

ANGULAR MOMENTUM AND THE ALGEBRA OF CURRENT COMPONENTS*

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It has recently been proposed¹ that the space integrals of all components of the vector current octet $\mathfrak{F}_{i\alpha}$ and the axial vector current octet $\mathfrak{F}_{i\alpha}^5$ obey the same commutation rules as if the currents were equal to $\mathfrak{G}_{i\alpha}$ and $\mathfrak{G}_{i\alpha}^5$, respectively, where, in terms of a quark field q , we have

$$\mathfrak{G}_{i\alpha} \equiv i\bar{q}\gamma_{\alpha}\lambda_i q/2, \quad \mathfrak{G}_{i\alpha}^5 \equiv i\bar{q}\gamma_{\alpha}\gamma_5\lambda_i q/2. \quad (1)$$

The system is algebraically closed if a ninth pair of currents is added; thus we let the index i run from 0 to 8, with $\lambda_0 = (\frac{2}{3})^{1/2}1$. We may rearrange the 72 components of $\int \mathfrak{G}_{i\alpha} d^3x$ and $\int \mathfrak{G}_{i\alpha}^5 d^3x$ to obtain the 72 Hermitian operators

$$G_{ij}^{\pm} = (\frac{3}{32})^{1/2} \int q^{\dagger} \lambda_i \sigma_j (1 \pm \gamma_5) q d^3x \quad (2)$$

$$(i=0, \dots, 8; j=0, \dots, 3),$$

where $\sigma_0 \equiv 1$. The corresponding linear combinations of the $\int \mathfrak{F}_{i\alpha} d^3x$ and $\int \mathfrak{F}_{i\alpha}^5 d^3x$ are called A_{ij}^{\pm} ; either the G_{ij}^{\pm} or the A_{ij}^{\pm} generate the algebra of $U(6) \otimes U(6)$.²

It was suggested that the A_{ij}^{+} and A_{ij}^{-} form a system of very approximate symmetries of the hadrons. In the energy density $\mathcal{H} = -\theta_{44}$, there is a large term \mathcal{H}' that breaks the symmetry down to that of the quantities

$$A_{ij} \equiv A_{ij}^{+} + A_{ij}^{-} \quad (3)$$

$$(i=0, \dots, 8; j=0, \dots, 3),$$

which generate the algebra of $U(6)$. The A_{ij} are apparently a good set of hadron symmetries, and we have thus interpreted the algebra of $U(6)$ discovered by Gürsey and Radicati,³ Sakita,⁴ and Zweig⁵ and further developed in several recent papers.⁶⁻¹² In this Letter we consider the relation of the algebra to the angular momentum \vec{J} , as well as the manner in which the symmetry $U(6)$ is broken down to $U(3)$. [The way in which $U(3)$ is violated will be treated elsewhere.] Some features of the $U(6)$ theory that have been obscure are clarified here.

Among the generators A_{ij} , we note that A_{00} is proportional to the baryon number (or the triplet number if the triplet q does not refer

to quarks) and that it commutes with the other 35 generators, which give $SU(6)$. The generators A_{i0} ($i=1, \dots, 8$) are just $(\frac{3}{2})^{1/2}F_i$, where the F_i are the components of the F spin connected with $SU(3)$ symmetry. The generators¹³

$$S_j \equiv A_{0j} \quad (j=1, 2, 3)$$

act exactly like a spin angular momentum since they obey the rules

$$[S_i, S_j] = ie_{ijk} S_k; \quad [J_i, S_j] = ie_{ijk} S_k. \quad (4)$$

(In the quark model, of course, \vec{S} is just the total spin of quarks, including any number of quark-antiquark pairs.) Naturally, \vec{S} is not equal to the total angular momentum \vec{J} , since it does not include the orbital angular momentum; we may define the difference

$$\vec{L} = \vec{J} - \vec{S}, \quad (5)$$

and note that \vec{L} obeys the rules

$$[L_i, L_j] = ie_{ijk} L_k, \quad [J_i, L_j] = ie_{ijk} L_k, \quad [L_i, S_j] = 0, \quad (6)$$

by virtue of (4). Thus, \vec{L} acts like an orbital angular momentum.¹⁴ (In a pure quark model, that is what it is. In a model with quarks and other basic particles, \vec{L} would include the intrinsic spins of the extra particles.)

It should perhaps be emphasized that the remaining generators A_{ij} ($i=1, \dots, 8; j=1, \dots, 3$) are not proportional to $F_i S_j$, but they do transform like $F_i S_j$ under the group $SU(6)$.

