CP NONCONSERVATION IN WEAK INTERACTIONS*

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The recent experiments[1] have shown that the long-lived component of K^0, K_L^0, decays into two pions. The branching ratio to the short-lived component K_S^0 is

$$R = \frac{\Gamma(K_L^0 \to \pi^+ + \pi^-)}{\Gamma(K_S^0 \to \pi^+ + \pi^-)} \approx 3.3 \times 10^{-6}. \tag{1}$$

Earlier experimental results were understood on the basis of the identification $K_1^0 \equiv K_S^0$ and $K_2^0 \equiv K_L^0$ and a purely $CP = +1$ weak interaction

$$\mathcal{L}_+ = (G/\sqrt{2})\{J_\alpha^\dagger, J_\alpha\}, \tag{2}$$

where $\{\}$ stands for the anticommutator. The current $J_\alpha = J_\alpha^l + J_\alpha^h$ where J_α^l is the leptonic part and the hadronic part is

$$J_\alpha^h = (\mathfrak{F}_{1\alpha}^+ + i\mathfrak{F}_{2\alpha}^+)\cos\theta + (\mathfrak{F}_{4\alpha}^+ + i\mathfrak{F}_{5\alpha}^+)\sin\theta, \tag{3}$$

where $\mathfrak{F}_{i\alpha}^+ = \mathfrak{F}_{i\alpha} + \mathfrak{F}_{i\alpha}^5$ in the notation of Gell-Mann[2]; i is the SU(3) index and α the space-time index. Since K_2^0 has $CP = -1$, the experiment clearly requires that CP invariance is violated by the weak interactions.

Many theoretical attempts have been given to explain this violation. Most of these attempts introduce an additional parameter suited to fit the observed CP nonconservation.[3] We make the observation that the commutator $\mathcal{L}_- = (G/\sqrt{2}) \times [J_\alpha^\dagger, J_\alpha]$ has $CP = -1$ and propose that the weak interaction be simply

$$\mathcal{L} = \mathcal{L}_+ + \mathcal{L}_-$$

or

$$\sqrt{2}GJ_\alpha^\dagger J_\alpha = (G/\sqrt{2})\{J_\alpha^\dagger, J_\alpha\} + (G/\sqrt{2})[J_\alpha^\dagger, J_\alpha]. \tag{4}$$

The commutator $\mathcal{L}_-$ seems the most natural way to introduce a $CP = -1$ part in a current $\times$ current picture. Furthermore, the $CP = +1$ and -1 terms are on an equal footing in our interaction to start with. However, the $\mathcal{L}_-$ term is highly singular and needs to be suitably defined. In fact, the evaluation of the matrix elements of a commutator like $\mathcal{L}_-$ is a very difficult problem[4] and needs to be solved. We will bypass this problem and attempt to discuss the consequences of the $\mathcal{L}_-$ term which follow directly from its structure and symmetry properties.

The structure of $\mathcal{L}_-$, the $CP = -1$ term, is as follows:

(a) The purely leptonic part is $[J_\alpha^{l\dagger}, J_\alpha^l] = [J_\alpha^{e\dagger}, J_\alpha^e] + [J_\alpha^{\mu\dagger}, J_\alpha^\mu]$, where J_α^e and J_α^μ are the electron and muon currents and these commute with eath other. Thus, $\mathcal{L}_-$ will give no contribution to muon decay.

To see the effect on lepton-lepton scattering, consider $[J_\alpha^{e\dagger}(x), J_\alpha^e(y)]$ where $J_\alpha^e(x) = \bar{e}(x)\Gamma_\alpha\nu(x)$ with $\Gamma_\alpha = \frac{1}{2}(1 - i\gamma_5)\gamma_\alpha$. Treating the electron and neutrino as free fields, which is not an unsensible thing to do, this would reduce to a term like $\sim m_e\bar{e}(x)\gamma_5 e(y)\Delta(x-y)$. Thus one may guess that the effective local contribution of the commutator is given by an effective pseudoscalar mass term[5] which has $CP = -1$. Such two-field terms cannot contribute to electron-neutrino scattering except in combination with $\mathcal{L}_+$. So one would expect that the CP-nonconservation effects in lepton-lepton scattering would be too small to be observed.

(b) Since J_α^l and J_α^h commute, $\mathcal{L}_-$ will not give any contribution to leptonic decays of the hadrons. Thus, the observation of any CP-invariance violation in the leptonic decays of mesons and baryons would be in contradiction to our theory.

(c) The remaining part of $\mathcal{L}_-$ is $[J_\alpha^{h\dagger}, J_\alpha^h]$, which is purely nonleptonic. To see what symmetry properties this has and what effects it may have, we make use of the equal-time commutators of $\mathfrak{F}_{i\alpha}^\pm$ given by Gell-Mann.[2] We have

$$[J_\alpha^{h\dagger}(x), J_\alpha^h(x')] = \{-4\cos^2\theta\,\mathfrak{F}_{34}^+ - 2\sin^2\theta(\mathfrak{F}_{34}^+ + \sqrt{3}\,\mathfrak{F}_{84}^+) + 4\sin\theta\cos\theta\,\mathfrak{F}_{64}^+\}\delta^3(x-x'). \tag{5}$$

Thus we expect the commutator to give purely octet terms and contributions to processes to which $\mathfrak{F}_{34}^+$, $\mathfrak{F}_{84}^+$, and $\mathfrak{F}_{64}^+$ can contribute.

