NONEXISTENCE OF PARITY EXPERIMENTS IN MULTIPARTICLE REACTIONS*

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In recent years various experiments have been proposed in which reactions with four particles participating are used to determine the parity of one of these particles in the absence of any dynamical information about the reaction. Some general properties of such parity experiments are by now well understood.[1] In particular, fairly stringent requirements are imposed on such experiments in terms of the polarizations of the participating particles. With the recent increase in importance of reactions involving more than four particles one might wonder, therefore, if such multiparticle reactions might not represent an easier way to determine parities.

The purpose of this Letter is to show that not only is this not the case, but that in fact there are essentially no parity experiments involving more than four particles. In particular, we want to prove the following:

Theorem.—In a reaction involving more than four particles, and in the absence of dynamical information, it is impossible to determine the parity of any of the particles unless all particles are coplanar in the reaction center-of-mass frame.

Proof.—Consider the reaction

$$A_1+A_2+\cdots+A_\mu\rightarrow B_1+B_2+\cdots+B_\nu \quad (\mu+\nu>4), \tag{1}$$

where the spins of the particles are arbitrary, μ and ν may or may not be equal, and the parities of all particles are assumed to be known, except that of B_ν. We will now specify the coordinate system in which the reaction is studied. Let us choose three momenta in the center-of-mass system of the reaction: $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$. They can be selected arbitrarily out of the possibly large number of independent momenta occurring in Reaction (1). Using these three momenta we define three unit vectors

$$l\equiv\frac{\vec{k}_1-\vec{k}_2}{|\vec{k}_1-\vec{k}_2|},\quad m'\equiv\frac{\vec{k}_1\cdot\vec{k}_2\times\vec{k}_3}{|\vec{k}_1\cdot\vec{k}_2\times\vec{k}_3|}\,\frac{\vec{k}_1\times\vec{k}_2}{|\vec{k}_1\times\vec{k}_2|},$$
$$n\equiv\frac{(\vec{k}_1-\vec{k}_2)\times(\vec{k}_1\times\vec{k}_2)}{|(\vec{k}_1-\vec{k}_2)\times(\vec{k}_1\times\vec{k}_2)|}. \tag{2}$$

These are three mutually orthogonal true (not pseudo) vectors. They can always be defined as long as there are at least three noncoplanar momenta in the problem (so that the $\vec{k}_1\cdot\vec{k}_2\times\vec{k}_3$ appearing in m' is nonzero). This is always the case for Reaction (1), unless all particles in it are coplanar. It is never the case for a type-(1) reaction with $\mu+\nu\leqslant4$, because in that case there are only two independent momenta.

Let us now denote by M^+ and M^- the M matrices of Reaction (1) in the cases when the product of the intrinsic parities of all particles appearing in the reaction is +1 and −1, respectively. We can write

$$M^+=\sum_{j=1}^{N}a_j^-\zeta_j \tag{3}$$

$$M^-=\sum_{j=1}^{N}a_j^+\zeta_j, \tag{4}$$

where the ζ_j's are a complete set of matrices connecting the spin space of all incoming particles with that of all outgoing particles, expressed in the coordinate frame defined above. Thus all ζ_j's are constructed only out of scalar products of angular momentum operators (which therefore are invariant under space reflection) and of l, m', or n (which do change sign). Hence the ζ_j's will change sign under space reflection. The a_j^+ and a_j^- are scalar and pseudoscalar amplitudes (form factors), respectively, composed of momentum vectors only. Unlike the ζ_j's, the detailed form of the a_j's depends on the dynamics of the reaction. The upper limit N is given by

$$\prod_{\alpha=1}^{\mu+\nu}(2S_\alpha+1),$$

where S_α is the spin of the αth particle in Reaction (1).

Let us consider now an arbitrary observable $L_1^\pm$ for Reaction (1), whose superscript denotes whether it refers to Eq. (1) with the product of all intrinsic parities +1 or −1. This observable can be written in the form

$$L_1^{\pm} = \mathrm{Tr}(M^{\pm *} S_f M^{\pm} S_i), \tag{5}$$

where S_i and S_f are the initial and final spin operators to be measured in the reaction. Substituting Eq. (3) and (4) into (5), we get

$$L_1^{\pm} = \sum_{j=1}^{N} \sum_{j'=1}^{N} a_j^{\mp *} a_{j'}^{\mp} \mathrm{Tr}(\xi_j^* S_f \xi_j S_i). \tag{6}$$

But in the absence of dynamical information, the only property we can ascertain about the bilinear combinations of the two a's is whether they are scalar or pseudoscalar. Since both $a_j^{-*}a_j^{-}$ and $a_j^{+*}a_j^{+}$ are scalar, we cannot distinguish between them, and hence, indeed, in the absence of dynamical information it is impossible to distinguish L_1^{+} from L_1^{-}, q.e.d.

Although the above proof is complete as it stands, a few clarifying comments might be helpful. If all momenta are coplanar, the present proof does not hold, because there is no way then to span the space with true vectors made up of momentum vectors. This case then becomes identical with the situation which prevails in reactions containing four or fewer particles and the theorems in reference 1 hold.

One might think at first sight that one could tell a^- from a^+ by the fact that a^-, being a pseudoscalar formed only of momentum vectors, must depend on some triple product of momenta, and thus an amplitude which does not vanish no matter which three momenta in the reaction are chosen coplanar must be a^+. This, however, is not so, because a^- might depend on $(\vec{k}_1+\vec{k}_2)\cdot\vec{k}_3\times\vec{k}_4$ only, in which case it would not vanish no matter which three momenta are made coplanar.

Finally, in order to clarify what we mean by complete absence of dynamical information, we cite the example of the reaction

$$A+B \to C+D+E, \tag{7}$$

and assume that we have a dynamical theory which tells us that Eq. (7) always proceeds through the chain of reactions

$$A+B \to C+X, \tag{8}$$

$$X \to D+E. \tag{9}$$

Since each of the reactions in the chain involves four or fewer particles, parity determination might be possible. If, however, we cannot be sure that Eq. (7) always proceeds as described by Eqs. (8) and (9), parity determination is not possible.

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[1]P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Phys. Letters 15, 353 (1965).

SMALL-ANGLE ELASTIC PION-PROTON SCATTERING AT HIGH ENERGIES AND THE REAL PART OF THE SCATTERING AMPLITUDE*

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This Letter reports results of a measurement of high-energy $\pi^{\pm}$-p elastic scattering in the Coulomb interference region. We find conclusive evidence that there is a sizable real part of the scattering amplitude in the region of incident momentum 8 to 14 BeV/c for both π^--p and π^+-p scattering. The magnitudes of the real amplitudes range from 10 to 35% of the imaginary amplitude, and in each case the sign (negative) corresponds to repulsive force. A preliminary report of part of these data was presented at the 1964 International Conference at Dubna,[1] where similar conclusions were reported. We also include results of preliminary measurements of 16-, 20-, and 24-BeV/c π^--p scattering. These higher momentum π^--p data show a decrease in the magnitude of the real part with increasing energy. Comparisons with the predictions of the forward dispersion relations are also made.