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OPTICAL THEOREM AS APPLIED TO CHARGED-PARTICLE SCATTERING TO YIELD TOTAL REACTION CROSS SECTIONS FROM ELASTIC-SCATTERING DATA*

J. T. Holdeman and R. M. Thaler
Department of Physics, Case Institute of Technology, Cleveland, Ohio
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Total reaction cross sections for charged-particle scattering may be obtained by a careful examination of the elastic-scattering data. The procedure for so doing arises from the optical theorem and is outlined below. The elastic-scattering amplitude for charged-particle scattering may be written as

$$f(\theta)=f_{\rm Coul}(\theta)+f'(\theta), \tag{1}$$

where $f_{\rm Coul}(\theta)$ is the Coulomb scattering amplitude, and is given by

$$f_{\rm Coul}(\theta)=(\eta/2k\sin^2\tfrac{1}{2}\theta)\exp[i(2\sigma_0+\pi-2\eta\ln\sin\tfrac{1}{2}\theta)]$$

$$=k^{-1}\sum_{l=0}^{\infty}(2l+1)\exp(i\sigma_l)\sin\sigma_l P_l(\cos\theta), \tag{2}$$

with $\sigma_l\equiv\arg\Gamma(l+1+i\eta)$. The residual amplitude $f'(\theta)$ is given by

$$f'(\theta)=f(\theta)-f_{\rm Coul}(\theta)=(2ik)^{-1}\sum_{l=0}^{L}(2l+1)$$

$$\times\exp(2i\sigma_l)[\exp(2i\delta_l)-1]P_l(\cos\theta). \tag{3}$$

The total cross section is infinite because the Coulomb cross section is infinite. We nevertheless apply the optical theorem to Eq. (1), to obtain

$$\sigma'^{\rm tot}\equiv(\sigma^{\rm tot}-\sigma_{\rm Coul}{}^{\rm tot})=(4\pi/k)\,{\rm Im}f'(0). \tag{4}$$

Now $\sigma'^{\rm tot}$ is just

$$\sigma'^{\rm tot}=\sigma_r{}^{\rm tot}+\sigma_{\rm el}{}'^{\rm tot}, \tag{5}$$

where

$$\sigma_{\rm el}{}'^{\rm tot}=\int\sigma_{\rm el}{}'(\theta)d\Omega=\int\{\sigma_{\rm el}(\theta)-\sigma_{\rm Coul}(\theta)\}d\Omega$$

$$=\int\{|f'(\theta)|^2+2\,{\rm Re}f'(\theta)f_{\rm Coul}{}^*(\theta)\}d\Omega. \tag{6}$$

Thus we have obtained for the total reaction cross section

$$\sigma_r{}^{\rm tot}=(4\pi/k)\,{\rm Im}f'(0)-\sigma_{\rm el}{}'^{\rm tot}, \tag{7}$$

with $\sigma_{\rm el}{}'^{\rm tot}$ given by Eq. (6). We shall show below that both $f'(0)$ and $\sigma_{\rm el}{}'^{\rm tot}$ and hence the total reaction cross section may be obtained from the measured charged-particle elastic-scattering angular distribution.

Because of the finite range of the non-Coulombic force, $f'(\theta)$ may be expressed as a finite sum in angular momentum, $l\leq L$, and hence for small angles

$$f'(\theta)=f'(0)+O(\theta^2). \tag{8}$$

For angles very much smaller than $\theta\approx1/L$, $f'(\theta)$ may be taken to be constant. The Coulomb amplitude is rapidly varying in the small angle region and is completely known. A knowledge of the elastic-scattering cross section at any two different angles in the small-angle region can then serve to determine the complex amplitude, $f'(0)$.

We may also determine $\sigma_{\rm el}{}'^{\rm tot}$ from the experimental elastic-scattering data. To this end we rewrite Eq. (6) as

$$\sigma_{\rm el}{}'^{\rm tot}=2\pi\int_{\theta_0}^{\pi}\{\sigma_{\rm el}(\theta)-\sigma_{\rm Coul}(\theta)\}\sin\theta d\theta$$

$$+2\pi\int_0^{\theta_0}\{|f'(\theta)|^2+2\,{\rm Re}f'(\theta)f_{\rm Coul}{}^*(\theta)\}$$

$$\times\sin\theta d\theta. \tag{9}$$

If θ_0 is so small that $f'(\theta)$ is constant for $\theta<\theta_0$, then Eq. (9) becomes

$$\sigma_{\rm el}{}'^{\rm tot}=2\pi\int_{\theta_0}^{\pi}\{\sigma_{\rm el}(\theta)-\sigma_{\rm Coul}(\theta)\}\sin\theta d\theta$$

$$+4\pi|f'(0)|^2\sin^2\tfrac{1}{2}\theta_0+4\pi\,{\rm Re}\{f'(0)$$

$$\times\lim_{\epsilon\to0}(\int_\epsilon^{\theta_0}f_{\rm Coul}(\theta)\sin\theta d\theta)\}. \tag{10}$$

