cation of Eq. (1) to reactions involving vector-meson exchange has not been very successful; for the case of pion-nucleon charge exchange (via ρ-meson exchange) treated by Barger and Ebel, it fails completely.[13] Presumably, matters would be improved if the exchange meson were treated as a Regge pole, but it is not clear how to include absorption corrections in that case.

In conclusion, we have seen that there exists a set of fairly plausible assumptions which lead to Eq. (1) in S-matrix theory, when the exchanged meson has spin zero. The approximations leading to Eq. (1) are easier to justify for the high partial waves than for the low ones, for which the case is rather weak. We see no prospect, however, of making any stronger case for the validity of Eq. (1) within the framework of S-matrix theory. The truncating of the unitarity series, Eq. (3), is clearly an essential part of the model. From this approximation Eq. (13) follows. The two remaining approximations are Eqs. (17) and (20). We can see no reason to believe that there should be any cancellation between the terms neglected in these approximations. It should be possible, however, to relax one or both of these approximations in reactions where the low-energy dynamics are rather well understood (such as pion-nucleon-nucleon scattering). In such a case one could evaluate Eq. (19) or perhaps Eq. (13) directly.

---

*Work supported in part by the U. S. Atomic Energy Commission and the National Science Foundation.

[1]K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964).

[2]L. Durand and Y. T. Chiu, Phys. Rev. Letters 12, 399 (1964); 13, 45 (1964); Proceedings of the International Conference on Particles and High Energy Physics, Boulder, Colorado, 1964 (to be published).

[3]M. H. Ross and G. L. Shaw, Phys. Rev. Letters 12, 627 (1964).

[4]N. J. Sopkovich, Nuovo Cimento 26, 186 (1962).

[5]M. Baker and R. Blankenbecler, Phys. Rev. 128, 415, (1962).

[6]G. A. Ringland and R. J. N. Phillips, Phys. Letters 12, 62 (1964).

[7]I. Derado, V. P. Kenney, and W. D. Shephard, Phys. Rev. Letters 13, 505 (1964).

[8]It has been shown by E. J. Squires [Nuovo Cimento 34, 1328 (1964)] that Eq. (1) results from random-phase N/D model.

[9]Arguments in favor of Eq. (1) have been given by R. Omnes [Phys. Rev. 137, B649 (1965)] by studying eigenphases, but it was necessary in his treatment to make assumptions about the behavior of the eigenphases in the complex-energy plane, a subject which has not been studied.

[10]This is, of course, exact in the energy range $W < W_3$. For a considerable range above W_3 the phase-space factor ρ_3 will still suppress the neglected term. At higher energies where many channels are open and absorption is strong, the diagonal elements of M should be much larger than the off-diagonal ones; moreover, the incoherence of the neglected terms in Eq. (2) should result in substantial cancellations.

[11]N. G. van Kampen, Phys. Rev. 91, 1267 (1953).

[12]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960). See p. 1610.

[13]V. Barger and M. Ebel, to be published.

DETERMINING THE CP ADMIXTURE OF NEUTRAL K MESONS

R. R. Lewis

University of Michigan, Ann Arbor, Michigan, and Université de Genève, Geneva, Switzerland

(Received 25 March 1965)

In a recent Letter,[1] we discussed the problem of determining the total mass matrix of the neutral kaons. If CP invariance is broken by the weak interactions, then in addition to the widths and the mass difference of the two "mass eigenstates" $|K_S\rangle, |K_L\rangle$, there are two further complex parameters r, ρ which measure the strength of the violation of T, and of TCP. These parameters determine the admixtures in $|K_S\rangle, |K_L\rangle$ of the CP eigenstates $|K_1\rangle, |K_2\rangle$. In our earlier Letter we pointed out the possibility of measuring these admixtures by observation of the CP nonconservation in a coherent beam of kaons in matter. The forward scattering in the medium induces CP admixtures of known phase and with amplitudes proportional to the density, which add coherently to the intrinsic CP admixtures. Hence, by measuring the magnitude of any CP-nonconserving effect versus the density of the medium, these two admixtures can be made to interfere, and both the phase and magnitude of the intrinsic admixture can be determined.

