# NONLEPTONIC HYPERON DECAY AND THE RELATIVISTIC GENERALIZATION OF SU(6)*

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When an interaction is not invariant under a particular symmetry, it is natural to assume, in the absence of any other criterion, that its transformation properties will be as simple as possible. For nonleptonic hyperon decay, this approach is empirically justified in the isospin and unitary-symmetry schemes, but it does not appear to be valid in SU(6). A recent study has shown that although the $\Delta T=\frac{1}{2}$ rule and octet dominance[1] are compatible with experiment, the assignment of the interaction to a 35-plet is not.[2,3] Before this result is accepted at face value, however, it would be well to examine a serious flaw in the argument.

In its original form,[4] SU(6) is to be regarded as the nonrelativistic limit of a higher symmetry which combines unitary symmetry and Lorentz covariance in a nontrivial manner. Under these circumstances, we would expect that the theory could be applied directly to such nonrelativistic processes as nonleptonic hyperon decay. This is indeed the case for parity-nonconserving ($S$-wave) interactions, because the leading contributions from covariant forms $\bar{\psi}\psi\varphi$ and $\bar{\psi}\gamma_\mu\psi\partial_\mu\varphi$ are both independent of $v/c$. The parity-conserving ($P$-wave) interactions $\bar{\psi}\gamma_5\psi\varphi$ and $\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\varphi$ are, however, both proportional to $v/c$, and so, by going to the nonrelativistic limit without a detailed knowledge of the relativistic interaction, we may unwittingly neglect important terms. Clearly, a fully relativistic calculation is needed to check this point.

Another way of stating this argument is to note that SU(6) contains no clear-cut prescription for dealing with orbital angular momentum. If it is independent of SU(6), a $P$-wave pion will transform under the group in exactly the same way as an $S$-wave pion. If, on the other hand, there exists a more intimate relation between orbital angular momentum and the intrinsic-spin subgroup, it may not be unreasonable to suppose that a $P$-wave pion behaves in SU(6) like the vector meson $\rho$. Since SU(6) provides no method for deciding between these alternatives, the consequences of the theory for $P$-, and higher, wave interactions are bound to be ambiguous. The ambiguity can only be resolved in the relativistic limit, which yields the true relation between orbital angular momentum and spin.

To make the relativistic calculation we use the generalization of SU(6) proposed by Sakita and Wali[5] and based upon the group $SL(6,c)$. We assume that the effective interaction for nonleptonic decays has minimal transformation properties under the group, and, after constructing its most general form, we proceed to the nonrelativistic limit. As expected, the $S$-wave amplitudes are exactly the same as those computed in SU(6),[2,3] but the $P$-wave amplitudes include several new terms which do not appear in SU(6). These additional terms enable us to remove the inconsistency with experiment which was noted above,[2] and to make a firm prediction that $\Sigma^+\to n+\pi^+$ is pure $P$ wave.[6]

The meson field is represented by a mixed tensor[7]

$$\Phi_B{}^A \equiv \Phi_{j\beta}{}^{i\alpha}, \tag{1}$$

and the baryon field by a totally symmetric tensor[7]

$$\Psi_{ABC} \equiv \Psi_{i\alpha,j\beta,k\gamma}. \tag{2}$$

Roman indices are Dirac indices with values running from 1 to 4; Greek indices refer to SU(3) and run from 1 to 3. From these fields, we can construct two types of interactions with minimal transformation properties:

$$H_1 \sim \bar{\Psi}^{i3,j\beta,k\gamma}(A)_j{}^g(B)_k{}^h\Psi_{f\alpha,g\beta,h\gamma}(E\Phi)_{i2}{}^{f\alpha}, \tag{3}$$

$$H_2 \sim \bar{\Psi}^{i3,j\beta,k\gamma}(A)_k{}^h(B)_i{}^f\Psi_{f2,g\delta,h\gamma}(E\Phi)_{j\beta}{}^{g\delta}. \tag{4}$$

Each of the matrices $A, B, E$ is either a unit matrix or $\gamma_5$, and the parity of the interaction depends upon how many are chosen to be $\gamma_5$. If the number is even, the interaction conserves parity,[8] and if it is odd, the interaction does not.

