a small background shift (1%), the data do not agree with the model. (4) Empirically, it has been found that in the lead salts, the incremental zero-bias resistance exhibits a two-segment logarithmic temperature dependence.

Finally, the spikes and the lack of fit to Wyatt's model are characteristic not only of lead salt diodes but also of III-V diodes.[9] If the effects being observed here are explainable within the present framework of tunneling theory,[10] then at low temperatures the measured conductance is the product of the density of states and the tunnel probability. Although previous tunnel phenomena have been interpreted as density-of-states effects, we cannot eliminate the tunnel probability on the basis of existing data. Both the smooth effects and the spike resonance spectroscopy are being pursued experimentally and theoretically.

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BAND POPULATION EFFECT ON THE INTERBAND FARADAY ROTATION IN SOLIDS: PbS

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The presence of a strongly temperature-dependent "paramagnetic" contribution to the Faraday rotation associated with localized centers in a magnetic field has long been recognized.[1,2] This contribution arises from the thermally induced difference in population between the ground and low-lying excited states of the system from which the left- and right-circularly polarized transitions (lcp and rcp) originate. More recently, strongly temperature- and carrier concentration-dependent contributions to the interband rotation have been observed experimentally in the semiconductors PbS[3,4] and GaSb.[5] In these materials, the interband rotation reverses sign at some low temperature. This temperature increases with increasing free carrier concentration. An explanation for this reversal in terms of the difference in the Burstein-Moss edge shift for lcp and rcp radiation has been given for PbS[4] and suggested for GaSb.[6] An additional feature of this effect which has not been recognized previously is that the interband rotation is modified by the free carrier population in such a way as to approach a finite value at low frequencies, in contrast to the interband rotation in insulating solids which approaches zero at zero

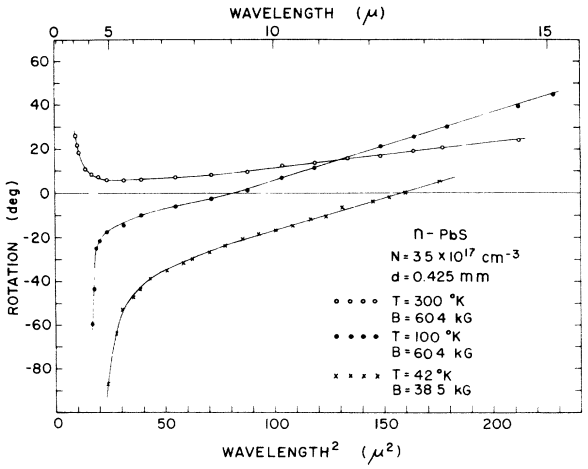

FIG. 1. Measured Faraday rotation in n-type PbS including both interband and free carrier contributions.

frequency. In this Letter, we present a calculation of the temperature-dependent interband Faraday rotation for simple bands with a Fermi distribution of carriers in one of the bands. Also presented are experimental data for the temperature-dependent Faraday rotation in n-type PbS measured over an extended frequency range. A comparison is made with the calculated rotation.

The reversal of the interband rotation at low temperatures in PbS is shown in Fig. 1. In addition, the presence of a low-frequency tail to the interband rotation is indicated in these data by the failure of the long-wavelength "free-carrier" rotation to extrapolate to zero at zero wavelength. We suggest that such a term is present in the published data for other materials which also fail this test.[7] This experimental feature of published data has been the subject of frequent comment but has not previously been adequately explained.

Dispersion relations for isotropic media in the presence of a magnetic field have been developed by Bennett and by Boswarva in unpublished theses. Alternative relations in terms of the dielectric tensor or conductivity tensor have subsequently been published.[8,9] In insulating crystals, the alternative forms for the dispersion relations are equally valid.[9] If free carriers are present then the dielectric dispersion relation requires modification. This is discussed by Landau and Lifshitz[10] for the zero-field dispersion relations. The complication stems from the additional pole at zero frequency introduced into the polarizability by the free carriers. Physically, this corresponds to the infinite polarizability of free carriers. A similar term appears in the dielectric dispersion relation for the magnetic field case. For this reason we use the dispersion relation derived from the conductivity tensor,

$$[n_r^2(\omega)-n_l^2(\omega)]-[\kappa_r^2(\omega)-\kappa_l^2(\omega)]=\frac{2c}{\omega\pi}\int_0^\infty \frac{\omega'[n_r(\omega')\alpha_r(\omega')-n_l(\omega')\alpha_l(\omega')]d\omega'}{\omega'^2-\omega^2}, \tag{1}$$

