be compared with the experimental value[3]

$$R_{\mathrm{expt}} = (3.3 \pm 0.6) \times 10^{-6}. \qquad (24)$$

(3) Hyperon nonleptonic decays should exhibit a 0.2% $T$-nonconserving effect. This is evident since the strangeness-changing $T$-invariance violation may proceed through the $\underline{27}$ piece of $s'p' \sim s_3 p_6 \cos\theta \sin^2\theta + \cdots$, which is a factor $\lambda \sin^2\theta \sim 1/500$ smaller than the usual $T$-conserving interaction. The $\Delta S = 0$ weak nuclear processes, on the other hand, should exhibit a $T$-violating effect of order 4%, since here there is an octet part to the weak $T$-violating Hamiltonian, and thus we may expect the $T$-violating term to be only $\sin^2\theta \sim 1/25$ smaller than the $T$-conserving one.

(4) The $T$-conserving $\Delta S = 0$ WI amplitudes satisfy an approximate $\Delta I = 0$ rule. Explicitly,

$$\frac{|\Delta I| = 1}{|\Delta I| = 0} \sim \sin^2\theta.$$

For the $\Delta S = 0$ $T$-nonconserving amplitudes, however,

$$\frac{|\Delta I| = 1}{|\Delta I| = 0} \sim 1.$$

Consequently, in $\Delta S = 0$, $|\Delta I| = 1$ transitions, $T$ violation and conservation are comparable.

(5) Finally, the vector coupling constant for strangeness-conserving $\beta$ decays is not $G$, but $G \cos\theta = 0.98G$. (Remember that our $\theta$ is smaller than Cabibbo's angle by a factor of $\sim\sqrt{2}$.) This is to be compared with the experimental value of $(0.978 \pm 0.0015)G$.[11]

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# RELATIVISTIC TREATMENT OF SPIN INDEPENDENCE

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There have been several interesting attempts[1-4] made recently to combine within one relativistic framework the internal symmetries of the strong interactions with their spin independence. The purpose of this Letter is to present for rapid evaluation and assimilation a somewhat different treatment of the spin aspects of the problem. A brief survey of the difficulties involved in the usual approach to the problem is given in order to clarify the subsequent derivation of the new technique, but detailed applications to the problem of combining spin and internal symmetries will be reserved for presentation at greater length in some more suitable publication.

In most usual treatments the infinitesimal transformations which may be performed on a spinor $U(p)$ (in the absence of internal symmetries) are taken to be the $\tilde{U}(4)$ transformations or some subset of these. That is,

$$U(p)\rightarrow SU(p)=[1+i\alpha_\mu\Gamma^\mu]U(p)=[1+i(a_I 1+a_5\gamma_5+a_\mu\gamma^\mu+a_{\mu5}i\gamma^\mu\gamma_5+\tfrac{1}{2}a_{\mu\nu}\sigma^{\mu\nu})]\mu(p), \tag{1}$$

where, if the parameters $\alpha_\mu$ are real, the scalar product $\overline{U}(p)U(p)$ is invariant. If the spinor is to represent a free Dirac particle, then $S$ must commute with $(\not{p}-m)$, and one obtains immediately

$$a_5=a_\mu=p^\nu a_{\nu5}=p^\nu a_{\mu\nu}=0. \tag{2}$$

The existence of this subset of transformations allows, of course, for the definition of relativistic spin operators, for example, the Pauli-Lubanski pseudovector $\omega_\mu=\frac{1}{4}\epsilon_{\mu\nu\rho\lambda}\sigma^{\nu\rho}p^\lambda$ and the tensor $\omega_{\mu\nu}=i\epsilon_{\mu\nu\rho\lambda}\gamma^\rho\gamma_5 p^\lambda$ devised by Calogero.[5] However, if now some interaction is envisaged, the above considerations must be further applied to ensure that the outgoing particle described by $\overline{U}(p')$ remains a solution of the Dirac equation $\overline{U}(p')(\not{p}'-m)=0$. Thus the final intersection of allowed transformations is specified by

$$a_5=a_\mu=p^\nu a_{\nu5}=p'^\nu a_{\nu5}=p^\nu a_{\mu\nu}=p'^\nu a_{\mu\nu}=0, \tag{3}$$

and forms only a four-parameter subset. The remaining generators still yield a closed algebra with a $\tilde{U}(2)$ structure, which may be interpreted as a representation of a spin group, and this circumstance has been employed[6] by the present author and co-workers to obtain an SU(6) theory which gives nonstatic results. A desire for a larger and more flexible set of generators has prompted several authors[1-4,7] to advocate that the transformations given in Eq. (1) be employed without the above-mentioned restrictions, and that the nonzero commutator of $\not{p}$ and $S$ be interpreted as an "intrinsic breaking of the symmetry by the kinetic-energy terms." This concept of a relativistic symmetry which is intrinsically broken in all except possibly static situations seems to the present author not entirely satisfactory.

