no indication of asymmetry in the $\pi^+ \to \mu^+$ decay angle for π^+'s in the energy range 0-12 MeV originating in τ^+ decay. Furthermore, the data show no evidence for an anomalously large branching ratio for $\pi^+ \to e^+$ decay for pions originating in τ^+ decay. Hence the experiment provides no evidence for the existence of a new particle with nonzero spin, of approximately the pion mass.

We wish to thank Dr. L. Lederman for forwarding to us a private communication from Dr. G. B. Cvijanovich. We gratefully acknowledge the invaluable help of our scanning and computing staff, especially D. Moran, G. Taplin, O. Wayne, L. Frank, and E. Dulberg.

*Work supported in part by grants from the National Science Foundation and by an equipment loan contract with the U. S. Office of Naval Research.

†Supported by a National Aeronautics and Space Administration Traineeship.

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ABSORPTIVE CORRECTIONS TO THE ONE-MESON-EXCHANGE MODEL IN S-MATRIX THEORY*

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The one-meson-exchange (OME) model modified by the inclusion of absorptive corrections has met with impressive success, at least for scalar meson exchanges.[1-7] The formula used by Sopkovich,[4] Durand and Chiu,[2] and Gottfried and Jackson[1] is to expand the OME scattering amplitude $B(W)$ in partial waves $B^J(W)$ and to make the replacement

$$B^J(W) \to [S_{11}^J(W)]^{1/2} B^J(W) [S_{22}^J(W)]^{1/2}, \qquad (1)$$

where W is the total energy in the c.m. system, S_{11}^J is the partial-wave S-matrix in the incident channel (1), and S_{22}^J is the partial-wave S matrix in the final channel (2). This formula has been derived under the following assumptions[1]: (i) the applicability of a complex potential to high-energy elementary-particle reactions; (ii) the neglect of "indirect" reactions from channel 1 to 2 via an intermediate state of channel n, $n > 2$; (iii) the neglect of the effect of the reaction $1 \to 2$ on the 11 and 22 amplitudes; (iv) the validity of the high-energy (eikonal) approximation; and either (v) that the interactions in channel 1 and 2 are the same, $S_{11} = S_{22}$, or (v′) that the range of the interaction giving rise to B is small compared to the range of the interactions in channel 1 and 2. Whether one questions assumption (i) is a matter of taste, but assumption (v) is quite questionable. In fact, (v′) is not satisfied in any of the reactions studied, and (v) seems arbitrary.

We present here a derivation of Eq. (1) within the framework of S-matrix theory. Our reasons for presenting this derivation are twofold: Firstly, the dubious assumption (v) or (v′) is not necessary; and secondly, more accurate formulas are found which should permit one to evaluate some corrections to Eq. (1).[8,9]

Let the reaction amplitude that we wish to calculate be designated M_{12}, which we analyze into partial waves M_{12}^J, suppressing for simplicity the labels referring to helicity and isospin. The unitary condition for M_{12}^J can be written in the form

$$\begin{aligned}\mathrm{Im} M_{12}^J = {} & M_{11}^{J*} \rho_1 M_{12}^J \theta(W - W_1) \\ & + M_{12}^{J*} \rho_2 M_{22}^J \theta(W - W_2) \\ & + \sum_{n>2} M_{1n}^{J*} \rho_n M_{n2}^{*} \theta(W - W_n),\end{aligned} \qquad (2)$$

where the ρ_i are phase-space factors, W_i is the threshold for the ith channel, and where the abbreviation

$$\sum_{n>2}$$

includes integrations over the phase space of many-particle states. Now we introduce assumption (ii) by neglecting the last term in Eq. (2); i.e., taking[10]

$$\mathrm{Im} M_{12}^{J} \approx M_{11}^{J*} \rho_1 M_{12}^{J} \theta(W-W_1) + M_{12}^{J*} \rho_2 M_{22}^{J} \theta(W-W_2). \tag{3}$$

We thereby neglect all "indirect" coupling between channels 1 and 2. To relax this assumption one necessarily goes beyond the OME model. The exact unitarity condition is retained for M_{11}^J and M_{22}^J; assumption (iii) is unnecessary in our treatment.

In the presence of inelastic scattering the general form of the diagonal elements of M is the following:

$$M_{ii} = [\eta_i \exp(2i\delta_R^{(i)}) - 1]/2i\rho_i, \tag{4}$$

where

$$\eta_i = \exp(-2\delta_I^{(i)}), \tag{5}$$

and where $\delta_R^{(i)}$ and $\delta_I^{(i)}$ are the real and imaginary parts of the phase shift. Substituting Eq. (4) into Eq. (3) one finds (for $W > W_2$)

$$\eta_1 \exp(-2i\delta_R^{(1)}) M_{12} = \eta_2 \exp(2i\delta_R^{(2)}) M_{12}^{*}. \tag{6}$$

This implies that $\eta_1 = \eta_2$, and that

$$M_{12} = |M_{12}| \exp[i(\delta_R^{(1)} + \delta_R^{(2)})]. \tag{7}$$

Thus the approximate unitarity equation (3) is a condition on the phase of M_{12}^J, analogous to the well-known phase relation between low-energy photoproduction and pion-nucleon scattering.

