For any real positive λ, we obtain

$$\int_0^{E_\lambda} \frac{[\mathrm{Re}f(E'+i0)-f(0)]}{E'^2}dE' \geq \frac{1}{\lambda}|f(i\lambda)-f(0)|, \quad (8)$$

where E_λ and λ are related by

$$\int_\mu^{E_\lambda} \frac{\mathrm{Im}f(E')}{E'^2}dE' = |g(i\lambda)|. \quad (9)$$

From the dispersion relation (2) one can always get a lower bound for the right-hand side of (8) which is independent of the value of the total cross section for $E'>E_m$. Namely, one can write

$$\frac{|f(i\lambda)-f(0)|}{\lambda} > \frac{2\lambda}{\pi}\int_\mu^{E_m} \frac{\mathrm{Im}f(E')}{E'(E'^2+\lambda^2)}dE'. \quad (10)$$

Although (8) is obtained from a general inequality and (6) is obtained by truncating the equality (5), it is not necessarily true that (6) is much better than (8). For example, one may take the simple form $\mathrm{Im}f(E')=cE'$ and compute the values of the quantities appearing on the right-hand sides of (6) and (8). For this simple Ansatz for Imf, it turns out that (6) is better than (8) by only about 50%. For the actual case we have the contribution from the 33 resonance which (6) tends to de-emphasize, while (8) does not.

A detailed numerical analysis of (6), (7), and (8) is clearly called for. However, the simple results of this short note make it clear that data on Ref should be obtained at all available energies before any meaningful comparison could be made.

*Work supported in part by the Office of Naval Research.

[1]K. J. Foley et al., Proceedings of the International Conference on High Energy Physics, Dubna, 1964 (to be published); G. Belletini, ibid.

[2]See, for example, N. N. Khuri and T. Kinoshita, Phys. Rev. (to be published).

[3]These are widely known to follow from the Lehmann-Symanzik-Zimmermann formalism. The recent work of K. Hepp (to be published) shows that they also follow from the Wightman axioms of quantum field theory. The fact that only two subtractions are necessary follows from the improvement of the Greenberg-Low upper bound given in reference 2.

[4]The integral is convergent at zero since $f'(0)=0$.

[5]S. Adler, Phys. Rev. (to be published).

[6]For a proof of this property of $g(E)$ see N. N. Khuri and T. Kinoshita, to be published. A more detailed paper is in preparation.

[7]See, for instance, W. K. Hayman, Multivalent Functions (Cambridge University Press, Cambridge, England, 1958), p. 3.

NONLEPTONIC BARYON DECAYS IN SU(6)

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(Received 8 December 1964)

It has been shown by a number of authors[1] that octet transformation properties of nonleptonic weak interactions with R or RP invariance lead to sum rules among four independent observable amplitudes of the nonleptonic baryon decays. In particular, the following sum rules are in good agreement with experiment:

p-v amplitudes (S wave),

$$A(\Lambda\to p+\pi^-)+2A(\Xi^-\to\Lambda+\pi^-) = \sqrt{3}A(\Sigma^+\to p+\pi^0); \quad (1)$$

p-c amplitudes (P wave),

$$B(\Lambda\to p+\pi^-)+2B(\Xi^-\to\Lambda+\pi^-) = \sqrt{3}B(\Sigma^+\to p+\pi^0). \quad (2)$$

The first relation can be proved without assuming R or RP invariance if we assume that the effective nonleptonic weak interactions are of the current×current form and of nonderivative type.[2]

In this note, we wish to report some consequences arising from the assumptions that (i) strong interactions are approximately SU(6) invariant; (ii) nonleptonic weak interactions transform like a member of an adjoint representation under SU(6), i.e., parity-nonconserving (p-v) and parity-conserving (p-c) interactions transform as $(\underline{8},\underline{1})$ and $(\underline{8},\underline{3})$ members of the 35-dimensional representations, respectively; and (iii) weak interactions are invariant under the operation of CP.