The technique which is proposed here is to accept that any symmetry which exists must be dictated by the physics of the situation (i.e., the momenta of the particles involved, and their interactions), and to accept the transformations defined by Eqs. (1) and (3) as the most general set possible. Furthermore, it is proposed that the free spinors of higher rank be assumed to obey the Bargmann-Wigner[8] equations

$$\begin{aligned}\not{p}_\alpha{}^\sigma\psi_{\sigma\beta\gamma\cdots}&=m\psi_{\alpha\beta\gamma\cdots},\\ \not{p}_\beta{}^\sigma\psi_{\alpha\sigma\gamma\cdots}&=m\psi_{\alpha\beta\gamma\cdots},\\ &\text{etc.},\end{aligned} \tag{4}$$

which are effectively the Dirac equation applied separately to each index, so that the above arguments may be generalized in an obvious manner for the higher dimensional representations. In particular, the second-rank mixed spinor $\Phi_\alpha{}^\beta(q)$, which will be assumed to transform in the same manner[1] as $\psi_\alpha(q)\overline{\psi}^\beta(q)$, and which may be written in the form

$$\Phi_\alpha{}^\beta=[\theta+\varphi\gamma_5+i\gamma^\mu\gamma_5 F_{\mu5}+\gamma^\mu\varphi_\mu+\tfrac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}]_\alpha{}^\beta, \tag{5}$$

on application of the Bargmann-Wigner equations (with mass $\mu$ and momentum $q$) yields the set of relations

$$\begin{aligned}&\theta=0,\quad i\mu F_{\mu5}=q_\mu\varphi,\quad \mu\varphi=iq^\mu F_{\mu5},\\ &i\mu F_{\mu\nu}=q_\mu\varphi_\nu-q_\nu\varphi_\mu,\quad i\mu\varphi_\mu=q^\nu F_{\mu\nu},\end{aligned} \tag{6}$$

and hence provides[9] a possible description[10] of pseudoscalar and vector particles[11] described, respectively, by the sets of functions $(\varphi, F_{\mu5})$ and $(\varphi_\mu, F_{\mu\nu})$. Finally, it is proposed that the interaction of free fermions with a virtual boson shall be specified by the Lorentz-invariant

$$g(q^2)\overline{\psi}^\alpha(p')\Lambda_\alpha{}^\beta(q)\psi_\beta(p), \tag{7}$$

where the second-rank mixed spinor $\Lambda_\alpha{}^\beta$ is to be formed from $\Phi_\alpha{}^\beta$, $\not{p}$, $\not{p}'$, and scalar functions of $q^2$, so that the interaction is further invariant under the spin transformations (whose generators commute with both $\not{p}$ and $\not{p}'$).

To finally determine $\Lambda_\alpha{}^\beta$, it is speculated that it must be formed from generators of the full set of transformations

$$U(p) \to S_p(p,p')U(p), \tag{8}$$

where $S_p$ is defined by the following properties: (a) $(\not p-m)S_pU(p)=0=\bar U(p')S_p(\not p'-m)$; (b) $\bar U(p) \times U(p)$ and $\bar U(p')U(p)$ are invariants; (c) the product of any two generators of these transformations shall again be a generator. Thus the closed set of generators defines a set of transformations [hereafter referred to as the $P\tilde U(4)$ transformations] which not only is compatible with the free-particle Dirac equations and preserves scalar products, but also has the property that the generators of the corresponding transformations

$$\bar U(P') \to \bar U(p')S_{p'}{}^{-1}(p,p') \tag{9}$$

are identical to those in $S_p$. This is, of course, essential if these generators are to be identified as couplings of bosons to fermions, as suggested above.