The purpose of this Letter is to develop this idea further, especially pointing out the possibility of providing a reference admixture which is adjustable in both phase and magnitude. This makes it possible to consider the exact cancellation of the CP admixture in $|K_S\rangle$ or in $|K_L\rangle$. The additional degree of freedom in varying the medium is provided by using a periodic (stratified) medium, rather than a homogeneous one; both the period and the average density can be varied. If the period is short compared to the decay length of the short-lived mode $1/\gamma_S$, then the medium can be considered homogeneous and the results depend only on the average properties of the constituents. But if the period is comparable to this decay length (typically a few centimeters), then we will show that, due to the phase differences caused by propagation between the successive strata, the length of the period will determine the phase of the induced CP admixture, while the density determines its magnitude.

Physically, such a medium can probably be realized by a spark chamber with counting regions of very low density (and spacing short compared to $1/\gamma_S$), separated by thicker metal plates to provide the correct average density. It should then be possible to measure the spatial distribution of decay products of both modes. Adequate separation of the coherent and incoherent mesons must be guaranteed by maintaining the sharp angular resolution of the beam and of the detection apparatus. This is possible by using a well-defined neutral kaon beam from a distant source, incident on a regenerator placed in front of the spark chamber, giving a mixture of the two modes as initial state.

If Λ is the mass matrix for a free meson, then the mass eigenstates in a medium are obtained by diagonalization of

$$\Lambda' = \Lambda - 2\pi N \begin{pmatrix} f & 0 \\ 0 & \bar{f} \end{pmatrix} /M,$$

where N is the number density of scatterers, M is the kaon mass, and $f, \bar{f}$ are the forward scattering amplitudes of $K, \bar{K}$. Since the form of Λ is assumed to be completely general, the mass eigenstates inside and outside the medium have the same form, with quantities inside the medium denoted with a prime. Writing the eigenvectors of Λ as

$$|K_S(\tau)\rangle = \begin{pmatrix} 1 \\ \rho r \end{pmatrix} \exp(-\lambda_S \tau/\sqrt{2})$$

and

$$|K_L(\tau)\rangle = \begin{pmatrix} \rho \\ -r \end{pmatrix} \exp(-i\lambda_L \tau/\sqrt{2}),$$

the parameters can be related to the mass matrix elements by

$$r = (\Lambda_{21}/\Lambda_{12})^{1/2}, \quad \rho = \eta + (1+\eta^2)^{1/2}, \quad \eta = \frac{\Lambda_{22}-\Lambda_{11}}{2(\Lambda_{12}\Lambda_{21})^{1/2}},$$

$$\lambda_S = \Lambda_{11} + \Lambda_{12} r\rho, \quad \lambda_L = \Lambda_{11} - \Lambda_{12} r/\rho.$$

Comparing the elements of Λ and Λ', we find $r' = r$ and $\eta' = \eta + \pi N(f-\bar{f})/M(\Lambda_{12}\Lambda_{21})^{1/2}$. To first order in $z = 1-r$ and $\zeta = 1-\rho$, we obtain

$$\lambda_{S,L}' \cong \lambda_{S,L}, \quad z' = z, \quad \zeta' \cong \zeta - 2\delta,$$

where $\delta = \pi N(f-\bar{f})/M(\lambda_S - \lambda_L)$ is the single parameter which determines the effect of the scattering medium. Notice that δ adds to ζ rather than to z, since the effect of the medium is that of a CP-nonconserving, T-conserving interaction.