In the case of $P$-wave decays, there are four possible choices for $A, B, E$:

$$A \equiv B \equiv E \equiv 1; \tag{5a}$$

$$A \equiv 1,\ B \equiv E \equiv \gamma_5; \tag{5b}$$

$$B \equiv 1,\ E \equiv A \equiv \gamma_5; \tag{5c}$$

$$E \equiv 1,\ A \equiv B \equiv \gamma_5. \tag{5d}$$

They are all equally important in the nonrelativistic limit, and they enable us to express the contributions from $H_1$ and $H_2$ as direct products of a kinematical factor and an SU(3) factor.

$H_1$ engenders two independent observable terms[9]:

$$P_1[3(DM)_2^{\,3} - 2(FM)_2^{\,3}], \tag{6}$$

and

$$P_1'(FM)_2^{\,3}, \tag{7}$$

where

$$(ST)_\beta^{\ \alpha} \equiv S_\lambda^{\ \alpha} T_\beta^{\ \lambda}.$$

$P_1$ and $P_1'$ are arbitrary $P$-wave amplitudes which may depend on the masses of parent and daughter particles. $F_\beta^{\ \alpha}$ and $D_\beta^{\ \alpha}$ denote the usual $R$-symmetric and $R$-antisymmetric octet couplings of baryons and antibaryons, and $M_\beta^{\ \alpha}$ is the pseudoscalar meson octet. From $H_2$ we obtain two more observable terms

$$P_2\{\tfrac{3}{4}[\underline{10}^*]_2^{\,3}M - \tfrac{3}{4}[\underline{10}]_2^{\,3}M - \tfrac{1}{2}[\underline{27}]_2^{\,3}M + \tfrac{3}{10}(DM)_2^{\,3} - \tfrac{1}{2}(FM)_2^{\,3}\}, \tag{8}$$

and

$$P_2'\{\tfrac{3}{4}[\underline{10}]_2^{\,3}M - \tfrac{3}{4}[\underline{10}^*]_2^{\,3}M - \tfrac{1}{2}[\underline{27}]_2^{\,3}M + \tfrac{3}{10}(DM)_2^{\,3} - \tfrac{1}{2}(FM)_2^{\,3}\}, \tag{9}$$

where

$$[X]_2^{\,3}M \equiv [X]_{2\mu}^{\ \ 3\lambda} M_\lambda^{\ \mu},$$

and $X \equiv [\underline{10}], [\underline{10}^*], [\underline{27}]$ denotes the appropriate SU(3) coupling of baryons and antibaryons.[10] The contributions of (6)-(9) to observable decays are given in Table I.

Notice that other interactions can be derived from $H_1$ and $H_2$ by replacing $\Phi$ with $\gamma_5\Phi\gamma_5$ in Eqs. (3) and (4). This replacement will change only the kinematical factors,[11] and its effects can be absorbed in the arbitrary amplitudes $P_1$, $P_1'$, $P_2$, and $P_2'$.

Table I. Matrix elements for observable decays.

| | Amplitudes | | | | | |
|---|---|---|---|---|---|---|
| | $S$-wave | | $P$-wave | | | |
| Decay mode | $S_1$ | $S_2$ | $P_1$ | $P_1'$ | $P_2$ | $P_2'$ |
| $\Sigma^+ \to p + \pi^0$ | 1 | $-2$ | $\frac{1}{3}$ | 1 | $-2$ | $-4$ |
| $\Sigma^+ \to n + \pi^+$ | 0 | $\sqrt{2}$ | 0 | 0 | $+\sqrt{2}$ | $5\sqrt{2}$ |
| $\Sigma^- \to n + \pi^-$ | $\sqrt{2}$ | $-\sqrt{2}$ | $\frac{1}{3}\sqrt{2}$ | $\sqrt{2}$ | $-\sqrt{2}$ | $\sqrt{2}$ |
| $\Lambda \to p + \pi^-$ | $\sqrt{3}$ | 0 | $-\sqrt{3}$ | $\sqrt{3}$ | 0 | $2\sqrt{3}$ |
| $\Xi^- \to \Lambda + \pi^-$ | $\sqrt{3}$ | $-\sqrt{3}$ | $-1/\sqrt{3}$ | $\sqrt{3}$ | $\sqrt{3}$ | $\sqrt{3}$ |