It has been proposed by Gürsey, Radicati, and Pais[3] that octet and decuplet baryons are grouped together forming the bases of a 56-

dimensional representation in SU(6), while Sakita[4] considered the 20-dimensional representation as a possible representation of the octet baryons (plus singlet $\frac{3}{2}^+$ baryon). The states for these two representations are described by tensors $B^{\alpha\beta\gamma}$ completely symmetric and antisymmetric with respect to the indices α, β, and γ, respectively. These tensors are reducible under SU(3)⊗SU(2), being expressed, in the rest frame, in terms of spin wave functions and SU(3) tensors[5]: for $\underline{56}$,

$$B^{\alpha\beta\gamma}=\chi^{ijk}d^{ABC}+\frac{1}{3\sqrt{2}}[(2\epsilon^{ij}\chi^{k}+\epsilon^{jk}\chi^{i})\epsilon^{ABD}b_D{}^C+(\epsilon^{ij}\chi^{k}+2\epsilon^{jk}\chi^{i})\epsilon^{BCD}b_D{}^A]; \quad (3)$$

for $\underline{20}$,

$$B^{\alpha\beta\gamma}=\frac{1}{\sqrt{6}}\chi^{ijk}\epsilon^{ABC}+\frac{1}{\sqrt{6}}[\chi^{i}\epsilon^{jk}\epsilon^{ABD}b_D{}^C-\epsilon^{ij}\chi^{k}\epsilon^{BCD}b_D{}^A]. \quad (4)$$

Here χ^i and χ^{ijk} stand for spin-$\frac{1}{2}$ and -$\frac{3}{2}$ wave functions, respectively, $b_B{}^A$ is the baryon-octet tensor, and d^{ABC} is the decuplet tensor.

According to the assumption (ii), the effective nonleptonic weak interaction transforms as the $T_3{}^2$ component of the octet under the subgroup SU(3). These effective interactions in general involve four parameters because $\underline{56}^*\otimes\underline{56}\otimes\underline{35}$ or $\underline{20}^*\otimes\underline{20}\otimes\underline{35}$ contains $\underline{35}$ four times. However, only two of these contribute to physically observable decay processes. In the nonrelativistic limit, they are expressed as[6]

$$H_{\text{p-v}}=f_{\text{p-v}}\bar{B}_{\alpha\beta(3,m)}B^{\alpha\beta\gamma}M_\gamma{}^{(2,m)}+g_{\text{p-v}}\bar{B}_{\alpha\beta(3,m)}B^{\alpha\beta'(2,m)}M_{\beta'}{}^{\beta}+\text{H.c.}, \quad (5)$$

$$H_{\text{p-c}}=f_{\text{p-c}}\bar{B}_{\alpha\beta(3,m)}B^{\alpha\beta\gamma}M_\gamma{}^{(2,m)}+g_{\text{p-c}}\bar{B}_{\alpha\beta(3,m)}B^{\alpha\beta'(2,m)}M_{\beta'}{}^{\beta}+\text{H.c.}, \quad (6)$$

where, for p-v interactions, the relevant pseudoscalar meson components of $M_\gamma{}^\beta$ are written as

$$M_\gamma{}^\beta\approx\delta_l{}^mP_C{}^D,\quad \beta=(D,m)\text{ and }\gamma=(C,l); \quad (7)$$

and for p-c interactions,

$$M_\gamma{}^\beta\approx i\vec{\sigma}_l{}^m\cdot(\vec{q}/|q|)P_C{}^D. \quad (8)$$

$P_C{}^D$ are the usual octet pseudoscalar mesons.

We remark that for p-v amplitudes there is no recoupling of spin and orbital angular momentum by decomposing $\underline{56}^*\otimes\underline{56}\otimes\underline{35}$ or $\underline{20}^*\otimes\underline{20}\otimes\underline{35}$ into the irreducible representation of $\underline{\text{SU}(6)}$ (see discussions below). Since only two parameters for each p-v and p-c interaction are involved, we may expect stronger relations than those given in (1) and (2).

