The first integral in Eq. (10) may be determined directly from the elastic angular distribution. Since $f'(0)$ is now supposed known, we have obtained the desired result provided that a unique finite limit exists for the final integral in Eq. (10). This integral is readily performed analytically and yields

$$\int_\epsilon^{\theta_0} f_{\rm Coul}^*(\theta)\sin\theta d\theta$$
$$= -(\eta/2k)\exp(-2i\sigma_0)\int_\epsilon^{\theta_0}(\sin\tfrac{1}{2}\theta)^{-2} \times\exp(2i\eta\ln\sin\tfrac{1}{2}\theta)\sin\theta d\theta$$
$$= (i/k)\exp(-2i\sigma_0)\{\exp(2i\eta\ln\sin\tfrac{1}{2}\theta_0) -\exp(2i\eta\ln\sin\tfrac{1}{2}\epsilon)\}. \qquad (11)$$

The limit $\epsilon\to 0$ does not exist as a uniform limit. However, from Eq. (2) we see that

$$\lim_{\epsilon\to 0}\int_\epsilon^{\pi} f_{\rm Coul}^*(\theta)\sin\theta d\theta$$
$$= \lim_{\epsilon\to 0}(i/k)\{\exp(-2i\sigma_0)-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\epsilon)\}$$
$$= (1/-2ik)\sum_{l=0}^{\infty}(2l+1)[\exp(-2i\sigma_0)-1] \times\int_0^{\pi}P_l(\cos\theta)P_0(\cos\theta)\sin\theta d\theta$$
$$= (i/k)[\exp(-2i\sigma_0)-1], \qquad (12)$$

so that the nonuniform limit of interest is obtained by choosing a sequence of ϵ_n such that

$$\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\epsilon_n)=1 \qquad (13)$$

as $\epsilon_n\to 0$. Thus the required limit in Eq. (11) is

$$\lim_{\epsilon\to 0}\{\int_\epsilon^{\theta_0} f_{\rm Coul}^*\sin\theta d\theta\}$$
$$= (ik)^{-1}[1-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\theta_0)]. \qquad (14)$$

We may now rewrite Eq. (10) as

$$\sigma_{\rm el}{}'^{\rm tot} = 2\pi\int_{\theta_0}^{\pi}\{\sigma_{\rm el}(\theta)-\sigma_{\rm Coul}(\theta)\} \times\sin\theta d\theta + 4\pi|f'(0)|^2\sin^2\tfrac{1}{2}\theta_0 + (4\pi/k) \times{\rm Im}\{f'(0)[1-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\theta_0]\}, \qquad (15)$$

and finally we obtain for the total reaction cross section

$$\sigma_{\rm r}^{\rm tot} = (4\pi/k){\rm Im}\{f'(0)\exp(-2i\sigma_0+2i\eta\ln\tfrac{1}{2}\sin\theta_0)\} -2\pi\int_{\theta_0}^{\pi}\{\sigma_{\rm el}(\theta)-\sigma_{\rm Coul}(\theta)\}\sin\theta d\theta -4\pi|f'(0)|^2\sin\tfrac{1}{2}\theta_0. \qquad (16)$$

In the limit as the charge goes to zero, Eq. (16) yields the familiar result that the total reaction cross section is the total cross section, as given by the optical theorem, minus the total elastic cross section. However, for the scattering of uncharged particles, the amplitude $f'(0)$ cannot, in general, be determined from the elastic-scattering data. For charged-particle scattering, $f'(0)$ can be determined, and hence Eq. (16) can be used to obtain the reaction cross section from the elastic-scattering data. The feasibility of the procedure described was checked by using an optical-model code to generate "experimental" data. We have also determined reaction cross sections for a few cases in which sufficient experimental data exist. Further details will be given in a paper now in preparation.

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SU(6) AND THE ELECTROMAGNETIC FORM FACTORS*

K. J. Barnes, P. Carruthers,† and Frank von Hippel
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
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Recently an interesting attempt has been made[1] to combine internal symmetries with spin degrees of freedom in a nontrivial way using the group SU(6). The striking correlations that have been obtained[2] are to be contrasted with the difficulties that have arisen in the interpretation of the symmetry.[3,4] To date all successes have been confined to the correlation of essentially static properties of the elementary particles. We discuss here a nonstatic problem in which the possibility of an SU(6) symmetry arises in a natural way. In the nonrelativistic limit the generators of this SU(6) coincide with those previously used.[1,2]

Specifically, we show that if an SU(6) invariance of the type discussed below actually holds, then the magnetic moment ratio derived by Bég, Lee, and Pais[2] generalizes in a simple way to the form-factor ratio

$$G_M^{\,n}(q^2)/G_M^{\,p}(q^2)=\mu_n/\mu_p=-\tfrac{2}{3}, \qquad (1)$$

where q^2 is the square of the four-momentum transfer. This relationship agrees well with experiment[5] over the measured range of q^2 ($0<-q^2<45$ F^{-2}). Similarly the fact that the neutron has zero charge generalizes under this SU(6) to

$$G_E^{\,n}(q^2)/G_E^{\,p}(q^2)=0. \qquad (2)$$

Experimentally $G_E^{\,n}(q^2)$ is substantially smaller than $G_E^{\,p}(q^2)$ for $0<-q^2<20$ F^{-2}, but the data are not sufficiently accurate[5] to show how small the ratio is or even that it is smaller than one for $-q^2>20$ F^{-2}.

