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[3]Each of the four S chambers was observed by 10 phototubes. These phototubes were optically isolated, and each observed 0.8 radiation length of the S chamber. The 10 signals were added electronically, and the sum pulse had been established experimentally to be proportional to the energy of an electron passing through the S chamber.

[4]We wish to thank the Columbia-Brookhaven neutrino group for the loan of the Al spark chambers.

[5]The separation of ρ and ω events is analogous to the separation of the two-pion decays of those particles. This has proved to be extremely difficult. An experiment of high statistical significance may be able to separate these events experimentally if the mass resolution is <10 MeV. If a mixing angle near 38° is correct, then SU(3) would predict nearly equal branching ratios for ρ and ω.

[6]J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

[7]If we assume a φ cross section of 50 μb, then the branching ratio of φ to electron pairs is $R(\varphi \rightarrow e^+ + e^-) = (6 \pm 3) \times 10^{-3}$.

MUON-PROTON SCATTERING AT HIGH MOMENTUM TRANSFERS*

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(Received 22 March 1965)

We report in this Letter the result of an experiment to observe the scattering of muons by protons at high momentum transfers. A comparison of the results with similar electron-scattering data provides a sensitive test of any presumed difference in the charge structure of the leptons. The data reported here consist of about 500 scattering events with momentum transfer in the interval 700 to 1100 MeV/c. They indicate no significant difference between μ-p and e-p scattering. This is in agreement with earlier studies,[1,2] but extends considerably the limits on any presumed anomaly.

The experiment was performed at the Brookhaven AGS, using the experimental arrangement shown in Fig. 1. A specially designed internal target ($G9$) permitted negative pions produced near 0° to be deflected out of the machine by one section of the AGS magnet ring. Pions of 6- to 10-BeV/c momentum were accepted and transported for a distance of about 170 ft by a 14-quadrupole magnet array. The decay muons were then separated from the pions by transmission through a concrete filter filling the first 32 ft of an iron collimator 45 ft in length. The transmitted muon beam had a momentum spectrum extending from approximately 1.5 to 6.0 BeV/c (peaked at 2.8 BeV/c), an angular divergence of less than 1°, and an intensity of about 2×10^6 per pulse for a flux of 3×10^{11} protons per pulse. From our measurements[3] of the nuclear attenuation of pions in concrete, we estimated that the ratio of pions to muons transmitted by the filter was less than 10^{-6}. This estimate of the contamination was corroborated by direct measurements (see below).

The detectors consisted of an array of scintillation counters and spark chambers. Incident muons were counted by a gas Cherenkov counter (C) 12 in. in diameter, and by a 10×10-in. scintillation counter (S), and were then incident on a liquid-hydrogen target 72 in. long and 18 in. in diameter. Scattered muons traversed one or both of a pair of aluminum plate spark chambers (Mu-1 and Mu-2), a 40-ft-thick heavy concrete absorber, and a liquid scintillation

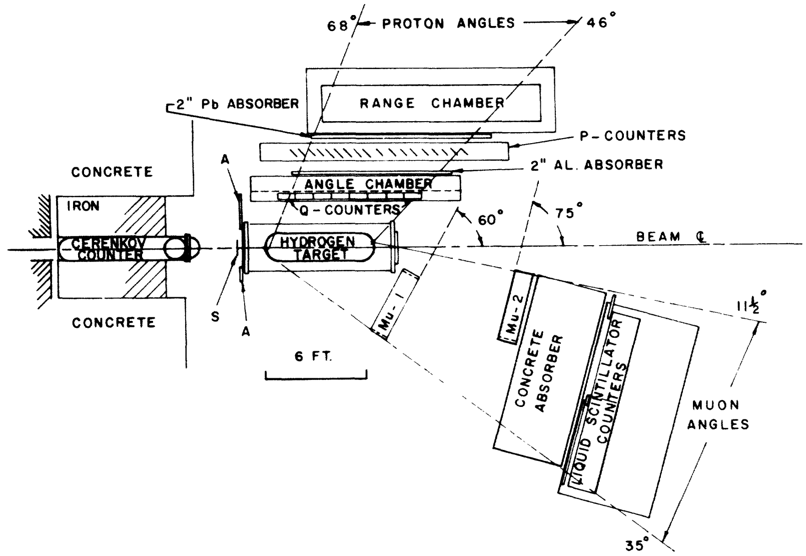

FIG. 1. Experimental arrangement for high-q^2 run (18 to 31 F^{-2}). In part of the run the 2-in. Pb absorber was removed (medium-q^2 run, 12 to 21 F^{-2}).

counter (R). The concrete absorber discriminated against pion-proton scattering events by a factor of at least 10. Recoil protons traversed a scintillator (Q), a thin-plate spark chamber (angle chamber), two more scintillators (P), and absorbers of lead and aluminum, and stopped in a thick aluminum plate spark chamber (range chamber). The spark chambers were pulsed when a coincidence $CSPQR$ was registered in anticoincidence with the guard counters A.

