tions). The $\Delta T \leq \frac{3}{2}$ rule then forbids $T=3$, so the decay amplitudes $a_{1,2}$ are purely symmetric, with $T=1$. Our theorem can therefore by applied, and gives

$$\frac{2\,\mathrm{Re}\,a_1a_2^*}{|a_1|^2+|a_2|^2}<1/50.$$

If $|a_1|\approx|a_2|$, then a_1 and a_2 must be relatively imaginary to within about 1°.

We are grateful for conversations with F. S. Crawford, Jr.

---

*Research supported in part by the U. S. Air Force Office of Scientific Research, Grant No. AF-AFOSR-232-63.

†Alfred P. Sloan Foundation Fellow.

[1]J. A. Anderson, F. S. Crawford, Jr., R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters 14, 475 (1965).

[2]T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957). We follow the convention of these authors in using K_1^0 and K_2^0 to denote the long- and short-lived components, which in general are not the same as $K^0 \pm \bar{K}^0$.

[3]The authors of reference 1 have, at our suggestion, reanalyzed their data with the additional constraint $\mathrm{Re}(a_1/a_2)=0$. They then find $\mathrm{Im}(a_1/a_2)=+0.90\pm0.50$. The corresponding intensity ratio is $\Gamma_1/\Gamma_2=0.81^{+1.15}_{-0.65}$, giving betting odds of 10 to 1 that Γ_1/Γ_2 is less than 2.5, and 100 to 1 that it is less than 5. The effect of the additional constraint is to reduce their upper limit on Γ_1/Γ_2 by a factor of two. They still cannot rule out $\Gamma_1/\Gamma_2=0$. (Private communication from F. S. Crawford, Jr.)

[4]J. H. Christensen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[5]F. S. Crawford, Jr., private communication.

MIXING MODEL FOR SCALAR MESONS*

Laurie M. Brown

Northwestern University, Evanston, Illinois

(Received 10 March 1965)

Indirect evidence has been presented for two isoscalar mesons of spin and parity 0^+: the sigma meson[1] of mass $m_\sigma \approx 400$ MeV and the epsilon meson[2] of mass $m_\epsilon \approx 750$ MeV. This situation is reminiscent of the ω-φ mixing problem and suggests that a similar model may be appropriate. It is well known that if the observed ω and φ mesons are assumed to be given by the orthogonal states

$$|\omega\rangle=\cos\lambda\,|\omega_0\rangle+\sin\lambda\,|\varphi_0\rangle,$$
$$|\varphi\rangle=\sin\lambda\,|\omega_0\rangle-\cos\lambda\,|\varphi_0\rangle, \qquad (1)$$

where ω_0 and φ_0 are hypothetical pure SU(3) singlet and pure SU(3) octet members, then there exists a "mixing angle" λ such that the decay properties of ω and φ are well explained and φ_0 has a calculated mass which satisifies the Gell-Mann–Okubo relation with the other members, ρ and K^*, of the vector octet.[3] We will show that if σ and ϵ are similarly assumed to be mixtures of hypothetical pure unitary singlet and octet isosinglet members σ_0 and ϵ_0, the $T=\frac{1}{2}$ member of the octet being $\kappa(725)$, then a $T=1$ meson ξ is predicted, decaying predominantly into $\eta+\pi$. The ordering of masses in this scheme is that suggested by various bootstrap calculations.[4]

We shall regard as input data the masses and widths of σ and κ. One additional datum is needed to completely determine the scheme and, for simplicity, we shall take for the mixing angle λ the value $\sin\lambda=1/\sqrt{3}$, which will be shown to be reasonably consistent with the not very well-established properties of ϵ.

Noting that the orthogonal matrix

$$S=\begin{pmatrix}\cos\lambda & \sin\lambda\\ \sin\lambda & -\cos\lambda\end{pmatrix}, \qquad (2)$$

formed of the eigenvectors (1), brings the mass-square matrix

$$M^2=\begin{pmatrix}\sigma_0 & x\\ x & \epsilon_0\end{pmatrix} \qquad (3)$$

to diagonal form (where σ_0 and ϵ_0 are squared masses of the particles and x is a mixing mass), we obtain

$$x=\sqrt{2}(\sigma_0-\epsilon_0),$$
$$\sigma=2\sigma_0-\epsilon_0,$$

and

$$\epsilon=2\epsilon_0-\sigma_0; \qquad (4)$$

hence, from the Gell-Mann–Okubo relation,

$$\xi=4\kappa-3\epsilon_0$$
$$=4\kappa-2\epsilon-\sigma. \qquad (5)$$

E.g., for $\epsilon \approx \kappa$, $\sqrt{\xi} \sim 950$ MeV; for $\epsilon \approx \rho$, on the other hand, $\sqrt{\xi} \sim 875$ MeV. In the latter case, the "equal-spacing rule"

$$K-\pi = K^* - \rho = \xi - \kappa \tag{6}$$

is well satisfied in the form appropriate for an inverted octet. The ordering of levels is in accord with predictions based on bootstrap calculations for scalar mesons.[4]

