required is a way of frequency modulating each sublattice by a different amount despite the fact that $g_1=g_2$. Although this appears to be impossible when a photon pump is used, it can in principle be accomplished by the employment of a phonon pump. Because of magnetoelastic coupling, the latter creates dissimilar time-varying torques that act upon the two sublattices. These torques can be expressed in terms of effective photon fields which carry the phonon energy and momentum[1]; with minor modification, the previously given photon-pump thresholds can be employed. Although the analysis has been carried out for a general ferrimagnet, we here specialize the solution to an unflopped antiferromagnet. The calculations are made within the molecular-field approximation for a cubic crystal magnetized along a principal axis and take into account the Zeeman, exchange, magnetic anisotropy, and magnetoelastic energies. To account for losses, phenomenological dissipation terms are expressed in terms of the spin-wave linewidths $2\Delta H_k(\omega_k, \vec{k})$.

The elastic displacement of the pump

$$\vec{\rho} = \vec{i}_z \rho_0 \cos(\omega_p t - \vec{k}\cdot\vec{r}) \tag{1}$$

propagates parallel to the magnetic field, $\vec{H}_{dc} = \vec{i}_z H_0$, with a velocity v_l. The magnetization vector of the ith sublattice is assumed to have the form

$$\vec{M}_i = \vec{m}_{i1}(\omega_{k1}t - \vec{k}_1\cdot\vec{r}) + \vec{m}_{i2}(\omega_{k2}t - \vec{k}_2\cdot\vec{r}) + \vec{i}_z M_i \quad (i=1,2). \tag{2}$$

Taken together, $\vec{M}_1$ and $\vec{M}_2$ represent a pair of traveling spin waves plus the dc magnetizations $M_1 = M + \frac{1}{2}\chi_{\parallel}H_0$ and $M_2 = -M + \frac{1}{2}\chi_{\parallel}H_0$, where $\chi_{\parallel}$ is the parallel susceptibility; the jth magnon wave vector is chosen to be

$$\vec{k}_j = k_j(\vec{i}_x \sin\psi_j + \vec{i}_z \cos\psi_j). \tag{3}$$

Since $\vec{i}_z$ is assumed parallel to either a $\langle 100\rangle$ or $\langle 111\rangle$ axis, the leading terms of the first-order magnetoelastic energy are of the form

$$U = [a(M_{1z}+M_{2z})^2 + 2bM_{1z}M_{2z}]\partial\rho_z/\partial z, \tag{4}$$

where the values of a and b are each expressible as a linear combination of the six fundamental constants for a cubic two-magnetic-sublattice crystal. Notice that the effective pumping field acting upon the ith sublattice is

$$\vec{h}_i = -\frac{1}{\mu_0}\frac{\partial U}{\partial M_{iz}}\vec{i}_z, \tag{5}$$

and that $\vec{h}_2 \approx -\vec{h}_1$. This implies that the two sublattices will be modulated out of phase; the pump is therefore maximally effective even though the sublattice g factors are equal. Consequently, the magnon pair can be parametrically "exchange-pumped" by the traveling elastic wave pump if the directions of magnon precession are opposite to one another and if energy and momentum are conserved;

$$\omega_{k1}+\omega_{k2}=\omega_p, \quad \vec{k}_1+\vec{k}_2=\vec{k}_p. \tag{6}$$

The first-order instability thresholds are obtained by the methods outlined previously,[3] and result in a critical strain, $(k_p\rho_0)$, given by

$$(k_p\rho_0)_{\rm crit}=\frac{\mu_0}{|b|M}\left\{\frac{2\Delta H_{k1}(\omega_{k1},\vec{k}_1)\Delta H_{k2}(\omega_{k2},\vec{k}_2)\omega_{01}\omega_{02}}{\omega_{A1}\omega_{A2}+\omega_{B1}\omega_{B2}-\omega_{01}\omega_{02}-[\frac{1}{2}(\chi_\parallel/\chi_\perp)\omega_H]^2}\right\}^{1/2}, \tag{7}$$

where

$$\omega_{Ai}=\omega_{\rm ex}+\omega_a+\lambda_{11}k_i^2\omega_M; \quad \omega_{Bi}=\omega_{\rm ex}+\lambda_{12}k_i^2\omega_M; \quad \omega_{0i}=\{\omega_{Ai}^2-\omega_{Bi}^2+[\tfrac{1}{2}(\chi_\parallel/\chi_\perp)\omega_H]^2\}^{1/2};$$

$$\omega_{ki}=\omega_{0i}\pm\left(1-\frac{1}{2}\frac{\chi_\parallel}{\chi_\perp}\right)\omega_H, \quad \omega_H<\left[\frac{\omega_a(\omega_a+2\omega_{\rm ex})}{(1-\chi_\parallel/\chi_\perp)}\right]^{1/2};$$

$$\omega_{\rm ex}=-w_{12}\omega_M; \quad \omega_a=-\gamma\mu_0H_a; \quad \omega_M=-\gamma\mu_0M; \quad \omega_H=-\gamma\mu_0H_0;$$

and

$$\chi_\perp=-1/w_{12}.$$

In these expressions, γ is the gyromagnetic ratio (negative), H_a the anisotropy field, and $w_{12}<0$, $\lambda_{12}<0$, and λ_{11} the appropriate exchange constants.

