the same as in the static SU(6), the ratios of the dynamic electric and magnetic moments are the same as found in the static case. This proves Eqs. (1) and (2). Equation (2) is a generalization, to the matrix element of G_E between neutron states, of the static result that the neutron charge is zero. Similar results can be obtained for other vertex functions. These results shall be presented elsewhere.

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SUM RULES FOR THE REAL PART OF THE FORWARD-SCATTERING AMPLITUDE

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Recent experiments[1] have shown that both the $\pi^{\pm}p$ and the pp forward-scattering amplitudes have a substantial real part at high energies (>6 BeV). While the data obtained so far are not yet in contradiction with the forward dispersion relation, the real part measured is larger than what was expected, and with the little data available it does not seem that $\mathrm{Re}f/E$ decreases with energy as it should if local field theory is valid.[2]

The purpose of this note is to present sum rules satisfied by the real part of a crossing-symmetric forward-scattering amplitude. These sum rules involve integration of the real part up to finite energies only and hold for any energy. Although these sum rules follow directly from the dispersion relations, they give for practical purposes a better tool for testing analyticity, crossing symmetry, and unitarity. They also show explicitly the fact that a large and repulsive real part at high energies, if maintained for a certain large energy range, will lead to a contradiction with the dispersion relation.

For concreteness we shall consider only pion-nucleon scattering. We let E be the total energy of the incident pion in the laboratory system, and let $f_{\pm}(E)$ be the forward amplitudes for $\pi^{\pm}p$ scattering, respectively. We define the symmetric amplitude $f(E)$ as follows:

$$f(E)=\tfrac{1}{2}[f_+(E)+f_-(E)]-[\text{nucleon-pole terms}]. \quad (1)$$

It follows from axiomatic field theory that $f(E)$ satisfies the forward dispersion relation[3]

$$f(E)-f(0)=\frac{2E^2}{\pi}\int_\mu^\infty dE'\frac{\operatorname{Im}f(E')}{E'(E'^2-E^2)}. \quad (2)$$

Our sum rules follow from considering the function $g(E)$ defined by

$$g(E)=\int_0^E\frac{[f(E')-f(0)]}{E'^2}dE', \quad \operatorname{Im}E\geq 0, \quad (3)$$

where the integration path should lie entirely in the upper-half E plane.[4] Now, dividing both sides of (2) by E^2, interchanging orders of integration, and integrating from 0 to E along the radial direction, we obtain, after taking the real parts,

$$\operatorname{Re}g(E)=\frac{1}{\pi}\int_\mu^\infty dE'\frac{\operatorname{Im}f(E')}{E'^2}\ln\left|\frac{E'+E}{E'-E}\right|, \quad (4)$$

$$0<\arg E<\pi.$$

We note that for $0\leq\arg E\leq\frac{1}{2}\pi$, $\ln|(E'+E)/(E'-E)|\geq 0$. Since $\operatorname{Im}f(E')>0$ for $E'>\mu$, we see that $\operatorname{Re}g(E)>0$ for all E such that $0\leq\arg E<\frac{1}{2}\pi$. In particular, for positive real E, we obtain

$$\int_0^E\frac{[\operatorname{Re}f(E')-f(0)]}{E'^2}dE' =\frac{1}{\pi}\int_\mu^\infty dE'\frac{\operatorname{Im}f(E')}{E'^2}\ln\left|\frac{E'+E}{E'-E}\right|. \quad (5)$$

The integrand on the right-hand side in (5) is always positive. If the integration is cut off at the maximum energy, E_m, for which one has data on the total cross section then one obtains an inequality which should be satisfied regardless of the actual value of the total cross sections at super-high energies:

$$\operatorname{Re}g(E)>\frac{1}{\pi}\int_\mu^{E_m}dE'\frac{\operatorname{Im}f(E')}{E'^2}\ln\left|\frac{E'+E}{E'-E}\right|. \quad (6)$$

It is already evident from (5) why a large and negative Ref is dangerous to analyticity. The present data give, roughly, $\operatorname{Re}f\sim -cE$ for energies between 7 and 12 BeV, where c is about $1/20\pi$ of the total cross section, $\frac{1}{2}(\sigma_+ +\sigma_-) =(4\pi/k)\operatorname{Im}f$. Clearly, such a behavior if maintained to higher energies will not only make the left-hand side of (5) smaller, but might even make it negative for large enough E.

In an actual comparison of (5) or (6) with the data, one has to know Ref in the unphysical region $0\leq E\leq\mu$. This can be obtained from the dispersion relation. It is well known that the dispersion relation is reliable for low energies. However, since we already know that the dispersion relation is fairly reliable for energies below a few BeV, we can subtract all the low-energy data from (5). For example, if for $E\leq E_1$ ($E_1\approx$ 1-3 BeV) the dispersion relation is known to be approximately valid, then a relation like (5) holds with $E=E_1$. Subtracting this relation from (5), we get

$$\int_{E_1}^E\frac{[\operatorname{Re}f(E')-f(0)]}{E'^2}dE' =\frac{1}{\pi}\int_\mu^\infty dE'\frac{\operatorname{Im}f(E')}{E'^2}\ln\left|\frac{(E'+E)(E'-E_1)}{(E'-E)(E'+E_1)}\right|, \quad (7)$$

where $E>E_1$. The integration on the left-hand side now involves only the high-energy domain. For $E'>E$ the integrand on the right-hand side is positive. We can therefore cut off the integration on the right at some $E_m\geq E>E_1$, and obtain a lower bound for the integral on the left. The only quantity in such an inequality which is not obtainable immediately from the data is $f(0)$. The value of $f(0)$ can be taken from the work of Adler.[5] This work also shows that the forward dispersion relation is very reliable for determining $f(0)$.