Terms transforming like $\mathfrak{F}_{34}^+$ and $\mathfrak{F}_{84}^+$ will give $|\Delta S|=0$ weak nuclear interactions with $|\Delta\vec{I}|=1$ and $|\Delta\vec{I}|=0$, respectively. Terms transforming like $\mathfrak{F}_{64}{}^{+}$, in $\mathfrak{L}_{-}$, will give pure $|\Delta\vec{I}|=\frac{1}{2}$ and $|\Delta S|=1$ transitions. Furthermore, $\mathfrak{F}_{34}$, $\mathfrak{F}_{84}$, and $\mathfrak{F}_{64}$ have $\mathfrak{C}=-1$, $C=-1$, $P=+1$, while $\mathfrak{F}_{34}{}^{5}$, $\mathfrak{F}_{84}{}^{5}$, and $\mathfrak{F}_{64}{}^{5}$ have $\mathfrak{C}=1$, $C=1$, $P=-1$, where C is charge-conjugation parity, P is parity, and $\mathfrak{C}$ has been defined by Gell-Mann.[6] It is clear that a term like $\mathfrak{F}_{64}{}^{5}$ can allow $K_L{}^0 \to 2\pi$.

In the normal theory, with $\mathfrak{L}_{+}$ only, the nonleptonic weak interactions are described at best phenomenologically because of our inability to calculate the strong interaction effects. The problem with $\mathfrak{L}_{-}$ is no better, and we will resort to symmetry and phenomenological arguments to estimate the nonleptonic effects of $\mathfrak{L}_{-}$. Let the effective amplitudes from $\mathfrak{L}_{+}$ and $\mathfrak{L}_{-}$ to $K^0 \to 2\pi$ be $A_{+}\cos\theta\sin\theta$ and $A_{-}\sin\theta\cos\theta$; we have lumped our ignorance in $A_{\pm}$. So if we assume that $A_{-}/A_{+}\approx 2\times 10^{-3}$, then we can explain the experimental ratio R given in (1) above.[7] Assuming this, we can make crude estimates of the effect of CP-invariance violation in other nonleptonic weak processes.

(1) For nonleptonic decays of baryons, we expect the CP-nonconserving effects to be of the order of A_{-}/A_{+}, i.e., about 0.2%.

(2) In the $\Delta S=0$ weak nuclear processes, we expect larger effects. The $\Delta S=0, CP=+1$ amplitudes are of order $A_{+}\cos^2\theta$ for $|\Delta\vec{I}|=0$ and $A_{+}\sin^2\theta$ for $|\Delta\vec{I}|=1$, while the $\Delta S=0, CP=-1$ amplitudes are of order $A_{-}\sin^2\theta$ for $|\Delta\vec{I}|=0$ and $A_{-}\cos^2\theta$ for $|\Delta\vec{I}|=1$. Thus, on these crude estimates, we expect the $|\Delta S|=0$, $|\Delta\vec{I}|=0$, CP-invariance violating effects to be ~0.01%. The $|\Delta S|=0$, $|\Delta\vec{I}|=1$, CP-invariance violating effects expected are larger, about 3%.

A measurement of this small effect and establishment that there are no CP-nonconserving effects in muon decay and leptonic decays would provide a test of our theory.

In conclusion, we may remark that our interaction seems most natural in a current×current picture, even though $\mathfrak{L}_{-}$ is so singular. Maybe one should look upon the commutator as providing a rationale for introducing a CP-invariance violation of a very definite kind.

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[1]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fischer, B. M. K. Nefkens, and J. H. Smith, Phys. Rev. Letters 13, 286 (1964); W. Galbraith, G. Manning, A. E. Taylor, B. D. Jones, J. Malos, A. Astbury, N. H. Lipman, and T. G. Walker, Phys. Rev. Letters 14, 383 (1965).

[2]M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[3]References and discussion of the various attempts can be found in Y. Ne'eman, Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energy, University of Miami, January 1965 (W. H. Freeman & Company, San Francisco, California, to be published).

[4]See, for example, J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

[5]Of course, these are heuristic arguments. Note that a pseudoscalar mass term $\sim g\bar{\psi}\gamma_5\psi$ can be transformed away by a redefinition of ψ, as long as ψ has only vector and axial-vector interactions. For example, this is true of the leptons.

[6]M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

[7]It is not clear that A_{-} should be smaller than A_{+}. Hopefully, a proper dynamical calculation will give the assumed ratio. If not, one will have to choose a coupling constant smaller than G, in Eq. (4), for the commutator term.