A precise definition of the CP admixtures for a beam which is initially a K is given by

$$\beta_S = \frac{\langle K_2|K_S\rangle\langle\tilde{K}_S|K\rangle}{\langle K_2|K_L\rangle\langle\tilde{K}_L|K\rangle} = \frac{1-\rho r}{\rho(\rho+r)} \simeq \frac{z+\zeta}{2},$$

$$\beta_L = \frac{\langle K_1|K_L\rangle\langle\tilde{K}_L|K\rangle}{\langle K_1|K_S\rangle\langle\tilde{K}_S|K\rangle} = \frac{\rho(\rho-r)}{1+\rho r} \cong \frac{z-\zeta}{2}.$$

These quantities are seen to be well defined, independent of any arbitrary phase or normalization conventions, and therefore measurable in principle. In fact, it is not yet known whether any of the common decay modes measure the CP of the decaying meson. If we assume that both K and $\bar{K}$ have decay amplitudes A and $\bar{A}$ for a given channel, then the partial decay rate of a beam initially K is

$$R(\tau) = \frac{R(0)}{|1+\rho^2|^2}\{\exp(-\gamma_S\tau)|1+\alpha\rho r|^2 + \exp[-(\gamma_S+\gamma_L)\tau/2][\rho(\rho-\alpha r)(1+\alpha\rho r)^* \exp(i\Delta m\tau) + \text{c.c.}]$$
$$+ \exp(-\gamma_L\tau)|\rho(\rho-\alpha r)|^2\},$$

where $\alpha = \bar{A}/A$. If the decay is through $|K_1\rangle$, then $\alpha = 1$ and the decay distribution is seen to depend on the magnitude and phase of β_L; if it is through $|K_2\rangle$, then $\alpha = -1$ and β_S appears. If, instead, the decay amplitudes weakly violate CP invariance, then the experiment measures only the combinations $\beta_L + (1-\alpha)$ or $\beta_S + (1+\alpha)$. The relative phase of β_S and β_L is determined by symmetry alone; TCP invariance implies $\beta_L/\beta_S = +1$, whereas T invariance implies $\beta_L/\beta_S = -\rho^2 \cong -1$.

In a periodic stratified medium, composed of two different homogeneous materials with mass matrices Λ_1 and Λ_2, the modes can be defined as the eigenvectors of the matrix

$$T = \exp(-i\Lambda_2\tau_2)\exp(-i\Lambda_1\tau_1).$$

In terms of this matrix, which is the S matrix for a single period, the meson state after one period is $T|\kappa\rangle$, and after n periods is $T^n|\kappa\rangle$, if the incident state is $|\kappa\rangle$. The meson is incident first on medium 1, then medium 2, and spends a proper time interval τ_1, τ_2 in each. The limit of a homogeneous medium is obtained by letting $\tau_1, \tau_2 \to 0$ and $n \to \infty$ with $n\tau_1, n\tau_2$ finite, in which case the S matrix for the medium becomes

$$\lim_{n\to\infty} T^n = \exp[-i(\Lambda_1\tau_1 n + \Lambda_2\tau_2 n)],$$

which describes a medium with a mass matrix which is the mean of Λ_1 and Λ_2.

By straightforward solution of the secular equations, the eigenvalues of T can be shown to be

$$\exp\{-\tfrac{1}{2}i[(\lambda_S+\lambda_L)_1\tau_1 + (\lambda_S+\lambda_L)_2\tau_2] \mp i\Theta\},$$

where

$$\cos\Theta = \frac{(1+\rho_1\rho_2)^2\cos(\varphi_1+\varphi_2) + (\rho_1-\rho_2)^2\cos(\varphi_1-\varphi_2)}{1+\rho_1^{\,2}+\rho_2^{\,2}+\rho_1^{\,2}+\rho_2^{\,2}},$$

and where

$$\varphi_{1,2} = \tfrac{1}{2}(\lambda_S-\lambda_L)_{1,2}\tau_{1,2}.$$

Using a double prime to designate the modes in a periodic medium, the eigenvectors of T can be written

$$|K_S''\rangle = \begin{pmatrix} 1-\rho_1 B_S \\ r\rho_1 + rB_S \end{pmatrix}/\sqrt{2},\quad |K_L''\rangle = \begin{pmatrix} \rho_1 + B_L \\ -r + r\rho_1 B_L \end{pmatrix}/\sqrt{2},$$

with

$$B_{S,L} = \left(\frac{\rho_2-\rho_1}{1+\rho_1\rho_2}\right)\left[\frac{e^{\mp i\Theta} - e^{\mp i(\varphi_1-\varphi_2)}}{e^{\mp i\Theta} - e^{\mp i(\varphi_1+\varphi_2)}}\right].$$