Turning now to the decay widths, we assume that for the four physical mesons these widths are determined by S-wave decays into two pseudoscalar mesons, namely $\sigma \to 2\pi$, $\epsilon \to 2\pi$, $\kappa \to K + \pi$, and $\xi \to \eta + \pi$. (Because of its even parity and odd G parity, ξ cannot decay strongly into a state of fewer than five pions.) For these decay modes the decay rate may be written

$$\Gamma_i = (g_i^2/4\pi m_i^2)(p_i/2), \tag{7}$$

where g_i is the effective coupling constant, m_i is the mass of the decaying particle, and p_i is the momentum of one of the decay products in the rest frame of the decaying particle. The factors $C_i = g_i^2/4\pi m_i^2$ are dimensionless and, as will become clear below, we treat them as the characteristic constants in the split nonet.[5]

The relevant parts of the SU(3) D-type couplings are given by

$$\epsilon_0 \to \pi^+ + \pi^-, \quad C_a = 8/3(g_8^2/4\pi\epsilon_0); \tag{8a}$$

$$\kappa^+ \to K^0 + \pi^+, \quad C_b = 4(g_8^2/4\pi\kappa); \tag{8b}$$

$$\xi^0 \to \eta + \pi^0, \quad C_c = 8/3(g_8^2/4\pi\xi); \tag{8c}$$

while for the singlet,

$$\sigma_0 \to \pi^+ + \pi^-, \quad C_d = 2(g_1^2 14\pi\sigma_0). \tag{8d}$$

For (8a), (8b), and (8d), the widths obtained with the C_i are two-thirds of the total widths of the particles, and for (8c) it is the total width. These constants are derived from an interaction Lagrangian

$$\mathcal{L}' = g_1 \sigma_0 T_r \varphi\varphi + g_8 T_r \varphi'\varphi\varphi, \tag{9}$$

where φ and φ' are 3×3 matrices representing the pseudoscalar and scalar octets, respectively.

Assuming again that $\sin\lambda = 1/\sqrt{3}$, Eq. (1) leads to

$$C_\sigma = \tfrac{2}{3}C_d + \tfrac{1}{3}C_a,$$

$$C_\epsilon = \tfrac{1}{3}C_d + \tfrac{2}{3}C_a; \tag{10}$$

or, in terms of the physical particles,

$$C_\epsilon = \tfrac{1}{2}C_\sigma + [\kappa/(\sigma + 2\epsilon)]C_\kappa, \tag{11}$$

where the C's in Eq. (11) all refer to two-thirds of the total widths. The total width of ϵ for $\epsilon \approx \rho$ or $\epsilon \approx \kappa$ is then found easily to be about 135 MeV (taking $\Gamma_\sigma = 95$ MeV, $\Gamma_\kappa = 23$ MeV).[1,6] By contrast, for the case of no mixing, $\Gamma_\epsilon = 1.5\Gamma_\kappa$, and so is about 35 MeV. In this way one should be able to tell whether the mixing picture is necessary, as for the vector mesons, or not necessary, as for the pseudoscalar mesons.

For the predicted $T = 1$ meson ξ, decaying into $\eta + \pi$, $\Gamma_\xi \approx \tfrac{1}{2}\Gamma_\kappa$ with mixing and, as stated before, its mass would be about 900 MeV. For no mixing, since $\epsilon \approx \kappa$, we would have $\xi \approx \kappa$ and $\Gamma_\xi \approx \tfrac{1}{4}\Gamma_\kappa$. Thus if ξ is observed, its mass and width will also determine the amount of mixing.

---

*Research supported by the National Science Foundation.

[1]L. M. Brown and P. Singer, Phys. Rev. 133, B812 (1964); L. M. Brown and H. Faier, Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1965 (W. H. Freeman & Company, San Francisco, California, to be published). A best fit to the π^0 decay spectrum of the η meson gives $m_\sigma = 400^{+15}_{-8}$ MeV and $\Gamma_\sigma = 95^{+12}_{-14}$ MeV.

[2]L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters 14, 329 (1965). These authors have found a good fit to the decay asymmetry of the ρ^0 meson by assuming $m_\epsilon = 770$ MeV, $\Gamma_\epsilon = 140$ MeV. More recently a $T = 0$ scalar meson called s^0 has been reported by the University of Pennsylvania spark chamber group with mass ~720 MeV and width ~50 MeV, and confirmatory evidence has been seen by the Penn-Saclay bubble chamber collaboration. We wish to thank Professor W. Selove for this information.

[3]In terms of the "folk model" based upon a fundamental SU(3) triplet (p, n, Λ), it is easily shown that this mixture has the property that $|\varphi\rangle \sim |\bar{\Lambda}\Lambda\rangle$ and $\omega \sim 1/\sqrt{2}|\bar{p}p + \bar{n}n\rangle$. A choice frequently made for λ is $\sin\lambda = 1/\sqrt{3}$.

[4]Chan Hong-Mo, P. C. De Celles, and J. E. Paton, Nuovo Cimento 33, 70 (1964); B. Diu and H. R. Rubinstein, Phys. Letters 9, 76 (1964); Chan Hong-Mo, Phys. Letters 11, 269 (1964); Chan Hong-Mo and C. Wilkin, Phys. Letters 13, 263 (1964).

[5]If, instead, we were to treat the g_i^2 as independent of mass, we would obtain the unacceptable result that the decay rate into a given mode would decrease as we increase the mass of the decaying particle.

[6]M. Ferro-Luzzi et al., Phys. Letters 12, 255 (1964).