where $n_{r,l}$ and $\kappa_{r,l}$ are the index and extinction coefficients for rcp and lcp, respectively, $\alpha_{r,l}$ are the absorption coefficients, and ω is the angular frequency of the radiation. In the limit of low magnetic fields and at frequencies ω where there is low absorption, the relation may be written[9]

$$n(\omega)\Theta(\omega)=\frac{1}{2\pi}\int_0^\infty \frac{\omega'[n_r(\omega')\alpha_r(\omega')-n_l(\omega')\alpha_l(\omega')]d\omega'}{\omega'^2-\omega^2}, \tag{2}$$

where $n(\omega)$ is the zero-field index and $\Theta(\omega)$ is the Faraday rotation per unit of thickness. In the low-frequency limit for insulating solids, the integral in Eq. (2) is zero.[9] Bennett and Stern have shown that it is nonzero when free carriers are present and have specifically calculated the intraband free carrier contribution in the limit $\omega \to 0$. In contrast to this, we calculate a finite value for the Faraday rotation in the "low-frequency limit" $\hbar\omega \ll E_g$ arising from the population effect on the interband transitions. Thus we are interested in the frequency interval that is high compared to the cyclotron frequency but is low compared to the interband transition frequencies. In this frequency range, the intraband transitions give rise to the usual λ^2 term for the Faraday rotation which we subtract from the data in order to obtain the interband contributions.

The valence and conduction bands in PbS are nondegenerate and are nearly spherical and parabolic in the vicinity of the band extrema.[4,13] The conduction band in PbS is split off from higher conduction bands by the spin-orbit interaction, which is quite large. Consequently, the Kramers degenerate pair of conduction-band states have both "spin-up" and "spin-down" components present so that electric dipole transitions with "spin flips" are allowed. A schematic diagram of the Landau levels and selection rules with added free carriers is shown in Fig. 2. This diagram illustrates the selective blocking of the transitions for one sense of circular polarization. This effect is the origin of the temperature-dependent terms for the interband Faraday rotation.

The absorption difference appearing in Eq. (2) was calculated for the simple level scheme of Fig. 2 by summing over the Landau levels with

a Fermi distribution of free carriers in the conduction band. Taking the low-field limit and neglecting oscillatory terms we obtain

$$[n_r(\omega)\alpha_r(\omega)-n_l(\omega)\alpha_l(\omega)] = 4\sqrt{2}M\frac{m_{cv}^{3/2}}{a_0}\frac{|P_{cv}|^2}{m_0E_g}\left(\frac{E_g}{m_0c^2}\right)^{1/2}\frac{\beta H}{2\hbar\omega}$$

$$\times\left\{\frac{(g_c+g_v)[(x-1)^{-1/2}-4(x-1)^{1/2}/x]}{1+\exp\{[E_F'-(\hbar\omega-E_g)]/kT'\}}-\frac{E_g[g_v-(m_c/m_v)g_c](x-1)^{1/2}}{2kT'\cosh^2\{[E_F'-(\hbar\omega-E_g)]/2kT'\}}\right\}, \quad (3)$$

where M is the number of equivalent bands, m_{vc} is the ratio of reduced effective mass to the free electron mass m_0, a_0 is the Bohr radius, and β is $\hbar$ times the Bohr magneton. E_F' and kT' are the Fermi energy and thermal energy multiplied by the scaling factor $[1+(m_c/m_v)]$ and x is the dimensionless ratio $(\hbar\omega/E_g)$. The remainder of the terms have their usual meaning.

In the derivation of Eq. (3), it is assumed that $P_{cv}{}^l/\omega_{cv}{}^l = P_{cv}{}^r/\omega_{cv}{}^r$, where $P_{cv}{}^{l,r}$ is the momentum matrix element and $\omega_{cv}{}^{l,r}$ is the frequency for a given lcp or rcp transition.[11] This assumption is not critical to this discussion since it only affects the first term in Eq. (3). This term has the form of the lcp-rcp absorption difference for a pure crystal multiplied by the Fermi occupation factor. The second term arises from the difference in the Burstein-Moss absorption edges for lcp and rcp radiation. This term is responsible for the reversal of the rotation and the finite contribution in the low-frequency limit. It should be noted that different combinations of the g factors occur in the two terms, so that the valence- and conduction-band g factors can be determined individually from the measurement of the interband Faraday rotation with added carriers in one band. Lax and Nishina[12] discussed contributions to the rotation for populated bands corresponding to the first term in Eq. (3), but derived incorrect equations due to the use of incorrect dispersion relations.