These results are exact, and can be seen to be consistent with our previous result for a homogeneous medium in the limit $\varphi_1, \varphi_2 \ll 1$. For practical applications, it suffices to drop terms of order z^2, ζ^2 leading to

$$(\lambda_{S,L})_1 \cong (\lambda_{S,L})_2,\quad \Theta \cong \varphi_1 + \varphi_2 \cong \tfrac{1}{2}(\lambda_S-\lambda_L)(\tau_1+\tau_2),$$

and

$$B_{S,L} \cong \left(\frac{\zeta_1-\zeta_2}{2}\right)e^{\mp i\varphi_1}\frac{\sin\varphi_2}{\sin(\varphi_1+\varphi_2)}.$$

For the CP admixtures we obtain

$$\beta_S'' \cong \tfrac{1}{2}(z+\zeta_1-2B_S),\quad \beta_L'' \cong \tfrac{1}{2}(z-\zeta_1+2B_L).$$

If we identify medium 1 with the vacuum and medium 2 with a metal, we obtain $\zeta_1-\zeta_2 = 2\delta$. Assuming $\varphi_2 \ll 1$, we get the final results

$$\beta_S'' \cong \tfrac{1}{2}(z+\zeta) - (\tau_2/\tau_1)\delta f_S(\tau_1),$$

$$\beta_L'' \cong \tfrac{1}{2}(z-\zeta) + (\tau_2/\tau_1)\delta f_L(\tau_1),$$

where $f_S(\tau) = 2i\varphi/[\exp(2i\varphi)-1]$ and $f_L(\tau) = 2i\varphi/[1-\exp(-2i\varphi)]$ are complex functions with the limiting value $f(0) = 1$.

These results justify our previous assertions about the effect of the periodicity of the medium. In the limit of a very short period, we regain the CP admixtures of a homogeneous medium, while for a period comparable to $1/\gamma_S$, the parameter δ is modified both in phase and in magnitude by the functions f_S and f_L (Fig. 1). The variation of these functions is roughly linear for $\gamma_S\tau_1 \lesssim 2$, with a total phase variation of about $45°$ in each.

A word of caution is necessary about the significance of the eigenvectors of T. In a homogeneous medium the eigenvectors are translation invariant and so the CP admixture in a pure mode does not vary along the beam. In a stratified medium this is no longer the case, since there is a spatial variation of the eigenvector within one period. Even if the CP admixture in $|K_L''\rangle$ were canceled exactly, corresponding to a pure $|K_2\rangle$ emerging from the metal plate, there would still be a variation of the CP admixture due to propagation in the individual strata, making it impossible to cancel the CP