Because the exchange frequency is usually dominant, Eq. (7) is well approximated by

$$(k_p\rho_0)_{\rm crit}=\frac{\mu_0}{|b|M}\frac{[\Delta H_{k1}(\omega_{k1},\vec{k}_1)\Delta H_{k2}(\omega_{k2},\vec{k}_2)\omega_{01}\omega_{02}]^{1/2}}{\omega_{\rm ex}}. \tag{8}$$

Equations (7) and (8) are subject to the energy and momentum constraints Eqs. (6); since the directions of magnon precession are opposite, we let

$$\omega_{k1}=\omega_{01}+(1-\tfrac{1}{2}\chi_\parallel/\chi_\perp)\omega_H, \tag{9a}$$

$$\omega_{k2}=\omega_{02}-(1-\tfrac{1}{2}\chi_\parallel/\chi_\perp)\omega_H, \tag{9b}$$

and require

$$\omega_p=\omega_{01}+\omega_{02}. \tag{10}$$

If the ω and $\vec{k}$ dependences of ΔH_{k1} and ΔH_{k2} are ignored, magnon pairs with $\omega_{01}=\omega_{02}=\omega_0$ will have the minimum thresholds. For such modes, $k_1=k_2=k$, $-\psi_2=\psi_1=\psi$, and $\omega_p=2\omega_0=2v_lk\cos\psi$; only when $\omega_H=0$ are the magnons degenerate. In keeping with the approximations leading to Eq. (8),

$$\omega_0\simeq\{2\omega_{\rm ex}[\omega_a+(\lambda_{11}-\lambda_{12})k^2\omega_M]+[\tfrac{1}{2}(\chi_\parallel/\chi_\perp)\omega_H]^2\}^{1/2}; \tag{11}$$

when combined with the previous equations,

$$\cos\psi=\frac{1}{v_l}\left\{\frac{2\omega_{\rm ex}\omega_M(\lambda_{11}-\lambda_{12})}{1-\{[8\omega_{\rm ex}\omega_a+(\chi_\parallel/\chi_\perp)^2\omega_H^2]/\omega_p^2\}}\right\}^{1/2}. \tag{12}$$

Real values of ψ require that

$$\omega_p \geqslant 2\left\{\frac{2\omega_{ex}\omega_a+[\frac{1}{2}(\chi_{\|}/\chi_{\perp})\omega_H]^2}{1-2\omega_{ex}\omega_M(\lambda_{11}-\lambda_{12})/v_l^2}\right\}^{1/2}, \quad (13)$$

and exist only when

$$2w_{12}(\lambda_{12}-\lambda_{11})\omega_M^2<v_l^2. \quad (14)$$

When satisfied, (14) implies that the magnon and phonon spectra intersect for some value of k. Since ω_M decreases to zero as the Néel temperature is approached, there will always be some temperature range over which (14) is fulfilled.

Because the antiferromagnet $RbMnF_3$ has a large exchange constant and extremely small anisotropy energy,[5] it would appear to be very suitable for "phonon-exchange pumping"; we attempt an order-of-magnitude calculation of the critical strain. The numerical values of $w_{12}=-233$, $w_{11}=-95$, $\lambda_{12}=\frac{1}{8}w_{12}l^2$, $\lambda_{11}=\frac{1}{3}w_{11}l^2$, $l=4.24$ Å, $\omega_{ex}=1.57\times10^{13}$ rad/sec, and $\omega_a=1.17\times10^8$ rad/sec (includes hyperfine anisotropy) pertain to a temperature of 4.2°K and were deduced from the most recent published data.[5] The easy axis of $RbMnF_3$ is along a $\langle111\rangle$ direction, and although measurements of v_l and the magnetoelastic energy constants have not yet been reported, we estimate as reasonable values $v_l=4\times10^3$ m/sec and $|b_{\langle111\rangle}|M^2=10^5$ J/m^3. Although the spin-wave linewidths are also unknown, values of $2\Delta H_k$ for low k should certainly be less than 10 Oe. Indeed, certain measurements[6] on the related antiferromagnet $KMnF_3$ suggest that the values of ΔH_k for $RbMnF_3$ may be several orders of magnitude less than the normal antiferromagnetic resonance linewidth; accordingly, we estimate $2\Delta H_{k1}=2\Delta H_{k2}=1$ Oe $=10^3/4\pi$ A/m. At 4°K, the inequality (14) is satisfied by over a factor of 6 and $\chi_{\|}/\chi_{\perp}\simeq0$; therefore, according to (13), a pump frequency as low as 20 kMc/sec can be employed. It then follows from Eq. (8) that the critical strain corresponding to this minimum pumping frequency is approximately 10^{-6}. This value corresponds to a pump power of only 30 mW/cm^2. Even if the values of $2\Delta H_k$ are an order of magnitude larger or those of $|b_{\langle111\rangle}|$ an order of magnitude smaller than estimated, the required threshold pump power remains at a reasonable level. On the other hand, if the values of $2\Delta H_k$ should turn out to be in the range 10^{-3}-10^{-2} Oe, the threshold strains might be as low as 10^{-9}-10^{-8}!

The prospects for phonon-pumping $RbMnF_3$ (possible continuously) are very encouraging. At helium temperatures, pump frequencies above 20 kMc/sec will be required; at higher temperatures the minimum pump frequency is reduced because both ω_a and ω_M decline in value.

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