We counted 2×10^{11} muons in about 400 hours of data taking. Because of the loose triggering requirement, most of the approximately 100 000 photographs taken were caused by accidental coincidences, inelastic-scattering events, and elastic-scattering events with such low momentum transfer that the recoil proton stopped before entering the range chamber. The photographs which showed both scattered muon and recoil proton tracks (about 7 % of the total) were measured in stereo, and the information was processed by a digital computer program. Although no measurement was made of the tracks of individual incoming muons, beam studies showed that the incident muons could be regarded as originating from a point 455 in. upstream from the H_2 target (midway in the concrete filter). Thus, knowing the paths of the incident particle and both recoil particles, one could reconstruct a scattering event completely.

Since the expected number of elastic muon-proton scattering events was still a small fraction of the candidates which were processed as outlined above, special care was required to establish criteria for selecting elastic-scattering events. The principal geometrical constraints imposed were coplanarity (incident- and scattered-particle trajectories define a common plane), "copunctuality" (scattered-particle trajectories intersect at a point), and proper location of the scattering point (in the hydrogen).

Two additional constraints were derived by assuming two-body kinematics: (1) The calculated incident muon momentum must lie in the range 1.5 to 6.0 BeV/c, and (2) the recoil proton momentum (P_c) calculated from the scattering angles must agree, within the limits set by the experimental resolution, with the momentum (P_m) determined from the proton range.

The last requirement resulted in the loss of those events in which the recoil proton stopped by nuclear interaction before reaching the end of its range. This event loss is a function of recoil proton momentum, and varies between 40 and 75 %. The probability of observing a full-range proton was determined experimentally in a separate measurement performed at the Brookhaven Cosmotron,[4] in which monoenergetic protons were detected in a facsimile of the proton-detection apparatus used in the main experiment. The momentum of the protons was varied between 700 and 1050 MeV/c to cover the interval of observed momentum transfers.

The computations of the loss by nuclear interaction and of many less important corrections (such as small losses resulting from the imposition of geometrical constraints) were performed with the aid of Monte-Carlo programs. First, a program was used to interpret the results of the Cosmotron experiment. The optical model was used to fit the known proton elastic and total scattering cross sections and was generalized to include one free parameter adjusted to match the observed inelastic scattering range and angular distributions. A second Monte-Carlo program was used to compute the efficiency for detecting elastic-scattering events in the AGS experiment, including the effects of multiple scattering, proton nuclear scattering (using the optical-model fit), and measurement errors. Tracks generated by the program were traced through the detection apparatus, and the simulated events were then put through the same reconstruction program as were the real events.

The characteristics of Monte-Carlo-generated events and real events were then compared in detail in order to establish the validity of the method. For example, it was verified that the simulated and real events gave the same distributions in the coplanarity and copunctuality variables. Most definitive, however, was the comparison of the frequency distribution of events versus the difference $(P_c-P_m)/P_m$ (see Fig. 2). As is expected, both curves show a prominent peak near zero ($P_c=P_m$) and a "tail" for $P_c>P_m$ corresponding to events having nuclear-stopped protons. There are also a small number of "real" events having $P_c<P_m$. These can be attributed to accidental coincidences (simulation of a scattering event by two tracks having no real connection). Since accidentals are expected to have a flat distribution, they can be subtracted (the area under the peak is thus reduced by 6 %). After this subtraction has been made, the two distributions are in good agreement. This not only justifies the Monte-Carlo simulation procedure, but provides strong evidence that the selection criteria were highly effective in separating real events from back-

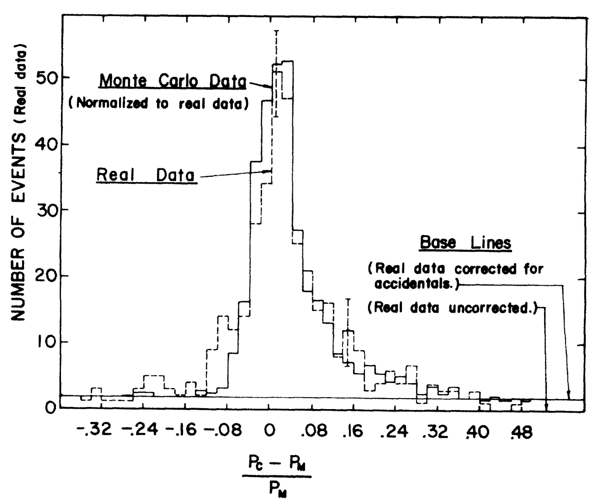

FIG. 2. Frequency distributions of real and simulated (Monte-Carlo) scattering events, versus $(P_c-P_m)/P_m$. The shift of baseline for the real events is the result of subtracting the accidental coincidences (see text), which have a flat distribution. The Monte-Carlo distribution has been normalized to have the same area as the real distribution after correction.