The key to our generalization is the recognition that the Breit frame for the vertex function (where the initial and final nucleon three-momenta are $\vec{p}$ and $-\vec{p}$) has all the features of the rest frame of a single particle requisite to the construction of an SU(6). In this frame we may consider the spin transformation of the Dirac spinor for a baryon of momentum $\vec{p}$ by means of the relativistic spin operator[6]

$$\vec{S}=\gamma_0(\vec{\sigma}\cdot\vec{e}_1)\vec{e}_1+\gamma_0(\vec{\sigma}\cdot\vec{e}_2)\vec{e}_2+(\vec{\sigma}\cdot\hat{p})\hat{p}. \qquad (3)$$

Here γ_0 and $\vec{\sigma}$ are the usual 4×4 Dirac matrices, and the unit vectors $\vec{e}_1, \vec{e}_2, \hat{p}$ form a right-handed set of coordinate axes in the three-dimensional space associated with the Breit frame. Calling these axes, respectively, 1, 2, 3 we see that the three matrices S_i are given by $\vec{S}=(\gamma_0\sigma_1, \gamma_0\sigma_2, \sigma_3)$. Because of the colinearity of initial and final momenta in the Breit frame, the S_i associated with both initial and final states are the same.

It is easy to check that the components of $\vec{S}$ have the same SU(2) algebra obeyed by their nonrelativistic counterparts, the 2×2 Pauli spin matrices. Another important property of the set S_i is that (in contrast to the ordinary matrices σ_i) they commute with the free-particle Dirac Hamiltonian. The transformation $\exp(i\theta\hat{n}\cdot\vec{S}/2)$ acting on a spinor corresponding to four-momentum p rotates the spin direction leaving $\vec{p}$ fixed. Similar considerations hold for the spin-$\frac{1}{2}$ constituents of the 56-dimensional representation.[7]

The 35 SU(6) generators may be written as products of spin and SU(3) operators $S_i\otimes1$, $1\otimes\lambda_j$, $S_i\otimes\lambda_j$ ($\vec{\lambda}$ is the usual eight-component Hermitian unitary-spin matrix vector) in direct imitation of the static-limit construction.

It is now possible to make the connection between this SU(6) and the baryon-photon vertex. The form factors of Eqs. (1) and (2) are defined by the following expression[8] for the electromagnetic current of a spin-$\frac{1}{2}$ particle labeled A:

$$j_\nu^{\,A}(P_f,P_i)=\frac{e}{(1-q^2/4M_A^{\,2})}\bar{u}(p_f)\left\{G_E^{\,A}(q^2)\frac{P_\nu}{2M_A}-G_M^{\,A}(q^2)\gamma_5\frac{\epsilon_{\kappa\lambda\mu\nu}P^\kappa q^\lambda\gamma^\mu}{4M_A^{\,2}}\right\}u(p_i), \qquad (4)$$

where p_i and p_f are the initial and final four-momenta of A, $P_\nu=(p_i+p_f)_\nu$, $q_\nu=(p_f-p_i)_\nu$, and $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$. For our purposes the merit of this decomposition is that in the Breit frame ($\vec{p}_i=-\vec{p}_f\equiv\vec{p}=-\frac{1}{2}\vec{q}$; $\vec{P}=0$, $q_0=0$) it reduces to

$$\{j_0(p_f,p_i),\vec{j}(p_f,p_i)\}=\frac{e}{(1-q^2/4M_A^{\,2})^{1/2}}u^\dagger(\vec{p})\{G_E^{\,A}(q^2),\; i\vec{S}\times\vec{q}G_M^{\,A}(q^2)/2M_A\}u(\vec{p}). \qquad (5)$$

It will be noted that $\bar{u}(P_f)=u^\dagger(p_f)\gamma_0=u^\dagger(-p_f)=u^\dagger(p)$. Moreover, only S_1 and S_2 contribute to the moment term of (5), so that the $\gamma_0\vec{\sigma}$ obtained from a direct reduction of Eq. (4) can be replaced by $\vec{S}$.

We assume, as suggested by Coleman and Glashow,[9] that the electromagnetic current transforms as the SU(3) octet charge operator Q. Then Eq. (5) is generalized to represent the effective electromagnetic vertex of the 56-dimensional representation composed of the baryon octet and decuplet. $G_E(q^2)$ and $G_M(q^2)$ become operators which must transform as appropriate tensor components of the 35-dimensional representation, as discussed by Bég, Lee, and Pais[2] and Sakita[2] for the static vertex. Since the construction of the SU(6) wave functions for the 56 representation is formally the same as in the static SU(6), the ratios of the dynamic electric and magnetic moments are the same as found in the static case. This proves Eqs. (1) and (2). Equation (2) is a generalization, to the matrix element of G_E between neutron states, of the static result that the neutron charge is zero. Similar results can be obtained for other vertex functions. These results shall be presented elsewhere.

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†Alfred P. Sloan Foundation Fellow.

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SUM RULES FOR THE REAL PART OF THE FORWARD-SCATTERING AMPLITUDE

N. N. Khuri
The Rockefeller Institute, New York, New York

and

T. Kinoshita*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

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Recent experiments[1] have shown that both the $\pi^{\pm}p$ and the pp forward-scattering amplitudes have a substantial real part at high energies (>6 BeV). While the data obtained so far are not yet in contradiction with the forward dispersion relation, the real part measured is larger than what was expected, and with the little data available it does not seem that Ref/E decreases with energy as it should if local field theory is valid.[2]

The purpose of this note is to present sum rules satisfied by the real part of a crossing-symmetric forward-scattering amplitude. These sum rules involve integration of the real part up to finite energies only and hold for any energy. Although these sum rules follow directly from the dispersion relations, they give for practical purposes a better tool for testing analyticity, crossing symmetry, and unitarity. They also show explicitly the fact that a large and repulsive real part at high energies, if maintained for a certain large energy range, will lead to a contradiction with the dispersion relation.

For concreteness we shall consider only pion-nucleon scattering. We let E be the total energy of the incident pion in the laboratory system, and let $f_{\pm}(E)$ be the forward amplitudes