In $S$-wave decays, there are also four choices of $A, B, E$, but only one,

$$E \equiv \gamma_5,\quad A \equiv B \equiv 1, \tag{10}$$

is significant in the nonrelativistic limit.[12] With this choice, $H_1$ and $H_2$ above [Eqs. (3) and (4)] yield exactly the same $S$-wave amplitudes as do the $H_1$ and $H_2$ couplings of Rosen and Pakvasa.[2] They are displayed in Table I.

It follows from Table I that the Lee triangle[13]

$$\sqrt{3}\langle \Sigma^+ | p\pi^0\rangle + \langle \Lambda | p\pi^-\rangle = 2\langle \Xi^- | \Lambda\pi^-\rangle \tag{11}$$

is automatically satisfied by the $S$-wave amplitudes and by $P_1$ and $P_1'$. If, in addition,

$$P_2 + P_2' = 0, \tag{12}$$

it will be satisfied by all amplitudes. To see why this is so, we note that (11) is automatically satisfied whenever the baryon-antibaryon system forms an octet of 27-plet[14]; decuplets, however, lead to (11) only when they occur in the combination $[\underline{10}]_2^{\,3}M + [\underline{10}^*]_2^{\,3}M$.

The $S$-wave amplitudes for $\Sigma^- \to n + \pi^-$ and $\Xi^- \to \Lambda + \pi^-$ are related by (see Table I)

$$\sqrt{3}\langle \Sigma^- | n\pi^-\rangle_S = \sqrt{2}\langle \Xi^- | \Lambda\pi^-\rangle_S, \tag{13}$$

but their $P$-wave amplitudes are independent of one another even when (12) holds. Because the asymmetry parameter for $\Xi$ decay does not vanish,[15] Eq. (13) implies that $\langle \Sigma^- | n\pi^-\rangle_S$ cannot vanish. Therefore, when the observed relations[15]

$$\alpha(\Sigma^- \to n + \pi^-) \approx \alpha(\Sigma^+ \to n + \pi^+) \approx 0$$

are combined with the $\Delta T = \frac{1}{2}$ rule,[16] we expect $\Sigma^- \to n + \pi^-$ to be pure $S$ wave and $\Sigma^+ \to n + \pi^+$ to be pure $P$ wave[6,17]; hence,

$$\tfrac{1}{3}P_1 + P_1' - P_2 + P_2' = 0, \tag{14}$$

$$S_2 = 0. \qquad (15)$$

Notice that this prediction is unambiguous; by contrast, other symmetry theories can accommodate $\Sigma^+ \to n + \pi^+$ either as an $S$-wave or as a $P$-wave decay.[18]

As shown by Rosen and Pakvasa,[2] the relative signs of $\alpha_0, \alpha_\Lambda, \alpha_\Xi$, are incompatible with minimal transformation properties in SU(6). This is not the case in SL(6,$c$). From (12), (14), and (15), and from Table I, the appropriate matrix elements are

$$\sqrt{3}\langle\Lambda\,|p\pi^-\rangle = 3S_1 - 4P_1,$$

$$3\langle\Sigma^+\,|p\pi^0\rangle = 3S_1 + 2P_1(1+3\lambda),$$

$$\sqrt{3}\langle\Xi^-\,|\Lambda\pi^-\rangle = 3S_1 - P_1(1-3\lambda), \qquad (16)$$

where

$$\lambda = P_1'/P_1. \qquad (17)$$

To satisfy the conditions[15]

$$\alpha_0\alpha_\Lambda < 0, \quad \alpha_\Lambda\alpha_\Xi < 0,$$

we merely choose

$$\lambda > \tfrac{1}{3}. \qquad (18)$$

This completes the demonstration that the empirical inconsistency associated with the 35-dimensional representation of SU(6) does not appear in SL(6,$c$).