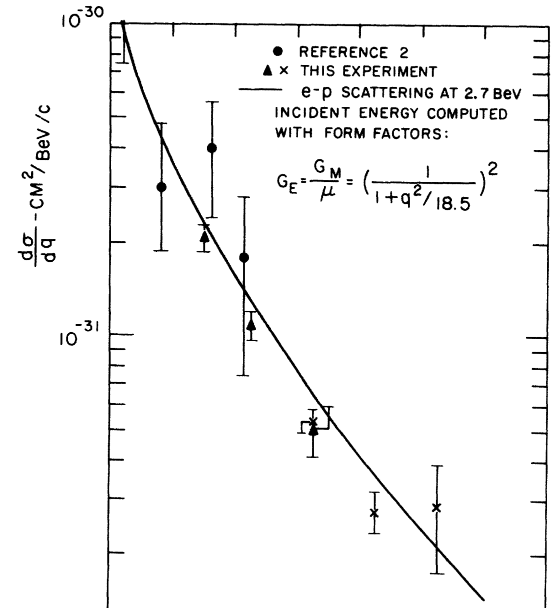

FIG. 3. Comparison of muon cross section $(d\sigma/dq)$ with the Rosenbluth prediction computed with a phenomenological fit to proton form factors. The two results at $q^2=20$ F^{-2} represent the overlap of two separate runs, medium-q^2 and high-q^2, differing in the thickness of absorber in front of the range chamber (Fig. 1). The expressions for G_E and G_M were chosen to fit low-q^2 e-p scattering data [L. N. Hand, D. G. Miller, and Richard Wilson, Rev. Mod. Phys. 35, 335 (1963)], and were found to fit data[7] with $q^2 \geq 20$ F^{-2} as well as the more commonly used form made up of resonance terms. Results of earlier experiments are also shown.

ground of all kinds; in particular, it seems very probable that there is no appreciable contamination of inelastic-scattering events[5] in the peak of the distribution.

The next step in the analysis was to compute the numbers of events which satisfied all the selection criteria, and tabulate them according to the momentum transfer ($q^2 = 2MT_p$, where T_p is the proton kinetic energy and is measured to an accuracy of $\approx 1\%$). The Monte-Carlo output was then used to evaluate the over-all detection efficiency for each value of q. Account was taken of small biases resulting from scanning losses, electronic inefficiencies, etc. (all effects independent of q). Scattering from material other than hydrogen was evaluated in an empty-target run and found to be negligible. The effect of pion contamination in the beam was demonstrated to be negligible, by measuring the yield of events with contamination artificially increased by a factor of about 1000. This is done by removing concrete in the collimator and in the scattered muon detector. Finally, the muon-proton scattering cross section was computed.[6]

The elastic-scattering cross section for structureless muons is predicted by the Rosenbluth formula, which can be written in the form

$$d\sigma/dq^2 = \frac{4\pi\alpha^2}{q^4}\frac{G(q^2)}{1+q^2/4M^2}(1-q^2/2Mp_0+\cdots), \quad (1)$$

where

$$G(q^2) = G_E^{\,2} + (q^2/4M^2)G_M^{\,2},$$

α is the fine-structure constant, G_E and G_M are proton form factors, M is the proton mass, and p_0 is the incident muon momentum. Equation (1) illustrates that, under the conditions of the present experiment where q is directly measured, the cross section is only weakly dependent on p_0, and to a good approximation is proportional only to the form-factor combination $G(q^2)$. Figure 3 shows a comparison of our results, the only previous observations in this region,[2] and the cross section computed from Eq. (1) after averaging over p_0, and using form factors obtained from e-p scattering experiments[7]; in Fig. 4 a comparison is made of the muon and electron[8] results for $G(q^2)$. It is evident from both figures that the muon results are somewhat lower than the electron data, although the q^2 dependence is very similar.

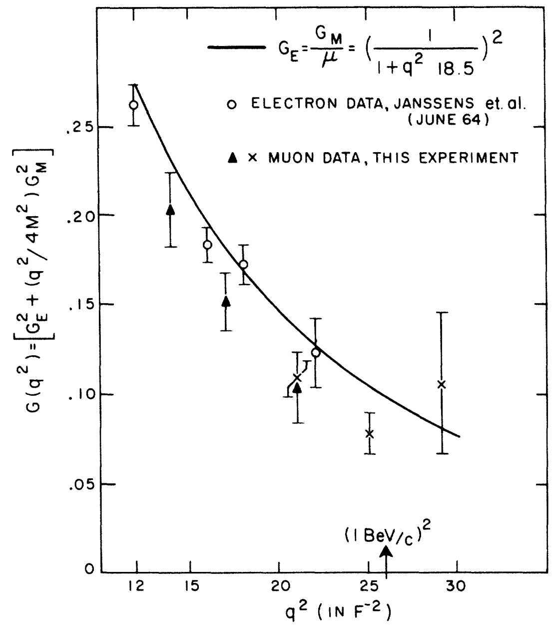

FIG. 4. Comparison of $G(q^2)$ as measured in μ-p and e-p scattering. The smooth curve is the phenomenological fit used to compute the cross section in Fig. 3.

