PHASE OF THE CP-INVARIANCE VIOLATION IN τ^0 DECAY*

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A recent experiment[1] indicates possible strong CP-invariance violation in the decay mode $K_1^0 \to \pi^+ + \pi^- + \pi^0$. The measured ratio of the K_1^0 and K_2^0 decay amplitudes,[2]

$$\mathrm{Re}(a_1/a_2) = +0.25 \pm 0.65,$$
$$\mathrm{Im}(a_1/a_2) = +1.00 \pm 0.65, \quad (1)$$

suggests that this ratio may have absolute magnitude of order unity and phase near 90°, though of course the statistics are hardly conclusive. (If CP were conserved then K_1^0 could only decay into asymmetric three-pion states with large centrifugal barriers, and a_1/a_2 would be very small.)

We wish to point out that if a_1/a_2 is actually not much less than one in magnitude, then CPT symmetry and the $\Delta T \leq \frac{3}{2}$ rule require its phase to be very close to ±90°. This constraint was not imposed in the analysis made in reference 1, so the fact that their analysis did give a mostly imaginary ratio lends additional credence to the existence of a strong $K_1^0 \to 3\pi$ mode.[3] If this mode does exist, then a refined measurement of the phase of a_1/a_2 can provide a sensitive test of CPT and/or $\Delta T \leq \frac{3}{2}$.

Our remarks are based on the following simple theorem: If A_1 and A_2 are the amplitudes for K_1^0 and K_2^0 decay into any self-CPT-conjugate channel which is an eigenstate of the strong-interaction S matrix [e.g., $(\pi\pi)_{T=0}$], then

$$\frac{2\,\mathrm{Re}A_1A_2^*}{|A_1|^2 + |A_2|^2} = \alpha, \quad (2)$$

with α a number independent of the particular channel being considered.

To prove this result, we recall that if CPT (but not necessarily CP) is conserved, then K_1^0 and K_2^0 are given as linear combinations,[2]

$$K_1^0 = pK^0 + q\bar{K}^0,$$
$$K_2^0 = pK^0 - q\bar{K}^0, \quad (3)$$

where p and q are in general model-dependent complex numbers satisfying

$$|p|^2 + |q|^2 = 1$$

but subject to no other general conditions. A simple calculation gives

$$\frac{2\,\mathrm{Re}A_1A_2^*}{|A_1|^2 + |A_2|^2} = \frac{|p|^2|A|^2 - |q|^2|\bar{A}|^2}{|p|^2|A|^2 + |q|^2|\bar{A}|^2}, \quad (4)$$

where A and $\bar{A}$ are the amplitudes for K^0 and $\bar{K}^0$ decay into the channel. But for a self-CPT-conjugate channel with phase shift δ (and no final-state mixing with other channels), CPT invariance tells us that

$$\bar{A} = \pm e^{2i\delta} A^*, \quad (5)$$

so that $|A| = |\bar{A}|$, and Eq. (3) gives Eq. (2) with α taking the channel-independent value

$$\alpha = |p|^2 - |q|^2. \quad (6)$$

We first apply this result to the 2π channels in order to set an upper limit on α. From the known rates[4] of $K_1^0 \to 2\pi$ and $K_2^0 \to \pi^+ + \pi^-$, we learn

$$|(\tfrac{2}{3})^{1/2} A_2^{T=0} + (\tfrac{1}{3})^{1/2} A_2^{T=2}| \simeq (1/450)(\tfrac{2}{3})^{1/2} |A_1^{T=0}|, \quad (7)$$

while the conservative estimate[5] that $\Gamma(K_2^0 \to 2\pi^0) < \frac{1}{2}\Gamma(K_2^0 \to 3\pi^0)$ yields

$$|(\tfrac{1}{3})^{1/2} A_2^{T=0} - (\tfrac{2}{3})^{1/2} A_2^{T=2}| < (1/55)(\tfrac{2}{3})^{1/2} |A_1^{T=0}|. \quad (8)$$

From (7) and (8) we conclude that $|A_2^{T=0}| < (1/100)|A_1^{T=0}|$, so (2) tells us that

$$|\alpha| < 1/50. \quad (9)$$

Actually, any reasonable model would require $\Gamma(K_2^0 \to 2\pi^0) \approx \Gamma(K_2^0 \to \pi^+ + \pi^-)$, in which case the actual value of α would be much smaller. For instance, if the $\Delta I = \frac{1}{2}$ rule applies in $K_2^0 \to 2\pi$ decay (which it well may not), then $|\alpha| \simeq 1/225$.

