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SMALL-ANGLE ELASTIC SCATTERING OF HIGH-ENERGY PROTONS FROM HYDROGEN*†

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We present the results of an experiment performed at the Brookhaven AGS to measure high-energy p-p scattering in the range of laboratory angle 2 to 20 mrad. Elastic scattering events were separated by momentum analysis. The prime purpose of the experiment was to investigate Coulomb interference effects. The range of incident particle momentum studied was 8 to 18 BeV/c.

The apparatus is shown in Fig. 1. A small-angle secondary beam from the AGS was momentum analyzed (±0.8% half-width at half-maximum) and focused on a liquid hydrogen target 18 in. long and 4 in. in diameter. The beam was defined by the scintillation counter $S2$ and the scintillation counter hodoscopes $HO2$ and $HO3$. The incident particle was selected by the differential Cherenkov counter C,[1] and its angle was measured to ±0.3 mrad (half-width at half-maximum) by $HO2$ and $HO3$. Each hodoscope was made up of $\frac{1}{4}$ in. wide scintillation counters in 4-element×4-element two-dimensional arrays. The hodoscopes $HO2$, $HO3$, and $H2$ determined the horizontal projection of the scattering angle to ±0.4 mrad while $HO2$, $HO3$, and $H4$ determined the vertical projection to ±0.4 mrad. The momentum of the final particle was measured to ±0.8% with $HO3$, $H2$, and $H4$. Hodoscope $H2$ comprised 80 counters, each $\frac{1}{4}$ in. wide×6 in. high. The 120 vertical elements of $H4$ were each $\frac{1}{2}$ in. wide×13 in. high and the 24 horizontal elements were each $\frac{1}{2}$ in. high×61 in. long. The absolute mean momentum of the incident particles was determined to within 0.2%.

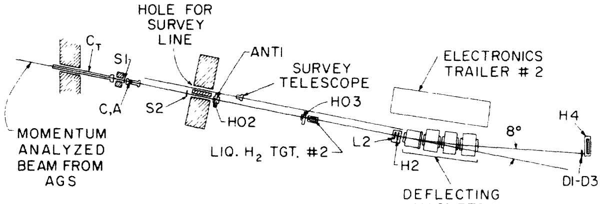

FIG. 1. The experimental arrangement.

The trigger system included the counters $L2$ and the sum of the outputs of the vertical elements in $H4$. $L2$ excluded the unscattered beam and $H4$ limited the range of momentum of the scattered particle. On a fast (~20-nsec) coincidence between these counters and a selected beam particle, 192 fast discriminator gates (~40 nsec) were opened and signals were detected from those counters which had fired. This information was then transferred to a buffer memory and, approximately 15 μsec later, the system was ready to accept another event. A signal from a ring counter (ANTI) behind the hodoscope $HO2$ was fed into the buffer memory at the same time so that the program could reject those events with an extra particle passing through the collimator but outside of $HO2$. The data-handling system was similar in principle to that described in earlier publications.[2] However, a new buffer memory with about 65 times the capacity[3] of the early system was used to take advantage of the high event rates available in this experiment. At the end of an AGS burst the information stored in the memory was transferred in parallel to magnetic tape and, via coaxial cables, to the Merlin computer. During the runs the computer analyzed events and stored them in a two-dimensional array of events versus angle and momentum of the scattered particle. At the end of each run the computer subtracted the inelastic background by linear interpolation and calculated the differential cross section. Unfortunately, the data-handling capacity of Merlin was inadequate at the high event rates, which at times exceeded 10^6/hour, so the data analysis was completed using the BNL IBM 7094. The Merlin analysis served as a monitor during the experiment, providing immediate feedback via scope display and printouts.

The target-empty background varied from

a maximum of 70% at the smallest $|t|$ to a minimum of 20%, decreasing rapidly with increasing $|t|$ where t is the negative square of four-momentum transfer. As this background was due to scattering from the material in the beam near the hydrogen target, it is insensitive to a variation of rate and other beam conditions and can be subtracted accurately and reliably. The inelastic background subtracted from the elastic peak was 5-10% both for target-full and target-empty runs. Corrections were also applied for the efficiency of the system, for accidental coincidences, and for nuclear absorption. The efficiency was determined experimentally by deflecting the beam onto the hodoscope $H4$. Variation of this efficiency with angle and momentum was measured to be less than 0.7%. The accidental corrections were less than 1.5%.

The cross-section results are shown in Fig. 2. The errors shown are the result of compounding counting statistics, the uncertainties in the mean scattering angle and the incident momentum, the uncertainty in the inelastic background subtraction, and the uncertainty in the accidental corrections.