We may draw more quantitative conclusions by adopting a model which incorporates deviations from the Rosenbluth formula. Following Barnes[9] and Drell,[10] we attribute to the leptons a single form factor, $f = (1+q^2/\Lambda^2)^{-1}$, where, however, Λ may be different for the electron and the muon. We define the quantity D^2 by the relation $1/D^2 = 1/\Lambda_\mu^{\,2} - 1/\Lambda_e^{\,2}$. Then, to first order, $f_\mu/f_e = [1+q^2/D^2]^{-1}$, and the ratio of muon and electron scattering cross sections (evaluated at the same values of q^2 and p_0) give a measure of D^2. Using the absolute cross sections determined in this experiment and the smoothed fit to electron data (illustrated in Fig. 3), we find

$$D^2 = 187\ \mathrm{F}^{-2}, \text{ i.e., } (2.7\ \mathrm{BeV}/c)^2.$$

The question which now arises is whether this result establishes a real difference between Λ_e and Λ_μ ($D \to \infty$ if $\Lambda_\mu = \Lambda_e$). Although statistically this result is more than two standard deviations away from $\Lambda_\mu = \Lambda_e$ (one must assign realistic errors to the electron points in this region), there is a systematic uncertainty in both the electron curve (estimated by experts as ~5%) and the muon points. All the likely sources of such errors in our experiment (flux normalization, scanning losses, etc.) are q^2

independent and could be as high as 10%. Thus the present observations are not adequate to establish firmly that $\Lambda_\mu \neq \Lambda_e$. However, the absolute rates can nevertheless be used to set a new limit on the size of any deviation:

$$D^2 > 95 \text{ F}^{-2}, \text{ i.e., } (1.9 \text{ BeV}/c)^2,$$

with 95% confidence.

In view of the q^2-independent nature of the systematic errors, it is fruitful to compare the dependence of the cross sections on q^2; the best fit is obtained by increasing the muon yields by a factor of 1.18, in which case one finds $D^2 = 5600 \text{ F}^{-2}$ and

$$|D^2| > 220 \text{ F}^{-2}, \text{ i.e., } (2.9 \text{ BeV}/c)^2,$$

with 95% confidence. This is evidently consistent with the only previous measurement[2] of μ-p scattering, which gave $D^2 > 36 \text{ F}^{-2}$.

Recent measurements of $(g-2)$ for the muon,[11] and of pair production of muons at large angles,[12] provide tests of possible anomalous muon properties with a sensitivity close to that achieved here. As has been pointed out by Drell and McClure,[13] however, the different types of experiments are not directly comparable. One may, of course, speculate that if there is any anomaly it should show up in all experiments in the same way, in which case the $(g-2)$ and pair-production results limit the muon vertex cutoff to $\Lambda_\mu^2 \gtrsim 40 \text{ F}^{-2}$. The results of the scattering experiment reported here then serve to set a new limit on the charge radius of the electron: $r_e = 6^{1/2}\Lambda_e \lesssim 3 \times 10^{-14}$ cm.[14]

We are grateful for the help of many graduate assistants at Columbia and Rochester during the preparation, performance, and analysis of the experiment. Professor L. Marshall was instrumental in the successful design of the muon transport system. We also acknowledge the unfailing support of the engineering and technical staffs at Brookhaven, Columbia, and Rochester. Finally, the experiment could not have been performed without the generous and continuing assistance of H. Blewett, E. Courant, and K. Green.

*Work supported in part by the U. S. Atomic Energy Commission.

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[5]We have considered, for example, the likelihood of observing the production of a single π^0 through the $\frac{3}{2}$-$\frac{3}{2}$ isobar; if the proton and recoil muon were to be coplanar with the incident particle and to have angles compatible with elastic scattering kinematics, the proton momentum would be considerably lower than would be required for elastic scattering. Thus such events would contribute to the spectrum in the region of $p_c > p_m$, but could not affect the population of events in the peak. As an additional check on the sensitivity of the selection procedure to inelastic contributions, we have computed the yield of events in the peak for different constraints on coplanarity; no appreciable change is found when the transverse momentum acceptance is increased from 100 to 200 MeV/c.

[6]A radiative correction of about 3% was included (W. Moran, private communication).

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[14]This result then also confirms the interpretation of e-p scattering results as to the rms radius of the proton ~0.8 F.