The sum rule (6) shares with the dispersion relation the disadvantageous feature, for large E, of giving more emphasis to the cross section at high energies. This might not be too serious. Nevertheless, it is possible to give another sum rule which is not sensitive to the value of the total cross section for large E'.

This second sum rule is only one of the several consequences of the fact that the function $g(E)$ defined by (3) is univalent (or schlicht) in the upper-half E plane. Thus $g(E)$ maps the upper-half E plane conformally into a domain of the upper-half g plane in a one-to-one fashion.[6] It is known that univalent functions satisfy various sharp inequalities, and we shall explore their consequences in a separate publication. Here, we restrict ourselves to mentioning one result similar to (6) which follows from Koebe's theorem on univalent functions.[7]

For any real positive λ, we obtain

$$\int_0^{E_\lambda} \frac{[\mathrm{Re}f(E'+i0)-f(0)]}{E'^2} dE' \geq \frac{1}{\lambda} |f(i\lambda)-f(0)|, \quad (8)$$

where E_λ and λ are related by

$$\int_\mu^{E_\lambda} \frac{\mathrm{Im}f(E')}{E'^2} dE' = |g(i\lambda)|. \quad (9)$$

From the dispersion relation (2) one can always get a lower bound for the right-hand side of (8) which is independent of the value of the total cross section for $E' > E_m$. Namely, one can write

$$\frac{|f(i\lambda)-f(0)|}{\lambda} > \frac{2\lambda}{\pi} \int_\mu^{E_m} \frac{\mathrm{Im}f(E')}{E'(E'^2+\lambda^2)} dE'. \quad (10)$$

Although (8) is obtained from a general inequality and (6) is obtained by truncating the equality (5), it is not necessarily true that (6) is much better than (8). For example, one may take the simple form $\mathrm{Im}f(E') = cE'$ and compute the values of the quantities appearing on the right-hand sides of (6) and (8). For this simple Ansatz for Im*f*, it turns out that (6) is better than (8) by only about 50%. For the actual case we have the contribution from the 33 resonance which (6) tends to de-emphasize, while (8) does not.

A detailed numerical analysis of (6), (7), and (8) is clearly called for. However, the simple results of this short note make it clear that data on Re*f* should be obtained at all available energies before any meaningful comparison could be made.

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[2]See, for example, N. N. Khuri and T. Kinoshita, Phys. Rev. (to be published).

[3]These are widely known to follow from the Lehmann-Symanzik-Zimmermann formalism. The recent work of K. Hepp (to be published) shows that they also follow from the Wightman axioms of quantum field theory. The fact that only two subtractions are necessary follows from the improvement of the Greenberg-Low upper bound given in reference 2.

[4]The integral is convergent at zero since $f'(0) = 0$.

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[6]For a proof of this property of $g(E)$ see N. N. Khuri and T. Kinoshita, to be published. A more detailed paper is in preparation.

[7]See, for instance, W. K. Hayman, Multivalent Functions (Cambridge University Press, Cambridge, England, 1958), p. 3.

NONLEPTONIC BARYON DECAYS IN SU(6)

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It has been shown by a number of authors[1] that octet transformation properties of nonleptonic weak interactions with R or RP invariance lead to sum rules among four independent observable amplitudes of the nonleptonic baryon decays. In particular, the following sum rules are in good agreement with experiment:

p-v amplitudes (S wave),

$$A(\Lambda \to p + \pi^-) + 2A(\Xi^- \to \Lambda + \pi^-) = \sqrt{3}A(\Sigma^+ \to p + \pi^0); \quad (1)$$

p-c amplitudes (P wave),

$$B(\Lambda \to p + \pi^-) + 2B(\Xi^- \to \Lambda + \pi^-) = \sqrt{3}B(\Sigma^+ \to p + \pi^0). \quad (2)$$

The first relation can be proved without assuming R or RP invariance if we assume that the effective nonleptonic weak interactions are of the current$\times$current form and of nonderivative type.[2]

In this note, we wish to report some consequences arising from the assumptions that (i) strong interactions are approximately SU(6) invariant; (ii) nonleptonic weak interactions transform like a member of an adjoint representation under SU(6), i.e., parity-nonconserving (p-v) and parity-conserving (p-c) interactions transform as $(\underline{8},\underline{1})$ and $(\underline{8},\underline{3})$ members of the 35-dimensional representations, respectively; and (iii) weak interactions are invariant under the operation of CP.

It has been proposed by Gürsey, Radicati, and Pais[3] that octet and decuplet baryons are grouped together forming the bases of a 56-