We state the main results obtained from the interactions (5) and (6).

(A) P-v amplitudes.—We get the following relations:

$$A(\Sigma^-\to n+\pi^-)+(\tfrac{2}{3})^{1/2}A(\Xi^-\to\Lambda+\pi^-)=0, \quad (9)$$

$$A(\Lambda\to p+\pi^-)+A(\Xi^-\to\Lambda+\pi^-)-(\tfrac{3}{2})^{1/2}A(\Sigma^+\to n+\pi^+)=0, \quad (10)$$

which are consistent with the sum rule (1).

It is of interest to note that both cases, i.e., the one in which octet baryons belong to the 56-dimensional representation and the other in which they belong to the 20-dimensional representation (hereafter abbreviated as cases I and II, respectively), give the same relations (9) and (10), although the F-to-D ratios of the axial vector currents are quite different: $F/D=\frac{2}{3}$ for case I and $F/D=0$ for case II.[3,4]

As we remarked before, there is no effect of spin and orbital angular momentum recoupling because for these decay processes only S waves are involved. Therefore, the relations (9) and (10) would be rather insensitive to the perturbation due to symmetry-breaking interaction of SU(6).

If we assume that the effective interactions are of the current×current form, we have an additional restriction by taking account of CP invariance,

$$A(\Sigma^+\to n+\pi^+)=0. \quad (11)$$

From (9), (10), and (11), we have the following sum rules for p-v amplitudes:

$$A(\Lambda\to p+\pi^-)=-A(\Xi^-\to\Lambda+\pi^-)=+(\tfrac{3}{2})^{1/2}A(\Sigma^-\to n+\pi^-), \quad (12)$$

$$A(\Sigma^+\to n+\pi^+)=0. \quad (13)$$

(B) P-c amplitudes.—We have different sum rules for case I and case II: for 56,

$$(\tfrac{2}{3})^{1/2}B(\Lambda\to p+\pi^-)-\tfrac{1}{3}B(\Sigma^-\to n+\pi^-)-B(\Sigma^+\to n+\pi^+)=0, \quad (14)$$

$$-(\tfrac{1}{6})^{1/2}B(\Xi^-\to\Lambda+\pi^-)+\tfrac{3}{10}B(\Sigma^-\to n+\pi^-)+B(\Sigma^+\to n+\pi^+)=0; \quad (15)$$

for 20,

$$(\tfrac{2}{3})^{1/2}B(\Lambda\to p+\pi^-)+\tfrac{1}{3}B(\Sigma^-\to n+\pi^-)-B(\Sigma^+\to n+\pi^+)=0, \quad (16)$$

$$-(\tfrac{3}{2})^{1/2}B(\Xi^-\to\Lambda+\pi^-)+\tfrac{1}{2}B(\Sigma^-\to n+\pi^-)+B(\Sigma^+\to n+\pi^+)=0. \quad (17)$$

The assumption that the nonleptonic weak interactions are of the current×current form imposes another restriction for case II,

$$B(\Sigma^+\to n+\pi^+)=0. \quad (18)$$

The sum rules (14) and (15) for case I do not satisfy (2), while (16) and (17) with the condition (18) are compatible with (2).[7] However, the decay $\Sigma^+\to n+\pi^+$ is forbidden for this case.[8]

These relations, as they stand, do not seem to be in agreement with the experimental data and strongly suggest that spin-orbital angular momentum recouplings are important for p-c amplitudes, which must necessarily be taken into account in order to reproduce observed mass splittings among super supermultiplets.[9]