A suitable form for $S_p$ subject to the above restrictions is

$$S_p(p,p') = 1 + if(q^2)\alpha_\mu(\not p+m)$$
$$\times\{(\not p'+m)\Gamma^\mu(\not p+m)+(4m^2-q^2)\Gamma^\mu+(4m^2-q^2)^{1/2}[\Gamma^\mu(\not p+m)+(\not p'+m)\Gamma^\mu]\}(\not p'+m), \tag{10}$$

where $f(q^2)$ is an arbitrary function of the invariant four-momentum transfer squared, and the normalization requirement that the generator with $\Gamma^\mu=1$ shall be the identity operator yields

$$f^{-1}(q^2)=2(4m^2-q^2)[(4m^2-q^2)+2m(4m^2-q^2)^{1/2}]. \tag{11}$$

The mechanism is now sufficient to treat spin symmetry within the context of a relativistic $S$-matrix theory dealing with free-particle states and three-particle vertices, where the spin symmetry is now invariance under the transformations defined in Eqs. (1) and (3), and the invariant coupling is defined by Eqs. (5) and (7), where

$$2[(4m^2-q^2)+2m(4m^2-q^2)^{1/2}]\Lambda=(4m^2-q^2)\Phi+(\not p+m)\Phi(\not p'+m)+(4m^2-q^2)^{1/2}[(\not p+m)\Phi+\Phi(\not p'+m)]. \tag{12}$$

Notice that the interaction may now be nonlocal.

The ultimate test of this technique is the comparison of its nonstatic predictions with experiment, and although the task of combining this type of spin symmetry with internal symmetries[12] lies beyond the scope of this Letter, it is perhaps appropriate to present the result of one such calculation[13] which does provide a crucial test of this kind. By assuming that the vector mesons dominate the electromagnetic interactions of the baryons, and that the basic symmetry underlying the strong interactions has as its generators all possible products of the generators of the spin transformations with those of the isotopic spin group,[14] the following relations between the electromagnetic form factors[15,16] of the proton and neutron have been obtained:

$$G_E{}^N(q^2)=0, \quad G_M{}^N(q^2)=-\tfrac{2}{3}G_M{}^P(q^2), \tag{13}$$

and

$$G_M{}^P(q^2)=(2m/\mu)G_E{}^P(q^2). \tag{14}$$

The form equality in this last relation [which is the only prediction actually dependent on the $P\tilde U(4)$ treatment of the couplings] has often been speculated[16] from the experimental data, and has recently been most impressively confirmed[17] by the Stanford electron scattering group.

It should be noted that the "spin" transformations defined in this work are uniquely determined by two independent four-momenta. There is no obvious extension of the technique to scattering amplitudes, and any restrictions which the theory may place on such amplitudes will be given by the implicit effects of three-particle vertices [e.g., through the decomposition of the scattering amplitude into single pole-exchange contributions, or through unitarity]. Such questions, and the related problem of consistently and uniquely specifying the three-meson vertices, are clearly matters for further detailed investigation.

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# INELASTIC $N/D$ PROBLEMS AND CASTILLEJO-DALITZ-DYSON CUTS*

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Several recent papers have discussed discrepancies between many-channel $N/D$ calculations and the corresponding single-channel calculation with a given inelasticity coefficient.[1-3] We have studied this question in a model-independent way for theories where the partial wave amplitude possesses a unique continuation in the angular-momentum variable $l$.[4] For realistic input forces (left-hand cut contributions to the amplitude) the $S$ matrix $S_{ij}\to 1$ and the corresponding amplitudes $A_{ij}\to 0$ as $l\to\infty$. This leads at once to the conclusion that for high $l$ (or weak coupling) the Frye-Warnock[5] equations and the Froissart[6] and Ball-Frazer[7] methods all give the same result as many-channel $N/D$.[4] This is consistent with findings of other authors.[1-3]

Once having established that the Frye-Warnock equations are correct at high $l$, we analytically continue the solutions to lower values of $l$. As Bander, Coulter, and Shaw[1] illustrated by specific examples, we also find the Frye-Warnock equations generally no longer give the right answer. Bander, Coulter, and Shaw, as well as Atkinson, Dietz, and Morgan,[3] suggest that this difficulty is connected with Castillejo-Dalitz-Dyson (CDD) ambiguities.

By means of our continuation in $l$, we find that generally cuts are introduced into the $D$ function. These cuts emerge gradually from the inelastic cuts as $l$ is decreased. These CDD cuts, as we indicate below, are strictly an inelastic phenomenon, and it is the purpose of this note to show how such cuts arise.

We write the elastic amplitude $A_{11}$ as follows:

$$A_{11}=(1/2i\rho)(S_{11}-1),$$
$$S_{11}=e^{2i\alpha(l,s)}=\eta(l,s)e^{2i\delta(l,s)}, \qquad (1)$$
$$0\leq\eta\leq 1, \quad \delta=\mathrm{Re}\,\alpha.$$

Here $\rho$ is the phase space; $\delta$ and $\eta$ are real