able can be written in the form

$$L_1^{\pm} = \mathrm{Tr}(M^{\pm *} S_f M^{\pm} S_i), \qquad (5)$$

where S_i and S_f are the initial and final spin operators to be measured in the reaction. Substituting Eq. (3) and (4) into (5), we get

$$L_1^{\pm} = \sum_{j=1}^{N} \sum_{j'=1}^{N} a_j^{\mp *} a_{j'}^{\mp} \mathrm{Tr}(\xi_j^* S_f \xi_j S_i). \qquad (6)$$

But in the absence of dynamical information, the only property we can ascertain about the bilinear combinations of the two a's is whether they are scalar or pseudoscalar. Since both $a_j^{-*} a_j^{-}$ and $a_j^{+*} a_j^{+}$ are scalar, we cannot distinguish between them, and hence, indeed, in the absence of dynamical information it is impossible to distinguish L_1^{+} from L_1^{-}, q.e.d.

Although the above proof is complete as it stands, a few clarifying comments might be helpful. If all momenta are coplanar, the present proof does not hold, because there is no way then to span the space with true vectors made up of momentum vectors. This case then becomes identical with the situation which prevails in reactions containing four or fewer particles and the theorems in reference 1 hold.

One might think at first sight that one could tell a^- from a^+ by the fact that a^-, being a pseudoscalar formed only of momentum vectors, must depend on some triple product of momenta, and thus an amplitude which does not vanish no matter which three momenta in the reaction are chosen coplanar must be a^+. This, however, is not so, because a^- might depend on $(\vec{k}_1 + \vec{k}_2)\cdot\vec{k}_3 \times \vec{k}_4$ only, in which case it would not vanish no matter which three momenta are made coplanar.

Finally, in order to clarify what we mean by complete absence of dynamical information, we cite the example of the reaction

$$A + B \to C + D + E, \qquad (7)$$

and assume that we have a dynamical theory which tells us that Eq. (7) always proceeds through the chain of reactions

$$A + B \to C + X, \qquad (8)$$

$$X \to D + E. \qquad (9)$$

Since each of the reactions in the chain involves four or fewer particles, parity determination might be possible. If, however, we cannot be sure that Eq. (7) always proceeds as described by Eqs. (8) and (9), parity determination is not possible.

We are grateful to Jerrold Franklin for an interesting discussion, and to Henry Stapp for some comments.

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*Work done under the auspices of the U. S. Atomic Energy Commission.

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SMALL-ANGLE ELASTIC PION-PROTON SCATTERING AT HIGH ENERGIES AND THE REAL PART OF THE SCATTERING AMPLITUDE*

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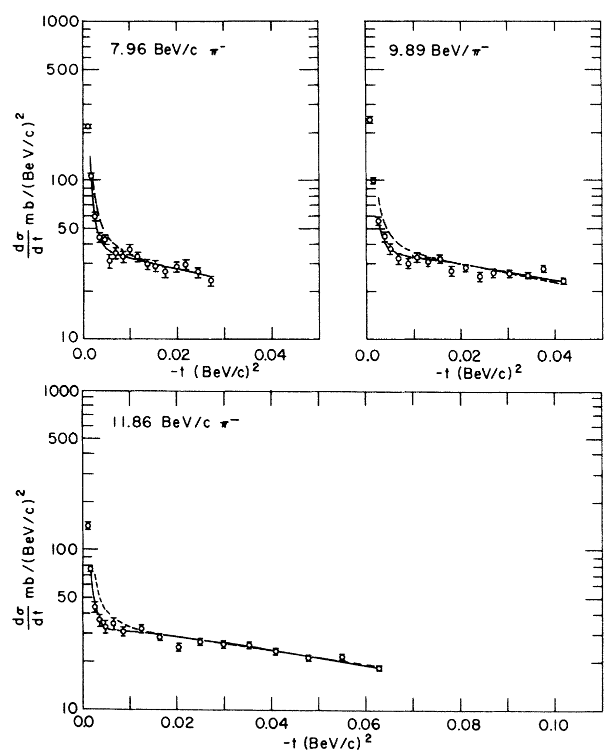
This Letter reports results of a measurement of high-energy $\pi^{\pm}$-p elastic scattering in the Coulomb interference region. We find conclusive evidence that there is a sizable real part of the scattering amplitude in the region of incident momentum 8 to 14 BeV/c for both π^--p and π^+-p scattering. The magnitudes of the real amplitudes range from 10 to 35% of the imaginary amplitude, and in each case the sign (negative) corresponds to repulsive force. A preliminary report of part of these data was presented at the 1964 International Conference at Dubna,[1] where similar conclusions were reported. We also include results of preliminary measurements of 16-, 20-, and 24-BeV/c π^--p scattering. These higher momentum π^--p data show a decrease in the magnitude of the real part with increasing energy. Comparisons with the predictions of the forward dispersion relations are also made.