The other consequence which immediately follows from the SU(6) symmetry model presented here is that for the case I, the p-c amplitudes of decuplet-baryon nonleptonic decays are related to those of octet-baryon decays. In particular, we find that the p-c amplitudes of $\Omega^-\to\Xi^{-,0}+\pi^{0,-}$ are proportional to that of $\Sigma^-\to n+\pi^-$, and the corresponding amplitude of $\Omega^-\to\Lambda+K^-$ is expressed in terms of $\Sigma^+\to n+\pi^+$ p-c amplitudes,

$$\Gamma_P(\Omega^-\to\Xi^0+\pi^-)=\left(\frac{12}{25}\right)\frac{m_\Sigma m_\Xi}{m_N m_\Omega}\left(\frac{k_\Xi}{k_n}\right)^3\Gamma_P(\Sigma^-\to n+\pi^-), \quad (19)$$

$$\Gamma_P(\Omega^-\to\Lambda+K^-)=18\frac{m_\Lambda m_\Sigma}{m_N m_\Omega}\left(\frac{k_\Lambda}{k_n}\right)^3\Gamma_P(\Sigma^+\to n+\pi^+), \quad (20)$$

where k_Ξ, k_Λ, k_n are spatial momenta of the resulting baryons.

Experimentally, it is known that one of the two decay processes, $\Sigma^+\to n+\pi^+$ and $\Sigma^-\to n+\pi^-$, is purely parity conserving, which according to (19) and (20) implies that one of the p-c decay amplitudes, $B(\Omega^-\to\Xi+\pi)$ or $B(\Omega^-\to\Lambda+K^-)$, should be small. Let us now assume that the p-v amplitudes of the octet-baryon decays are correctly described by the relations (12) and (13). Then $B(\Sigma^-\to n+\pi^-)$ must be small in any case, in order to satisfy the Gell-Mann–Rosenfeld triangle relation, and the decay $\Sigma^+\to n+\pi^+$ is parity conserving. This implies, according to (19), that p-c amplitudes of $\Omega^-\to\Xi^{-,0}+\pi^{0,-}$ are small.[10] In other words, our model predicts that the decay of $\Omega^-\to\Xi^{-,0}+\pi^{0,-}$ should be d-wave dominant.

The author is grateful to Professor Abdus Salam and the IAEA for the hospitality extended to him at the International Centre for Theoretical Physics. He would like to thank Dr. T. Sawada and Dr. R. White for their careful reading of the manuscript and for their comments.

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[1]B. W. Lee, Phys. Rev. Letters 12, 83 (1964); M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 213 (1964); S. Okubo, Phys. Letters 8, 362 (1964); B. Sakita, Phys. Rev. Letters 12, 379 (1964); S. P. Rosen, Phys. Rev. Letters 12, 408 (1964); R. H. Dalitz, Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures, 1964 (unpublished).

[2]Gell-Mann, reference 1; Okubo, reference 1.

[3]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[4]B. Sakita, to be published.

[5]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964). We use the same notations given in this Letter. (3) and (4) can be rewritten in more symmetrical form like, for 56,

$$B^{\alpha\beta\gamma}=\chi^{ijk}{}_\alpha{}^{ABC}+\frac{1}{3\sqrt{2}}[\epsilon^{ij}\chi^k\epsilon^{ABD}b_D{}^C+\epsilon^{jk}\chi^i\epsilon^{BCD}b_D{}^A+\epsilon^{ki}\chi^j\epsilon^{CAD}b_D{}^B];$$

for 20,

$$B^{\alpha\beta\gamma}=\frac{1}{\sqrt{6}}\chi^{ijk}\epsilon^{ABC}+\frac{1}{3\sqrt{6}}[\epsilon^{ij}\chi^k(\epsilon^{CAD}b_D{}^B-\epsilon^{BCD}b_D{}^A)+\epsilon^{jk}\chi^i(\epsilon^{ABD}b_D{}^C-\epsilon^{CAD}b_D{}^B)+\epsilon^{ki}\chi^j(\epsilon^{BCD}b_D{}^A-\epsilon^{ABC}b_D{}^C)],$$

which are reduced to (3) and (4) by making use of the following identities:

$$\epsilon^{ABD}b_D{}^C+\epsilon^{BCD}b_D{}^A+\epsilon^{CAD}b_D{}^B=0,$$

$$\epsilon^{ij}\chi^k+\epsilon^{jk}\chi^i+\epsilon^{ki}\chi^j=0.$$

[6]We assume that the Hamiltonians (5) and (6) are of nonderivative type in relativistic form.