In the approximation of conservation of all the A_{ij} , each degenerate particle multiplet at rest belongs to a definite $SU(6)$ representation and a definite value of L , with \vec{L} and \vec{S} added vectorially to give the spins J of the particles.¹⁵ The vector meson octet and singlet and the pseudoscalar meson octet presumably belong to $\underline{35}$ with $L=0$; the pseudoscalar meson singlet may belong to $\underline{1}$ with $L=0$; a $J=2^{+}$ meson singlet might belong to $\underline{1}$ with $L=2$; and so forth. The baryon $J=\frac{1}{2}^{+}$ octet and $J=\frac{3}{2}^{+}$ decimet presumably belong to $\underline{56}$ with $L=0$.

We may now discuss Regge recurrences. The first recurrence of the $\underline{56}$, for example, has $L=2$ added vectorially to $S=\frac{1}{2}$ for the oc-

tet and $S = \frac{3}{2}$ for the decimet; altogether, this multiplet has $5 \times 56 = 280$ states. The Regge recurrences of $J = \frac{1}{2}^+$ (8) and $J = \frac{3}{2}^+$ (10), with $J = \frac{5}{2}^+$ (8) and $J = \frac{7}{2}^+$ (10), respectively, are included; but so are particles with $J = \frac{3}{2}^+$ (8) and with $J = \frac{5}{2}^+$ (10), $\frac{3}{2}^+$ (10), and $\frac{1}{2}^+$ (10), all of which lie on trajectories that give "nonsense" (no real particles) at $L = 0$.

In a quark model, there is a term in the energy density $\mathcal{H} = -\theta_{44}$, namely,

$$(2i)^{-1}(q^\dagger \vec{\alpha} \cdot \nabla q - \nabla q^\dagger \cdot \vec{\alpha} q), \quad (7)$$

that breaks SU(6) down to SU(3). Let us assume that in the true theory the term \mathcal{H}'' that breaks SU(6) down to SU(3) transforms in the same manner, namely (1, 35) and (35, 1) under SU(6) \otimes SU(6), or 35 under SU(6), with $L = 1$. In first order, such a term \mathcal{H}'' can lead to an $\vec{L} \cdot \vec{S}$ splitting in a multiplet with $L > 0$, such as the Regge recurrence of the 56 discussed above. In second order, \mathcal{H}'' can split a multiplet with $L = 0$; we obtain splittings that transform like 405 and 189 under SU(6). These are the only representations in $35 \otimes 35$, besides the trivial representation 1, that contain a unitary singlet with $S = 0$.

Bég and Singh⁹ show that unitary singlet mass perturbations belonging to 405 and 189 are sufficient to explain the splitting between octet and decimet in 56 and between pseudoscalar and vector mesons in 35. In order to explain the φ - ω degeneracy before SU(3) breaking, we must have a particular linear combination of 405 and 189 that gives spin splitting but no unitary-spin splitting; such a combination cannot be required by symmetry under U(6) alone, but must be explained by approximate symmetry under a larger algebra, such as that of U(6) \otimes U(6).

An interesting application of the hypothesis that \mathcal{H}'' transforms like 35 with $L = 1$ is the study of the magnetic moment operator μ_j between states with $L = 0$. The operator μ_j transforms like $e_{jkl} \int x_k \mathfrak{T}_l d^3x$, where the index e refers to the charge direction in SU(3) space. Evidently, μ_j belongs to 35 with $L = 1$ and in the limit of U(6) symmetry it gives zero between $L = 0$ states. To first order in \mathcal{H}'' , we can get an effective μ_j operator that has $L = 0$ and transforms under U(6) like $35 \otimes 35$; it contains pieces that belong to 35, 189, 405, 280, and 280*. Now between the baryon multiplet

56 ($L = 0$) and itself, the only pieces that can contribute are 35 and 405, which are contained in $56 \otimes 56^*$; the effective μ_j in this case has the form

$$\mu_j = a_1 A_{ej} + a_2 e_{jkl} \{A_{ek}, A_{ol}\}. \quad (8)$$

The second term can be shown to vanish by time-reversal invariance. Thus the nucleon magnetic moments obey the rule $\mu_n/\mu_p = -\frac{2}{3}$ characteristic of the first term alone. This ratio was first presented in reference 11, where it was not explained why the effective μ_j transforms in this case according to 35 alone.

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¹³The symbol S is used for something different in references 9 and 10.

¹⁴We note that $[J_l, A_{ij}^\pm] = [S_l, A_{ij}^\pm] = ie_{ljk} A_{ik}^\pm$, so that \vec{L} commutes with all components of A_{ij}^\pm and A_{ij}^\mp .

¹⁵For each value of L_3 , there must be a complete representation of the algebra, including all values of S_3 ; thus we obtain all the values of J that come from vector addition. If we consider a different Lorentz frame, the definition of the A_{ij} (including the components of \vec{S}) changes accordingly and so do the definitions of \vec{L} and \vec{J} . In the new frame, we again consider states of hadrons at rest and apply the new operators to them.