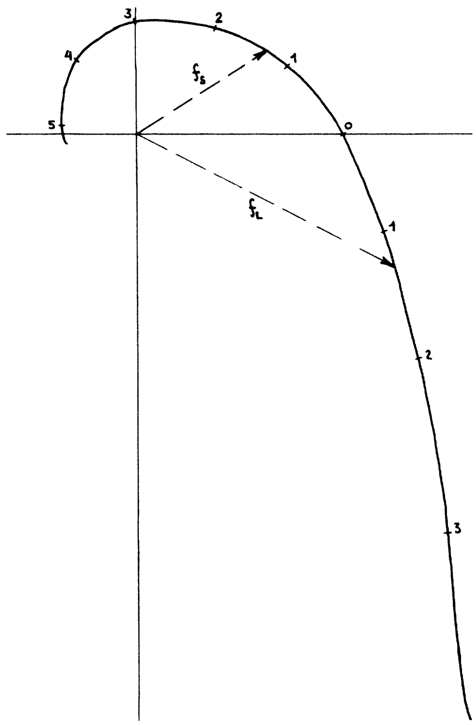

FIG. 1. The complex functions $f_S(\tau)$ and $f_L(\tau)$. The upper branch of the curve is f_S, and the lower branch is f_L, both coinciding at the initial value unity. The argument τ is indicated along the curve in units $1/\gamma_S$. The mass difference $|\Delta m/\gamma_S| = 0.82$ was used, with Δm assumed negative.

admixture everywhere. Obviously, if the vacuum portion becomes very long compared to $1/\gamma_S$, the meson state approaches the long-lived vacuum mode, with its definite CP admixture β_L. Therefore, application of this technique is restricted to media with periods $\gamma_S\tau_1 \lesssim 2$. This limits the available variation of phase, as noted above. Another alternative would be to restrict the fiducial volume to include only a small portion of one period.

One class of models for the mass matrix[2] which make definite predictions for the phase and magnitude of z and ζ are those in which CP invariance is violated only in the self-energy of the kaon. Then the observed rate[3] of $K_L \rightarrow \pi^+ + \pi^-$ fixes the magnitude of the relevant parameters, and T or TCP fixes their phase (to within a multiple of π). If TCP is conserved, as in the models of Sachs and Treiman[4] and of Wolfenstein and Lee,[5] then

$$z \cong 2\delta M_{12}/(\lambda_S - \lambda_L), \quad \zeta = 0,$$

where δM_{12} is imaginary. If, instead, T is conserved, as in the model of Bernstein, Cabibbo, and Lee, and of Bell and Perring,[6] then

$$z = 0, \quad \zeta \cong 2\delta M_{11}/(\lambda_S - \lambda_L),$$

with δM_{11} real. In either case the magnitude is fixed, $|\delta M/\gamma_S| \cong 2\times 10^{-3}$. Using the mass difference $|\Delta m/\gamma_S \cong 0.82$,[7] with the sign of $\Delta m = m_S - m_L$ negative,[8] we find that the phase of ζ is either $-30°$ or $+150°$, while the phase of z is $+60°$ or $+240°$. According to the calculations of Good and Pauli,[9] the phase of δ in carbon at 565 MeV is nearly real and positive. It follows from Fig. 1 that in a medium with period $\gamma_S\tau_1 \simeq 1$, ζ and δf_L will have relative phase $0°$ or $180°$, but z and δf_L will have relative phase $\pm 90°$. The density dependence of the magnitude of the CP admixtures for these two cases is characteristically different, and so for these models the rate of $K_L'' \rightarrow \pi^+ + \pi^-$ versus density is a test of T and TCP invariance.

The author wishes to gratefully acknowledge the discussion of this subject with Professor C. P. Enz, and to thank Professor J. M. Jauch for the hospitality of the Institut de Physique Théorique, where this work was performed.

---

[1]R. R. Lewis and C. P. Enz, to be published.

[2]Tai Tsun Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).

[3]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[4]R. G. Sachs and S. B. Treiman, Phys. Rev. Letters 8, 137 (1962); R. G. Sachs, Phys. Rev. Letters 13, 286 (1964).

[5]L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964); T. D. Lee and L. Wolfenstein, to be published.

[6]J. Bernstein, N. Cabibbo, and T. D. Lee, Phys. Letters 12, 146 (1964); J. S. Bell and J. K. Perring, Phys. Rev. Letters 13, 348 (1964).

[7]T. Fujii, J. V. Jovanovitch, F. Turkot, and G. T. Zorn, Phys. Rev. Letters 13, 253 (1964).

[8]F. S. Crawford, B. B. Crawford, R. L. Golden, and G. W. Meisner, Bull. Am. Phys. Soc. 9, 443 (1964).

[9]R. H. Good and E. Pauli, Phys. Rev. Letters 8, 223 (1962).