Assuming the phase shifts to be given, we can construct a solution for M_{12}^J in terms of the functions

$$D_i(W) \equiv \exp\left[-\frac{W-W_i}{\pi} \int_{W_i}^{\infty} dW' \frac{\delta_R^{(i)}(W')}{(W'-W_i)(W'-W)}\right]. \tag{8}$$

The quantity $D_1 M_{12} D_2$ is real on the real axis for $W > W_1$, and hence is analytic in that region. Therefore, we can write

$$M_{12}^{J}(W) = \frac{1}{D_1(W) D_2(W)} \frac{1}{2\pi i} \int_L dW' \frac{D_1(W') D_2(W') \,\mathrm{disc}[M_{12}^{J}(W')]}{W'-W}, \tag{9}$$

where the contour L extends over the unphysical singularities of M_{12}^J, and where $\mathrm{disc}[M_{12}^J]$ is (in the OME model) just the discontinuity of the OME term,

$$\mathrm{disc}[M_{12}^{J}(W)] = \mathrm{disc}[B^{J}(W)], \tag{10}$$

and

$$B^{J}(W) = \frac{1}{2\pi i} \int_L dW' \frac{\mathrm{disc}[B^{J}(W')]}{W'-W}. \tag{11}$$

If we define

$$\bar{B}^{J}(W) \equiv \frac{1}{2\pi i} \int_L dW' \frac{D_1(W') D_2(W') \,\mathrm{disc}[B^{J}(W')]}{W'-W}, \tag{12}$$

then we can write Eq. (9) in the OME approximation as

$$M_{12}^{J}(W) = [1/D_1(W)] \bar{B}^{J}(W) [1/D_2(W)]. \tag{13}$$

This form of solution is well known in low-energy problems (that is, at energies below inelastic thresholds) such as photoproduction and $N + \bar{N} \to \pi + \pi$, where a unitarity condition of the form of Eq. (3) is exact.

Equation (13) is similar in form to Eq. (1); the remaining questions are the relationship of $\bar{B}^J$ to B^J and the relationship of D_i to S_{ii}. The latter problem can be dealt with by considering the analytic properties of the phase shift, $\delta^{(i)}(W) = \ln(S_{ii}^J)/2i$. The phase shift will be singular wherever the scattering amplitude is singular, with additional branch points at the zeros of S_{ii}^J. These zeros are associated with resonances or other poles of S_{ii}^J in unphysical sheets. When such zeros are present, one can factor them out explicitly and define an effective phase shift $\bar{\delta}(W)$ in terms of the resulting zero-free amplitude. For example, if there is a pole in the second sheet, one can write

$S_{11}{}^J$ as[11]

$$S_{11}{}^J(W) = \frac{[(W-W_1)^{1/2}+\gamma][(W-W_1)^{1/2}-\gamma^*]}{[(W-W_1)^{1/2}-\gamma][(W-W_1)^{1/2}+\gamma^*]}\tilde{S}_{11}{}^J(W), \tag{14}$$

in which case the effective phase shift would be $\tilde{\delta}_1 = \ln(\tilde{S}_{11}{}^J)/2i$. Assuming that such zeros are absent, or that we have factored them out and are working with the effective phase shift $\tilde{\delta}$, we can then write a dispersion relation for the phase shift,

$$\delta^{(1)}(W) = \frac{1}{2\pi i}\int_L dW' \frac{\operatorname{disc}[\delta^{(1)}(W')]}{W'-W} + \frac{1}{2\pi i}\int_{W_1}^{\infty} dW' \frac{\operatorname{disc}[\delta^{(1)}(W')]}{W'-W}, \tag{15}$$

where the contour L extends over the unphysical singularities of $M_{11}{}^J(W)$. Since $i\delta$ is a real analytic function of W, it follows that on the real axis, $\operatorname{disc}[\delta] = 2\operatorname{Re}\delta$. Using this condition, and making a subtraction at threshold, we obtain

$$\delta^{(1)}(W) = \frac{W-W_1}{2\pi i}\int_L dW' \frac{\operatorname{disc}[\delta^{(1)}(W')]}{(W'-W_1)(W'-W)} - i\frac{W-W_1}{\pi}\int_{W_1}^{\infty} dW' \frac{\delta_R{}^{(1)}(W')}{(W'-W_1)(W'-W)}. \tag{16}$$

At high energies it seems plausible to assume that the right-hand integral in Eq. (16) is the dominant term, because of the usual argument about the dominance of nearby singularities. In more physical terms, we are assuming that the scattering is driven primarily by the absorption, rather than by the potential in channel 1. Making this assumption, we see from comparison of Eqs. (16) and (8) that

$$S_{11}{}^J(W) \approx 1/D_1{}^2(W). \tag{17}$$

Treating $\delta^{(2)}$ similarly, one finds

$$\delta^{(2)}(W) = \frac{W-W_2}{2\pi i}\int_L dW' \frac{\operatorname{disc}[\delta^{(2)}(w')]}{(W'-W_2)(W'-W)} - i\frac{W-W_2}{\pi}\int_{W_1}^{\infty} dW' \frac{\delta_R{}^{(2)}(W')}{(W'-W_2)(W'-W)}. \tag{18}$$