As in a previously reported study of small-angle p-p scattering,[2] a counter hodoscope-magnetic spectrometer setup with on-line computer analysis was used. The only difference in the apparatus was that a 24-ft.-long threshold Cherenkov counter was used in parallel with the differential counter to improve the separation of pions from kaons. The $\pi^{\pm}$-p and p-p experiments were all part of a run at the Brookhaven AGS.

The treatment of the data was similar to the p-p case apart from the pion decay corrections and the effects of muon contamination of the beam. The pion decay correction accounted for those pions which decayed between the hydrogen target and magnets $D5$-$D8$. It varied from 1.3 to 3.9%. The total muon contamination of the beam, measured at the hydrogen target, varied from 2.3 to 5.8%. A scale correction for each momentum was applied to the data to correct for this effect. The total muon contamination consists of muons of beam momentum as well as those of other than beam momentum. The momentum spectrum of the muon contamination was measured by bending the beam into the final hodoscope ($H4$ in reference 1, Fig. 1) and placing behind it 6 ft. of steel plus $1\frac{1}{2}$ ft. of concrete, followed by a large counter which was used to identify those events due to muons. The beam momentum contamination varied from 0.4 to 3.4% over the energy range studied. The contribution due to Coulomb scattering of these muons was subtracted from the data.

The cross sections are shown in Figs. 1 and 2. The errors shown are the result of compounding counting statistics, the uncertainties in the mean scattering angle and the incident momentum (as reflected as an uncertainty in the cross section), and the uncertainties in the accidental corrections and in the decay corrections. In calculating the value of t for each angular bin, an unfolding procedure was used which took account of the finite sizes of all counters and the target, the measured spatial, angular, and momentum distributions of the incident beam, and multiple Coulomb scattering. A least-squares fit to the differential cross section including nuclear and Coulomb scattering (single and multiple) was used in an iterative procedure to determine the effective t value



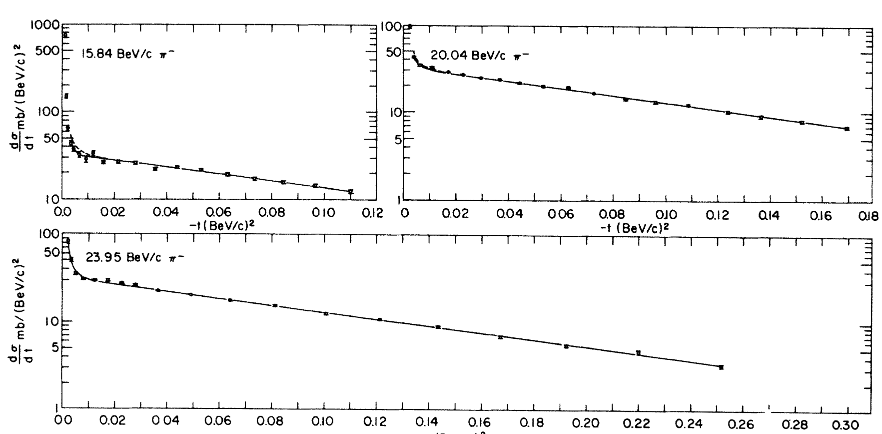


FIG. 1. Small-angle π^--p elastic scattering cross sections. The solid line is the best fit varying α and b. The dotted line is the best fit for $\alpha = 0$.

for each bin. Thus, we have transformed the data into a form suitable for direct comparison with theoretical calculations since all resolution effects were removed.