The first integral in Eq. (10) may be determined directly from the elastic angular distribution. Since $f'(0)$ is now supposed known, we have obtained the desired result provided that a unique finite limit exists for the final integral in Eq. (10). This integral is readily performed analytically and yields

$$\int_\epsilon^{\theta_0} f_{\mathrm{Coul}}^*(\theta)\sin\theta d\theta$$
$$= -(\eta/2k)\exp(-2i\sigma_0)\int_\epsilon^{\theta_0}(\sin\tfrac{1}{2}\theta)^{-2} \times\exp(2i\eta\ln\sin\tfrac{1}{2}\theta)\sin\theta d\theta$$
$$= (i/k)\exp(-2i\sigma_0)\{\exp(2i\eta\ln\sin\tfrac{1}{2}\theta_0) -\exp(2i\eta\ln\sin\tfrac{1}{2}\epsilon)\}. \quad (11)$$

The limit $\epsilon\to 0$ does not exist as a uniform limit. However, from Eq. (2) we see that

$$\lim_{\epsilon\to 0}\int_\epsilon^{\pi} f_{\mathrm{Coul}}^*(\theta)\sin\theta d\theta$$
$$= \lim_{\epsilon\to 0}(i/k)\{\exp(-2i\sigma_0)-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\epsilon)\}$$
$$= (1/-2ik)\sum_{l=0}^{\infty}(2l+1)[\exp(-2i\sigma_0)-1] \times\int_0^{\pi}P_l(\cos\theta)P_0(\cos\theta)\sin\theta d\theta$$
$$= (i/k)[\exp(-2i\sigma_0)-1], \quad (12)$$

so that the nonuniform limit of interest is obtained by choosing a sequence of ϵ_n such that

$$\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\epsilon_n)=1 \quad (13)$$

as $\epsilon_n\to 0$. Thus the required limit in Eq. (11) is

$$\lim_{\epsilon\to 0}\{\int_\epsilon^{\theta_0} f_{\mathrm{Coul}}^*\sin\theta d\theta\}$$
$$= (ik)^{-1}[1-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\theta_0)]. \quad (14)$$

We may now rewrite Eq. (10) as

$$\sigma_{\mathrm{el}}'^{\mathrm{tot}} = 2\pi\int_{\theta_0}^{\pi}\{\sigma_{\mathrm{el}}(\theta)-\sigma_{\mathrm{Coul}}(\theta)\} \times\sin\theta d\theta + 4\pi|f'(0)|^2\sin^2\tfrac{1}{2}\theta_0 + (4\pi/k) \times\mathrm{Im}\{f'(0)[1-\exp(-2i\sigma_0+2i\eta\ln\sin\tfrac{1}{2}\theta_0]\}, \quad (15)$$

and finally we obtain for the total reaction cross section

$$\sigma_r^{\mathrm{tot}} = (4\pi/k)\mathrm{Im}\{f'(0)\exp(-2i\sigma_0+2i\eta\ln\tfrac{1}{2}\sin\theta_0)\} -2\pi\int_{\theta_0}^{\pi}\{\sigma_{\mathrm{el}}(\theta)-\sigma_{\mathrm{Coul}}(\theta)\}\sin\theta d\theta -4\pi|f'(0)|^2\sin\tfrac{1}{2}\theta_0. \quad (16)$$

In the limit as the charge goes to zero, Eq. (16) yields the familiar result that the total reaction cross section is the total cross section, as given by the optical theorem, minus the total elastic cross section. However, for the scattering of uncharged particles, the amplitude $f'(0)$ cannot, in general, be determined from the elastic-scattering data. For charged-particle scattering, $f'(0)$ can be determined, and hence Eq. (16) can be used to obtain the reaction cross section from the elastic-scattering data. The feasibility of the procedure described was checked by using an optical-model code to generate "experimental" data. We have also determined reaction cross sections for a few cases in which sufficient experimental data exist. Further details will be given in a paper now in preparation.

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SU(6) AND THE ELECTROMAGNETIC FORM FACTORS*

K. J. Barnes, P. Carruthers,† and Frank von Hippel
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
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Recently an interesting attempt has been made[1] to combine internal symmetries with spin degrees of freedom in a nontrivial way using the group SU(6). The striking correlations that have been obtained[2] are to be contrasted with the difficulties that have arisen in the interpretation of the symmetry.[3,4] To date all successes have been confined to the correlation of essentially static properties of the elementary particles. We discuss here a nonstatic problem in which the possibility of an SU(6) symmetry arises in a natural way. In the nonrelativistic limit the generators of this SU(6) coincide with those previously used.[1,2]