[7]Since the vector and axial vector currents are *RP* invariant for the case II, the resulting effective nonleptonic Hamiltonian is necessarily *RP* invariant. Therefore, it is natural that the amplitudes (16), (17), and (18) satisfy the weaker form of the sum rule (2).

[8]It is interesting to note that the global-symmetry (or doublet-approximation) model also forbids the decay $\Sigma^+\rightarrow n+\pi^+$. See G. Taheda, Ann. Phys. (N.Y.) 18, 310 (1962).

[9]A. Pais, Phys. Rev. Letters 13, 175 (1964); T. K. Kuo and Tsu Yao, Phys. Rev. Letters 13, 415 (1964); M. A. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964); M. A. Bég, Phys. Rev. Letters 13, 509 (1964).

[10]For decuplet-baryon weak decays, it might be important to consider the final-state interactions which can be neglected for octet strange baryon decays. For example, in the decay of Ω^-, if the $\Xi_{1/2}{}^*$ intermediate state is dominant, the amplitudes become complex and the branching ratio $\Gamma(\Omega^-\rightarrow\Lambda+K^-)/\Gamma(\Omega^-\rightarrow\Xi+\pi)$ roughly equals $1/40\sim1/50$.

ERRATA

POLARIZED POSITRON ANNIHILATION IN FERROMAGNETS. Stephan Berko and Joel Zuckerman [Phys. Rev. Letters 13, 339a (1964)].

In the derivation of Eqs. (1) and (2), the time dependence of the positron polarization has been neglected. The corrected Eq. (2) should read

$$N_{\uparrow,\downarrow}(\theta)=\frac{(1+P_p)\sum_l(1\mp P_l)n_l(\theta)}{2\sum_l(1\mp P_l)w_l}+\frac{(1-P_p)\sum_l(1\pm P_l)n_l(\theta)}{2\sum_l(1\pm P_l)w_l}. \qquad (2)$$

In the sample computation of Fig. 2 the new theoretical value of $p(\theta)$ is within 5% of the old one. The corrected equations lead again to $\int N_\uparrow(\theta)d\theta=\int N_\downarrow(\theta)d\theta$ and do not change any of the conclusions of the Letter.

LOCALIZED MOMENTS OF MANGANESE IMPURITIES IN FERROMAGNETIC IRON. V. Jaccarino, L. R. Walker, and G. K. Wertheim [Phys. Rev. Letters 13, 752 (1964)].

The inequality which appeared in print as "$H_0{}^{Mn}/H_0{}^{Fe}>0.5$" in paragraph 4 on page 753 should read "$H_0{}^{Mn}/H_0{}^{Fe}<0.5$."

DISTORTED-WAVE CALCULATION OF ROTATIONAL EXCITATION OF N_2 BY SLOW ELECTRONS. Raymond C. Mjolsness and Douglas H. Sampson [Phys. Rev. Letters 13, 812 (1964)].

In the first column on page 813, line 3, "$Q=1.10$" should read "$Q=-1.10$"; and in lines 16 and 17, "1-eV electron energies" should read "low electron energies."

TRUE PHOTOELECTROMAGNETIC EFFECT IN BULK BISMUTH AT 4.2°K. R. N. Zitter [Phys. Rev. Letters 14, 14 (1965)].

Paragraph 4, line 4: The dimensions of the sample are $2.5\times2.4\times10$ mm^3, not "2.5×10 mm^3."

Equation (1): μ_1 should be replaced by $\mu_{1/2}$. This error was included in the calculation of lifetime; the correct value for τ is smaller by a factor of 2.5 than that published in the text.