In order to extract values for the real part of the scattering amplitude we have fit the data with functions of the form $d\sigma/dt = |A_c + A_n|^2$ where A_c and A_n are the complex scattering amplitudes due to Coulomb and nuclear forces, respectively. The form factors used for Coulomb scattering were calculated on the assumption that the same charge distribution can be used for the proton and the pion, an assumption supported by the similar shapes obtained for p-p and π-p high-energy scattering.[3] The nuclear amplitude A_n is much simpler than in the p-p case, where one has spin-dependent amplitudes which do not vanish at very small angles. For π-p elastic scattering, invariance with respect to rotation, parity, and time reversal requires that the complex nuclear amplitude for a particular charge state be of the form $A_{nf}(s,t) + A_{sf}(s,t)$, where the first term is the ordinary (spin-nonflip) and the second term the spin-flip amplitude, which contains a factor $(\vec{\sigma}\cdot\vec{k}_{\mathrm{inc}} \times \vec{k}_{\mathrm{final}})$ proportional to $\sin\theta$. Since in the region of interest, where the Coulomb amplitude is comparable with the nuclear amplitude, $\sin\theta \lesssim 10^{-2}$, we can quite safely neglect this term unless we admit the unlikely possibility that the spin-flip amplitude coefficient is anomalously large. Hence, a treatment using a single complex nuclear amplitude is well justified for the small-angle π-p scattering considered here. We follow the previous procedure of setting $A(t) = (\alpha + i)\exp(a + bt)$, which is in agreement with the characteristics of the data in these and previous experiments.[4] We thus assume that $\mathrm{Re}A_n/\mathrm{Im}A_n = \mathrm{const} \equiv \alpha$. The value of $\mathrm{Im}A_n(t=0)$ was deduced from the measured total cross sections,[5] while α and b were the free parameters in the fits. Because of the steeply rising cross section, the uncertainty in the calculation of the effective t increased rapidly with decreasing $|t|$; consequently, for the purpose of the fits, values of $|t|$ less than 0.0015 $(\mathrm{BeV}/c)^2$ were not used. The solid lines on the curves in Figs. 1 and 2 are the results of least-squares fits to the data. The dotted lines correspond to fits with $\alpha = 0$.

Values of α obtained are given in Table I together with the χ^2 and the statistical errors deduced from the fits. Also listed are the estimated limits of the systematic error due to

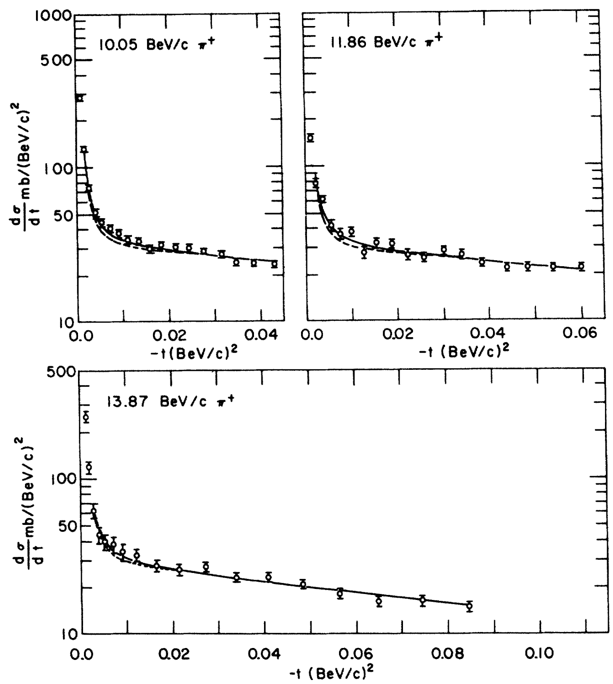


FIG. 2. Small-angle π^+-p elastic scattering cross sections. The solid line is the best fit varying α and b. The dotted line is the best fit for $\alpha = 0$.