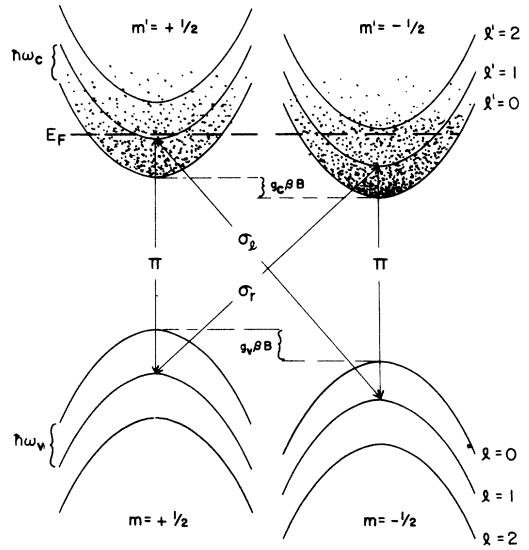

FIG. 2. E versus k_z diagram for simple and parabolic bands in the presence of a large magnetic field in the z direction. The spacing between adjacent Landau levels labeled by l or l' is the cyclotron energy $\hbar\omega_v$ or $\hbar\omega_c$. The gyromagnetic splitting of the individual Landau levels, labeled by the quantum numbers $m = \pm\frac{1}{2}$ or $m' = \pm\frac{1}{2}$, is given by $g_v\beta B$ or $g_c\beta B$ with the effective g factors both taken to be positive. The sign of g_v as determined experimentally in PbS is negative. The stippling represents the occupation of the conduction band states by free carriers with a Fermi level E_F. The $\sigma_{r,l}$ transitions are for rcp and lcp radiation with the plane of polarization perpendicular to the magnetic field. The π transitions are for radiation linearly polarized parallel to the magnetic field.

PbS is particularly well suited to show the effect of the second term, since the valence- and conduction-band g factors are large and approximately equal in magnitude but opposite in sign.[4] Thus the first term of Eq. (3) is reduced in magnitude while the second is enhanced. Equation (3) was evaluated for several temperatures using the band parameters established by previous studies[4,13] and the value 0.52 eV for the matrix element $|P_{cv}|^2/2m_0$ obtained from a fit to the zero field absorption. The variations of the band parameters and Fermi level with temperature are appreciable and have been included in the calculation. At the lowest temperatures, the first term in Eq. (3) is negligible and the second term is a strongly peaked function of frequency. We have used this fact in executing the numerical integration of Eq. (2). The calculated curve at 42°K is plotted in Fig. 3

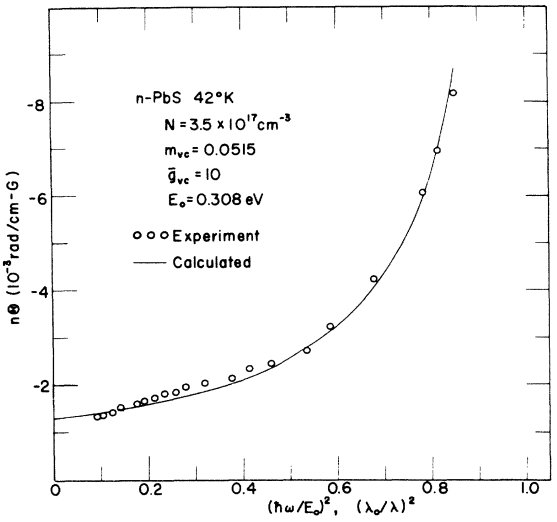

FIG. 3. Calculated and measured plot of the interband rotation in n-type PbS at low temperature. The calculated free carrier rotation has been subtracted from the data of Fig. 1 of this plot. Note the finite value obtained in the "low-frequency limit." The energy E_0 represents the center of the strongly peaked function in Eq. (3) which was utilized in the numerical integration of the dispersion relation. The wavelength $\lambda_0 = hc/E_0$. $\bar{g}_{vc}$ is one-half the g-factor combination which appears in the second term of Eq. (3).

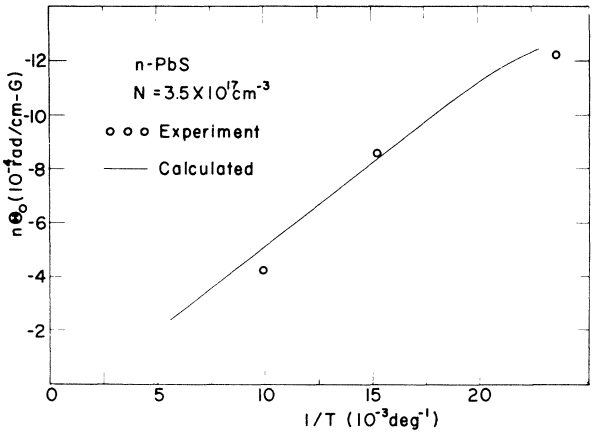

FIG. 4. Calculated and measured plot of the interband rotation $n\theta$ in the low-frequency limit. The calculated curve includes the temperature variation of the Fermi level and band parameters but ignores the band-edge contribution represented by the first term in Eq. (3). This is a good approximation below 150°K.