The differential cross section may be written as $d\sigma/dt = |A_c + A_n|^2$, where A_c and A_n are the complex scattering amplitudes due to the Coulomb and nuclear forces, respectively. The Coulomb amplitude including form factor may be written in the form[4]

$$A_c(t) = e^{2i\delta} \frac{2e^2\sqrt{\pi}}{\beta c t} \left[G_E^{\,2}(t) - \frac{t}{4M^2} G_M^{\,2}(t) \right] \left[\frac{1}{1-t/4M^2} \right].$$

The formula for the phase angle δ was derived by Bethe,[5] and an estimated radius of interaction equal to 0.92×10^{-13} cm was used. Measurements of electron-proton scattering[6] give $G_M \approx \mu G_E$ and $G_E = 1 + 2.77t$ for $-t < 0.2$ $(\mathrm{BeV}/c)^2$.

With the assumption of spin independence the nuclear amplitude at a given energy may be written in the form $A_n(t) = (\alpha + i)\exp(a + bt)$, as was suggested by the previous experiments at larger $|t|$ values[2] and is a good fit to the characteristics of the present data. If we make the assumption that the slopes of the real and imaginary parts of the amplitude are the same, then a single t-independent constant α represents the ratio $\mathrm{Re}A_n(t)/\mathrm{Im}A_n(t)$. The quantity a is determined by the optical theorem requirement using the most recent values of total cross

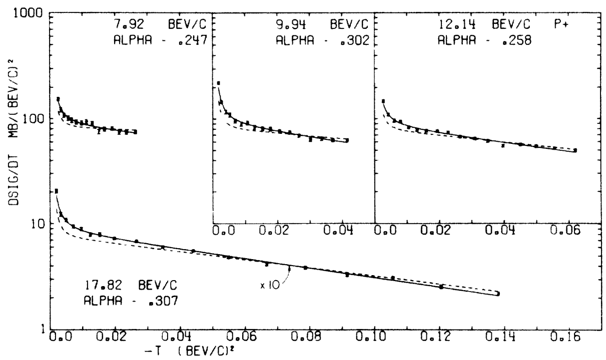

FIG. 2. The measured differential cross sections. The lines represent least-squares fits to the data as described in the text. Note that the 17.82-BeV/c results have been lowered one decade.

sections by Galbraith et al.[7]

In addition, a term (<4%) must be added to $d\sigma/dt$ to account for multiple Coulomb scattering in the liquid hydrogen target. A calculation based on Molière's theory[8] was used.

The quantities α and b are then fit by least squares to the measured differential cross sections. The solid line on Fig. 2 shows the fit obtained in this way. The dotted line is the best fit obtained setting $\alpha = 0$. The mean value of t for each point is corrected for the effects of the inclusion of a finite range of incident and final momentum and angle. These calculations are being refined and for the purpose of this Letter the data with $|t|$ smaller than 0.002 have been left out of the fits since the uncertainty in the t corrections is most important there. The values of α and the fitted errors, together with the value of χ^2 for the fits, are listed in Table I. The errors in the third column are the standard deviations obtained from the fit. The fourth column lists the estimated limit of the systematic error due to uncertainties in the absolute normalization, the total cross section used to evaluate $\mathrm{Im}A_n(0)$, and the liquid hydrogen absorption correction. For the purposes of this analysis, a cross section equal to the total inelastic cross section plus half of the total elastic cross section was used for the absorption correction. An error corresponding to half the elastic total cross section has been assigned to this correction. A more detailed study of this problem is in progress.

From Table I, it is clear that with the stated assumptions there is a real part of the p-p scattering amplitude in the region of 8 to 18 BeV/c

Table I. The fitted values of $\alpha = \mathrm{Re}A_n/\mathrm{Im}A_n$. The errors shown are described in the text.

Momentum	α	Error: Standard deviations	Error: Systematic (est. limit)	χ^2	χ^2 expected for two-parameter fit	χ^2 from the fit with $\alpha = 0$
7.92 BeV/c	−0.247	0.023	+0.059 −0.052	11.8	13	97.4
9.94 BeV/c	−0.302	0.018	+0.053 −0.051	12.9	14	222.5
12.14 BeV/c	−0.258	0.016	+0.050 −0.051	21.5	14	201.6
17.82 BeV/c	−0.307	0.016	+0.049 −0.044	15.9	15	254.1

laboratory momentum amounting to about 25% of the imaginary part and with a sign corresponding to a repulsive potential. Other recent experiments[9,10] have reached a similar conclusion in nearby energy regions. These results are in agreement with dispersion-relation calculations[11] but in view of the large experimental errors and the question of spin dependence, this is not considered very significant. It is of interest to note that we have obtained a similar result for α for 8- to 12-BeV/c π^--p and 10-BeV/c π^+-p scattering.[9]

One should note that a most important assumption in the p-p case is that of spin independence. Were we to allow different t dependence and amplitude for the singlet and triplet scattering amplitudes we could certainly get a good fit to the data with zero real part.

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