uncertainties in the absolute normalization, the total cross section used to evaluate $\mathrm{Im}A_n(t=0)$, and the liquid-hydrogen absorption correction. In order to reduce the sensitivity of the results to any possible incorrect parametrization, the systematic error also includes a contribution to reflect the effect of varying the minimum $|t|$ included in the fit. It is clear from Table I that at low momentum (8-14 BeV/c) there is a substantial real part of the scattering amplitude in both π^--p and π^+-p scattering, the average magnitude and sign being similar to that previously found in p-p scattering, corresponding to a repulsive potential. However, the π^--p results at high momenta show a rapid decrease in magnitude of the real part. The systematic errors are such that the average value for both π^+-p and π^--p at a particular momentum is better determined than either value alone. Most of the systematic uncertainties are coupled so that a variation tends to drive the α values for π^+ and π^- in opposite directions.

Figure 3 shows a comparison of our results with recent forward dispersion relation calculations.[6,7] The 8- to 12-BeV/c $\pi^- + p$ values of α do not overlap the dispersion-relation cal-

Table I. The fitted values of $\alpha = \mathrm{Re}A_n/\mathrm{Im}\,A_n$. The errors shown are described in the text.

Momentum (BeV/c)	α	Standard deviation	Est. systematic limits	χ^2	Degrees of freedom
			π^--p		
7.96	−0.330	0.035	−0.362 −0.252	11.0	14
9.89	−0.300	0.048	−0.338 −0.206	22.0	14
11.86	−0.408	0.038	−0.428 −0.339	18.5	14
15.84	−0.216	0.055	−0.230 −0.144	12.5	14
20.04	−0.106	0.029	−0.158 +0.023	28.8	16
23.95	−0.001	0.028	−0.064 +0.122	31.7	17
			π^+-p		
10.05	−0.109	0.024	−0.200 −0.074	23.4	15
11.86	−0.132	0.034	−0.225 −0.095	24.6	15
13.87	−0.066	0.052	−0.170 −0.019	9.9	14

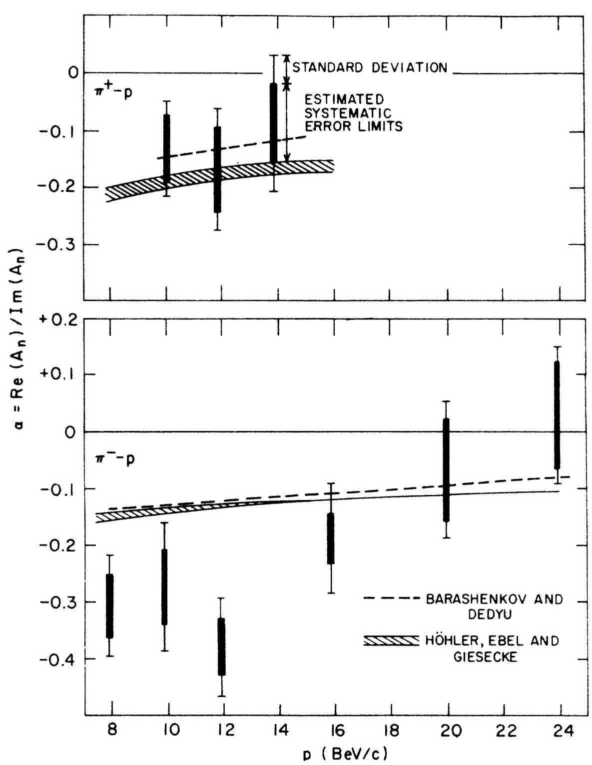


FIG. 3. A comparison of the ratio of the real part of the nuclear amplitude to the imaginary part with the predictions of the forward dispersion relation calculations. See references 6 and 7. The solid rectangles represent the estimated limits of the systematic errors; the flags add one standard deviation of the statistical errors.