together with the experimental data of Fig. 1 less the calculated free carrier rotation. There is excellent agreement in both magnitude and frequency dependence. It is to be noted that a finite value for the rotation is obtained in the low-frequency limit. The contributions from other interband transitions have been estimated using the results of a $\vec{k}\cdot\vec{p}$ calculation. The contributions are of the order of a few percent and have been neglected.

The experimental and calculated contributions to $n\theta$ in the low-frequency limit are plotted in Fig. 4 as a function of temperature. The curve is of limited accuracy above 150°K due to the neglect of the first term in Eq. (3). Omission of the low-frequency contribution indicated in Fig. 4 may lead to erroneous analysis of the ordinary free carrier Faraday rotation. The magnitude and sign of the low-frequency interband population rotation may be estimated from the extrapolation of the long-wavelength rotation to zero wavelength. We have experimentally observed contributions of the proper sign and magnitude in p-type PbS and PbTe. In these materials the population effect is of the same sign as the rotation in the pure crystal so that no reversal is observed. In GaAs and GaSb, a similar test for extrapolation of the long-wavelength rotation indicates the presence of a significant population effect on the interband rotation; however, a detailed calculation is difficult due to the complex valence bands of these materials.

In summary, we have calculated the interband Faraday rotation arising from the selective blocking of lcp or rcp transitions by free carriers in a spherical, parabolic band and obtain agreement with the experimental results for PbS. The most significant feature of the study is the presence of a low-frequency contribution to the rotation which has not been identified previously.

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TENSOR POLARIZATION OF DEUTERONS FROM p-d ELASTIC SCATTERING

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The elastic scattering of nucleons by deuterons provides a useful method for studying the fundamentally important three-nucleon system. In the present experiment, the second-rank tensor polarization moments of the recoil deuterons from p-d elastic scattering have been determined at several proton energies in the range between 4 and 8.7 MeV.

Satisfactory fits to n-d and p-d angular distribution data, based on the "resonating group" method, have been obtained by using velocity-independent central forces only and by assuming that the deuteron is not distorted in the interaction.[1] Such forces, however, result in predictions of zero polarization for both the nucleon and the recoil deuteron, whereas published data[2,3] appear to indicate the presence of small but nonzero values of the nucleon polarization at energies below 10 MeV and increasingly large values at higher energies. In addition, the present measurements indicate some small but nonzero values of the deuteron polarization tensors.

It would be desirable if numerical calculations were performed in which both a spin-orbit and a tensor term in the nucleon-nucleon interaction were included. Either of these terms could lead to nucleon and deuteron polarization. A resonating-group formalism in which tensor (but not spin-orbit) forces are included has been published, but numerical results have not yet appeared in the literature.[4] The problem has also been formulated with a spin-orbit (but not tensor) force, but again no numerical results have appeared.[5] A more approximate calculation, in which the exchange of the two like nucleons is not taken into account, has been published.[6] This calculation does take into account, to some extent, the distortion of the deuteron in the scattering and includes a tensor force. However, the model is a simple one and is expected to give only qualitative results.[7]

In the present experiment, the tensor polarization of deuterons from p-d scattering was determined for proton energies of 4.06, 5.06, and 5.84 MeV at a laboratory scattering angle of 30°, and for proton energies of 6.10, 7.59, and 8.69 MeV at a laboratory scattering angle of 45°. To make the measurements, a deuterated polyethylene target was bombarded with a proton beam, and the recoiling deuterons, after being slowed to about 800 keV, were used to initiate the reaction $He^3(d,p)He^4$.[8] From the observed angular distribution of the resulting protons, the second-rank spin moments[9] $\langle T_{20}\rangle$, $\langle T_{21}\rangle$, and $\langle T_{22}\rangle$ of the recoil deuterons were determined from the expression[10]

$$\sigma(\theta,\varphi)=\sigma_0[1-\tfrac{1}{4}\sqrt{2}\langle T_{20}\rangle(3\cos^2\theta-1) -\sqrt{3}\langle T_{21}\rangle\sin\theta\cos\theta\cos\varphi -\tfrac{1}{2}\sqrt{3}\langle T_{22}\rangle\sin^2\theta\cos 2\varphi], \quad (1)$$

where σ_0 is the cross section for scattering of