It is instructive to compare the $P$-wave amplitudes of Eqs. (6)-(9) with earlier SU(6) calculations.[2,3] We first note that the $P_2$ term of Eq. (8) can be derived from the $H_2$ coupling of Rosen and Pakvasa[2] if orbital angular momentum is assumed to commute with SU(6). To derive $P_2'$ [Eq. (9)] from $H_2$(RP),[19] however, it must be assumed that a $P$-wave pion behaves in SU(6) exactly like the vector meson $\rho$. $P_1$ can be obtained from $H_1$(RP) with either one of these assumptions, and $P_1'$ is a term which has exactly the same SU(3) structure as the $S$-wave amplitude engendered by $H_1$(RP). [Note that a corresponding term arises from the $H_2$ of Eqs. (4) and (5); it has been omitted because it is a linear combination of (6)-(9)]. As pointed out above, the $S$-wave amplitudes in SL(6,$c$) are the same as those in SU(6).

From this analysis, we may conclude that the 35-fold representation of SU(6) is incompatible with the data on nonleptonic hyperon decay in the following cases: (a) when orbital angular momentum is assumed to commute with SU(6), i.e., $P_1' = P_2' = 0$ (this is the case most frequently discussed in references 2 and 3); (b) when a $P$ wave is assumed to behave like a $\rho$ meson in SU(6), i.e., $P_1' = P_2 = 0$ [the inconsistency here is qualitatively the same as in case (a)]; and (c) when a $P$-wave pion behaves as an arbitrary linear combination of cases (a) and (b) [in this case, $P_1' = 0$, and so the conditions of (17) and (18) cannot be met]. We can now use this conclusion to compare SL(6,$c$) with $\tilde{\text{U}}(12)$ and $M(12)$.[20]

Minimal transformation properties in $\tilde{\text{U}}(12)$ and $M(12)$ lead to the interactions of Eqs. (3) and (4), except that the matrices $A, B, E$ are restricted to the choice (5a) for $P$ waves. In the nonrelativistic limit, these $P$-wave amplitudes are exactly the same as those of case (b) in the previous paragraph. Since the $S$-wave amplitudes are still the same as in SU(6), we see that $\tilde{\text{U}}(12)$ and $M(12)$ are inconsistent with the data on nonleptonic decay.[21]

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[1]See, for example, S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N.Y.) 30, 348 (1964).

[2]S. P. Rosen and S. Pakvasa, Phys. Rev. Letters 13, 773 (1964).

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[4]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964); B. Sakita, Phys. Rev. 136, B1765 (1964).

[5]B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965); W. Rühl, Phys. Letters 13, 349 (1964). Other approaches have been proposed by R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 13, 698 (1964), and 14, 48 (1965); A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. 138, B692 (1965); S. Okubo and R. E. Marshak, Phys. Rev. Letters 13, 818 (1964); T. Fulton and J. Wess, Phys. Letters 14, 57 (1965); P. Roman and J. J. Aghassi, Phys. Letters 14, 68 (1965).

[6]This result can be derived from SU(6) and $CP$ invariance; see the papers cited in reference 3. Our argument does not depend on $CP$.

[7]For the detailed structure of meson and baryon fields, see Sakita and Wali, reference 5.

[8]Because the definition of $\Phi$ includes a $\gamma_5$ matrix,[7] $\bar{\Psi}\Psi\Phi$ is a parity-conserving interaction.

[9]"Observable terms" is used to denote those terms that engender observable decays within the octet of $J=\frac{1}{2}^+$ baryons.

[10]$[X]_{\lambda\mu}{}^{\alpha\beta}$ is constructed from its symmetry properties with respect to $\alpha\beta$, and to $\lambda\mu$, but no normalization is imposed, e.g. $[\underline{27}]_{\lambda\mu}{}^{\alpha\beta}=\bar{B}_\lambda{}^\alpha B_\mu{}^\beta+\bar{B}_\mu{}^\alpha B_\lambda{}^\beta+\bar{B}_\lambda{}^\beta B_\mu{}^\alpha+\bar{B}_\mu{}^\beta B_\lambda{}^\alpha$ −trace terms. For the symmetries of $[\underline{10}]$ and $[\underline{10}^*]$, see S. Okubo, Progr. Theoret. Phys. (Kyoto) 28, 24 (1962). Notice also that $F_\beta{}^\alpha=\frac{1}{2}[(B\bar{B})_\beta{}^\alpha-(\bar{B}B)_\beta{}^\alpha]$, $D_\beta{}^\alpha=\frac{1}{2}[(B\bar{B})_\beta{}^\alpha+(\bar{B}B)_\beta{}^\alpha$ −trace term].