culations which are outside our systematic-error limits by several standard deviations. Furthermore, the rapid drop of α with increasing momentum is not predicted by the calculations. These results include the assumption that the real part has the same exponential t dependence as the imaginary part of the nuclear amplitude. Assuming the real part is a constant, independent of t, makes only a small change (~0.01) in the value of α at $t=0$. We have tried exponential slopes for the real part of up to[8] 120 $(\mathrm{BeV}/c)^{-2}$ [compared to ~5 $(\mathrm{BeV}/c)^{-2}$ for the imaginary part]. This increases the magnitude of α at $t=0$ by an average of 0.15 for π^- and 0.02 for π^+, not substantially affecting the conclusion.

Before a definite conclusion can be drawn as to the validity of the dispersion relations at these energies, a systematic study of the sensitivity of the dispersion-relation calculations to the asymptotic behavior of the total cross sections, and of the various low-energy parameters used, is necessary. We plan to repeat these measurements soon with better resolution and smaller systematic uncertainties, and to study the detailed shape of the real amplitude curve versus momentum, especially in the high-momentum region. An evaluation of the dispersion-relation predictions including the effect of additional subtractions to reduce sensitivity to the assumptions about the asymptotic behavior of the cross sections at high energy is in progress, so that a critical comparison of theory and experiment can be attempted.

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†Visitor from Rutherford High Energy Laboratory.

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BREAKING OF $U_{\mathcal{L}}(12)$ SYMMETRY*

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In a recent paper[1] we have proposed a spurion theory of broken $U_{\mathcal{L}}(12)$ symmetry. In this scheme the formal, initial $U_{\mathcal{L}}(12)$ invariance[2] of an amplitude is not only broken by the imposition of Bargmann-Wigner equations,[3] which define the asymptotic "states" of physical particles, but also by spurions.[4] It is the purpose of this note to give a more detailed discussion of these spurions and to report certain consequences of the spurion theory for meson-baryon amplitudes.

We note that the spurion theory of $U_{\mathcal{L}}(12)$ should be considered as an approximation scheme for S-matrix elements and vertex functions, and not as an exact supermultiplet theory which complies a priori with all the axioms of field theory or of dispersion theory. It is well known that a relativistic theory which incorporates the successful features of SU(6) models encounters difficulties because of the noncompact nature of the Lorentz group.[5] In $U_{\mathcal{L}}(12)$ schemes, the impact of these difficulties is softened because the transformations are not directly applied to the physical amplitudes; rather, they act on formal amplitudes which are generalizations of the "spinor space amplitudes" used in dispersion theory.[6] There are, however, difficulties with the unitarity condition, even after imposition of the Bargmann-Wigner equations.[7,8]

A priori, we introduce the spurions in such a way that they preserve the Lorentz invariance of the amplitudes, transform as SU(3) singlets, and do not split the masses of particles with different spin values which are contained in the same SU(6) supermultiplet. We write the general spurion in the form

$$S=(\Gamma+\Gamma_5\gamma_5+\Gamma_\mu\gamma_\mu+\Gamma_{5\mu}i\gamma_5 +\tfrac{1}{2}\Gamma_{\mu\nu}\sigma_{\mu\nu})\otimes 1, \qquad (1)$$

and if used in first order, the coefficients are Lorentz covariants which must be constructed out of the independent momenta available in the amplitude into which S is being inserted. In the second and higher order, we can proceed in different ways. If we continue to construct the coefficients in Eq. (1) out of external momenta only, then the masses of the SU(6)