Now consider the decay mode $K_{1,2}^0 \to \pi^+ + \pi^- + \pi^0$. In general, there would be seven different amplitudes with $T = 0$, 1, 2, and 3, but centrifugal barriers strongly suppress all but the two completely symmetric amplitudes, with $T = 1$ and $T = 3$ (and also prevent final-state mixing with the other asymmetric $T = 1$ configurations). The $\Delta T \leq \frac{3}{2}$ rule then forbids $T = 3$, so the decay amplitudes $a_{1,2}$ are purely symmetric, with $T = 1$. Our theorem can therefore by applied, and gives

$$\frac{2\,\mathrm{Re}\,a_1 a_2^*}{|a_1|^2 + |a_2|^2} < 1/50.$$

If $|a_1| \approx |a_2|$, then a_1 and a_2 must be relatively imaginary to within about 1°.

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[1]J. A. Anderson, F. S. Crawford, Jr., R. L. Golden, D. Stern, T. O. Binford, and V. G. Lind, Phys. Rev. Letters 14, 475 (1965).

[2]T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. 106, 340 (1957). We follow the convention of these authors in using K_1^0 and K_2^0 to denote the long- and short-lived components, which in general are not the same as $K^0 \pm \bar{K}^0$.

[3]The authors of reference 1 have, at our suggestion, reanalyzed their data with the additional constraint $\mathrm{Re}(a_1/a_2) = 0$. They then find $\mathrm{Im}(a_1/a_2) = +0.90 \pm 0.50$. The corresponding intensity ratio is $\Gamma_1/\Gamma_2 = 0.81^{+1.15}_{-0.65}$, giving betting odds of 10 to 1 that Γ_1/Γ_2 is less than 2.5, and 100 to 1 that it is less than 5. The effect of the additional constraint is to reduce their upper limit on Γ_1/Γ_2 by a factor of two. They still cannot rule out $\Gamma_1/\Gamma_2 = 0$. (Private communication from F. S. Crawford, Jr.)

[4]J. H. Christensen, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[5]F. S. Crawford, Jr., private communication.

MIXING MODEL FOR SCALAR MESONS*

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Indirect evidence has been presented for two isoscalar mesons of spin and parity 0^+: the sigma meson[1] of mass $m_\sigma \approx 400$ MeV and the epsilon meson[2] of mass $m_\epsilon \approx 750$ MeV. This situation is reminiscent of the ω-φ mixing problem and suggests that a similar model may be appropriate. It is well known that if the observed ω and φ mesons are assumed to be given by the orthogonal states

$$|\omega\rangle = \cos\lambda\,|\omega_0\rangle + \sin\lambda\,|\varphi_0\rangle,$$
$$|\varphi\rangle = \sin\lambda\,|\omega_0\rangle - \cos\lambda\,|\varphi_0\rangle, \qquad (1)$$

where ω_0 and φ_0 are hypothetical pure SU(3) singlet and pure SU(3) octet members, then there exists a "mixing angle" λ such that the decay properties of ω and φ are well explained and φ_0 has a calculated mass which satisifies the Gell-Mann–Okubo relation with the other members, ρ and K^*, of the vector octet.[3] We will show that if σ and ϵ are similarly assumed to be mixtures of hypothetical pure unitary singlet and octet isosinglet members σ_0 and ϵ_0, the $T = \frac{1}{2}$ member of the octet being $\kappa(725)$, then a $T = 1$ meson ξ is predicted, decaying predominantly into $\eta + \pi$. The ordering of masses in this scheme is that suggested by various bootstrap calculations.[4]

We shall regard as input data the masses and widths of σ and κ. One additional datum is needed to completely determine the scheme and, for simplicity, we shall take for the mixing angle λ the value $\sin\lambda = 1/\sqrt{3}$, which will be shown to be reasonably consistent with the not very well-established properties of ϵ.

Noting that the orthogonal matrix

$$S = \begin{pmatrix} \cos\lambda & \sin\lambda \\ \sin\lambda & -\cos\lambda \end{pmatrix}, \qquad (2)$$

formed of the eigenvectors (1), brings the mass-square matrix

$$M^2 = \begin{pmatrix} \sigma_0 & x \\ x & \epsilon_0 \end{pmatrix} \qquad (3)$$

to diagonal form (where σ_0 and ϵ_0 are squared masses of the particles and x is a mixing mass), we obtain

$$x = \sqrt{2}(\sigma_0 - \epsilon_0),$$
$$\sigma = 2\sigma_0 - \epsilon_0,$$

and

$$\epsilon = 2\epsilon_0 - \sigma_0; \qquad (4)$$

hence, from the Gell-Mann–Okubo relation,

$$\xi = 4\kappa - 3\epsilon_0$$
$$= 4\kappa - 2\epsilon - \sigma. \qquad (5)$$