[11]The combined effect of pre- and post-multiplying $\Phi$ by $\gamma_5$ is to change the sign of the mass of the pseudoscalar meson; see reference 7.

[12]To understand why other choices of $A,B,E$ do not contribute to the $S$-wave amplitudes, we note that $\gamma_5$ links the "large" components of one Dirac spinor to the "small" components of another. Thus any term with $A$ or $B$ equal to $\gamma_5$ is smaller by a factor $v/c$ than the corresponding term with $A$ and $B$ both equal to the unit matrix. By setting $E\equiv\gamma_5$, however, we merely cancel out a $\gamma_5$ appearing in the definition of $\Phi$ (see reference 7).

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[15]For a summary of the experimental data, see F. S. Crawford, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 827.

[16]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. 7, 407 (1957).

[17]A similar argument has been given by M. Suzuki.[2]

[18]S. P. Rosen, Phys. Rev. Letters 9, 186 (1962); Phys. Rev. 137, B431 (1965).

[19]We use $H_i$(RP), $i=1,2$, to denote the SU(6) couplings of Rosen and Pakvasa.[2]

[20]See the papers of Salam et al. and Bardakci et al., reference 5.

[21]Nonleptonic decay has recently been discussed in the $\tilde{\text{U}}(12)$ scheme by R. Gatto, L. Maiani, and G. Preparata, to be published; K. Kawarabayashi and R. White, to be published; and R. Oehme, to be published. See also M. Ademollo, G. Altarelli, and R. Gatto, Phys. Rev. Letters 14, 420 (1965) for a discussion in U(6) ⊗ U(6).

# EXISTENCE OF PIONS WITH SPIN

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It has recently been proposed[1] that one of the secondaries from $K_2{}^0\to\pi^++\pi^-$ decay, first observed by Christenson et al.,[2] and later by others,[3,4] is not a normal pion but a pion with spin (spion). The existence of such a spion could explain a long-standing asymmetry[5,6] in the $\pi\to\mu$ decay angular distribution for the lower part of the $\pi^+$ spectrum in $\tau$ decay. An admixture of at least 5% of spions in $\tau$ decay would be required. Several other consequences follow from this assumption, as pointed out in reference 1:

(a) The ratio of the decay rates for electronic and muonic modes should be comparable for the charged spion.

(b) If the spin of the spion is 1 and neutral spions exist, the preferred decay mode would be into an electron-positron pair plus a photon. Assuming a very short lifetime ($\lesssim 10^{-12}$ sec), one expects to observe in a bubble chamber an anomalous number of Dalitz pairs in $K^+$ decays.

We have checked hypothesis (a) in $\tau$ and $\tau'$ decay and hypothesis (b) in $\tau'$, in $K_{\mu 3}$, and in all $K^+$ decays involving a $\pi^0$. We have not found any evidence of the above effects, all the results being compatible with a completely normal behavior of the pions from $K^+$ decays.

The data were obtained from two exposures of the 81-cm Saclay CERN bubble chamber to beams of stopping $K^+$ mesons. The liquid in the chamber was $H_2$ for the first and $D_2$ for the second exposure. To check hypothesis (a) in $\tau$ decay, we studied the decays of the stopped positive secondaries. At least 3% of these should decay directly into positrons of 70-MeV energy to explain the observed magnitude of the $\pi\to\mu$ decay asymmetry. In the scan we examined all the positive secondaries of $\tau^+$ for apparently direct decays into a positron. We found 78 events out of 14 806 $\tau^+$'s. The range of the secondary and the momentum of the positron were measured for these events. A large background is expected among these