Note that the lower limit of the right-hand integral is W_1, whereas the integral in D_2 starts at W_2. At energies $W \gg W_2$ we may neglect, in addition to the left-hand singularities, the piece of the right-hand cut between W_1 and W_2, and thereby identify $S_{22}{}^J$ with $1/D_2{}^2$. Substituting Eq. (17) and the analogous equation for S_{22} into Eq. (13), one finds

$$M_{12}{}^J(W) \approx (S_{11}{}^J)^{1/2}\tilde{B}^J(S_{22}{}^J)^{1/2}. \tag{19}$$

This result is very close to the formula in question, Eq. (1); there remains only the question of the relation between $\tilde{B}^J$ and B^J, defined by Eq. (12). For the high partial waves, where the forces are weak, $D_i(W)$ does not differ significantly from one, and the approximate equality

$$B^J \approx \tilde{B}^J \tag{20}$$

follows from comparison of Eqs. (11) and (12). For the low partial waves the situation is much less clear. Again, if the phase shifts in channels 1 and 2 happen to be small in a given partial wave, Eq. (20) will be true. A third possibility is that $\operatorname{disc}[B^J(W)]$ is sufficiently sharply peaked on that portion of L which is nearest to the physical threshold that the integral in Eq. (11) and (12) is dominated by that portion. Since $D_i(W_i) = 1$, the integral in Eq. (12) will then be little affected by the D_i's and Eq. (20) will again follow. This is the case for one of the amplitudes in the process $N+\bar{N}\rightarrow\pi+\pi$, for example.[12] It is clear, however, that the arguments in favor of Eq. (20) are not compelling; we should not be surprised to find the application of Eq. (1) successful for some reactions and unsuccessful for others.

When the exchanged meson is a vector meson, it seems impossible to justify Eq. (20). If the exchanged vector meson is treated as an elementary particle, then $\operatorname{disc}[B^J(W)]$ behaves badly at infinity, and the integral in Eq. (11) does not exist. Even if we assume that a theory without subtractions does exist, there is no reason to believe that Eq. (20) would hold. This is consistent with the fact that the application of Eq. (1) to reactions involving vector-meson exchange has not been very successful; for the case of pion-nucleon charge exchange (via ρ-meson exchange) treated by Barger and Ebel, it fails completely.[13] Presumably, matters would be improved if the exchange meson were treated as a Regge pole, but it is not clear how to include absorption corrections in that case.

In conclusion, we have seen that there exists a set of fairly plausible assumptions which lead to Eq. (1) in S-matrix theory, when the exchanged meson has spin zero. The approximations leading to Eq. (1) are easier to justify for the high partial waves than for the low ones, for which the case is rather weak. We see no prospect, however, of making any stronger case for the validity of Eq. (1) within the framework of S-matrix theory. The truncating of the unitarity series, Eq. (3), is clearly an essential part of the model. From this approximation Eq. (13) follows. The two remaining approximations are Eqs. (17) and (20). We can see no reason to believe that there should be any cancellation between the terms neglected in these approximations. It should be possible, however, to relax one or both of these approximations in reactions where the low-energy dynamics are rather well understood (such as pion-nucleon-nucleon scattering). In such a case one could evaluate Eq. (19) or perhaps Eq. (13) directly.

*Work supported in part by the U. S. Atomic Energy Commission and the National Science Foundation.

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[10]This is, of course, exact in the energy range $W < W_3$. For a considerable range above W_3 the phase-space factor ρ_3 will still suppress the neglected term. At higher energies where many channels are open and absorption is strong, the diagonal elements of M should be much larger than the off-diagonal ones; moreover, the incoherence of the neglected terms in Eq. (2) should result in substantial cancellations.

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DETERMINING THE CP ADMIXTURE OF NEUTRAL K MESONS

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(Received 25 March 1965)

In a recent Letter,[1] we discussed the problem of determining the total mass matrix of the neutral kaons. If CP invariance is broken by the weak interactions, then in addition to the widths and the mass difference of the two "mass eigenstates" $|K_S\rangle, |K_L\rangle$, there are two further complex parameters r, ρ which measure the strength of the violation of T, and of TCP. These parameters determine the admixtures in $|K_S\rangle, |K_L\rangle$ of the CP eigenstates $|K_1\rangle, |K_2\rangle$. In our earlier Letter we pointed out the possibility of measuring these admixtures by observation of the CP nonconservation in a coherent beam of kaons in matter. The forward scattering in the medium induces CP admixtures of known phase and with amplitudes proportional to the density, which add coherently to the intrinsic CP admixtures. Hence, by measuring the magnitude of any CP-nonconserving effect versus the density of the medium, these two admixtures can be made to interfere, and both the phase and magnitude of the intrinsic admixture can be determined.

The purpose of this Letter is to develop this idea further, especially pointing out the possibility of providing a reference admixture which