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[7] In the notation $\Sigma_0{}^{+}$ the upper sign is that of the decaying Σ, while the lower one is the sign of the emitted π. $\Omega_{K^-}{}^{-}$ denotes the decay $\Omega^- \rightarrow K^- + \Lambda$. All amplitudes are defined for the polarization states one-half along the direction of the decay.

[8] The calculations were performed using SU(6) Clebsch-Gordan coefficients given by J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters 14, 523 (1965). C. L. Cook and G. Murtaza, to be published.

[9] B. W. Lee, Phys. Rev. Letters 12, 83 (1964); M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).

[10] This is analogous to the vanishing of $\rho \rightarrow 2\pi$ in $\mathrm{SU}(6)_S$.

SPIN-2^+ MESON DECAYS IN BROKEN $\tilde{\mathrm{U}}(12)$

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It has been emphasized of late[1] that unitarity of the S matrix is in direct conflict with strict invariance under the noncompact group $\tilde{\mathrm{U}}(12)$. Nevertheless, the unitarity property is immediately restored once the symmetry is intrinsically broken via the equations of motion and specifically through the kinetic-energy terms.[2] Ultimately, of course, the only remaining group of invariance is $\mathrm{U}(3)\otimes\mathfrak{L}_4$ or, for certain kinematical situations, perhaps something larger. However, the prime motivation in formulating a relativistic supermultiplet theory is the assumption that $\tilde{\mathrm{U}}(12)$ invariance provides a reasonable zeroth approximation[3] for the description of physical processes, and that the symmetry-breaking terms[4] act as "small corrections." Many of the resulting $\tilde{\mathrm{U}}(12)$ predictions appear to be receiving experimental confirmation, but there are certain experimental cases which disagree with this hypothesis.

We shall suppose that the disagreements with $\tilde{\mathrm{U}}(12)$ can be simulated by momentum spurion symmetry breakers of the type

$$P_B{}^A = P_{\beta q}{}^{\alpha p} = (\gamma\cdot p)_\beta{}^\alpha \delta_q{}^p,$$

whose effect is appreciable whenever the "zeroth-order" $\tilde{\mathrm{U}}(12)$ predictions are small. For $p\bar{p}$ annihilation into two pseudoscalar mesons [zero by strict $\tilde{\mathrm{U}}(12)$], this hypothesis can produce results in reasonably good agreement with the small observed cross sections.[5] Certain polarization predictions[6] [which vanish by strict $\tilde{\mathrm{U}}(12)$] might again be brought into agreement with experiments on the same basis, and calculations are now in progress to elucidate this question. As another example which has not hitherto been discussed, we cite the case of $B\bar{B} \rightarrow B\bar{B}$ scattering near threshold. A trivial calculation shows that only the elastic amplitudes survive of the possible $\tilde{\mathrm{U}}(12)$ interactions, and this situation is very well supported by the experiments on $p\bar{p}$ interactions. Momentum spurions again offer a possibility of explaining the small $n\bar{n}$ production cross sections.

In this Letter we shall consider in detail the decays of the spin-2^+ meson octet: $2^+ \rightarrow 0^- + 0^-$, $2^+ \rightarrow 0^- + 1^-$. It has already been noted[7] that these processes are forbidden by strict $\tilde{\mathrm{U}}(12)$, and this is approximately the experimental situation in that the associated coupling constants are indeed small when due account is taken of the D-wave nature of the decay. We attempt to show that the assumption of symmetry breaking by kinetic spurions leads to reasonably good comparisons with the known partial decay widths of these mesons. For the sake of definiteness we assign the 2^+ particles to the $\underline{4212}^+$ multiplet[2,7] of $\tilde{\mathrm{U}}(12)$, which multiplet accommodates a nonet of vector mesons. The $K\pi$ and $\rho\pi$ resonances[8] (at 1430 and 1310 MeV) seem to fit nicely here, and then the mass formula implies a mass of 1470 MeV for the isosinglet octet member. That the physical f^0 lies at 1250 MeV may be attributed to mixing with the unitary singlet; from the relative absence of the mode $f^0 \rightarrow K\bar{K}$ we may presume that the f^0 is the analog of the physical ω, viz. $f^0 = (\frac{2}{3})^{1/2} f_{(1)} + (\frac{1}{3})^{1/2} f_{(8)}$, thereby predicting the existence of another isosinglet 2^+ resonance at approximately 1650 MeV.

Now the only possible couplings that lead to the physically observed decays are

$$\mathfrak{L}=G(q-r)_A{}^C\Phi_{[CD]}{}^{[AB]}(p)[\Phi_B{}^E(q)\Phi_E{}^D(r)-\Phi_B{}^E(r)\Phi_E{}^D(q)],$$

$$\mathfrak{L}'=G'(q-r)_A{}^C\Phi_{[CD]}{}^{[AB]}(p)(q-r)_B{}^D\cdot\Phi_E{}^F(q)\Phi_F{}^E(r). \tag{1}$$

Since $\mathfrak{L}'$ describes the decays of just the unitary member and contains two momentum spurions, it will be neglected at present. Following the prescriptions[2] of $\tilde{\mathrm{U}}(12)$, we are led to the amplitude

$$\mathfrak{L}=g_{pp}(q-r)_\kappa(q-r)_\lambda\,\mathrm{Tr}[S_{\kappa\lambda}(p)\{\varphi_5(q),\varphi_5(r)\}]/\mu+g_{pv}\epsilon_{\kappa\lambda\nu\mu}p_\lambda(q-r)_\nu(q-r)_\rho\,\mathrm{Tr}\{S_{\kappa\rho}(p)[\varphi_\mu(q),\varphi_5(r)]\}/\mu M$$

$$-\text{similar term with } q\leftrightarrow r+1^-1^- \text{ term}, \tag{2}$$

where $g_{pp}/g_{vp}=m(M+2\mu)/\mu(M+m+\mu)$ and M, m, and μ stand for mean masses of the 2^+, 1^-, and 0^- nonets. Taking a common mass of 600 MeV for the $\underline{143}^-$ gives $g_{pp}=g_{vp}$.

From (2) we deduce the widths

$$\Gamma_{pp}=8g_{pp}{}^2|\vec{q}_{pp}|^5/3\pi\mu^2M^2,$$

$$\Gamma_{vp}=4g_{vp}{}^2|\vec{q}_{vp}|^5/\pi\mu^2M^2. \tag{3}$$

Note the d coupling of the 0^-0^- mesons and the f coupling of the 0^-1^- mesons, as prescribed by charge-conjugation invariance.

The present experimental situation regarding the observed 2^+ resonances is summarized in Table I. To make comparisons of (3) with this table we use physical masses, following stardard practice. Using the SU(3) Clebsch-Gordan coefficients, we derive the theoretical estimate

$$\Gamma_{A\rho\pi}:\Gamma_{AK\bar{K}}:\Gamma_{A\eta\pi}=4q_{A\rho\pi}{}^5:\tfrac{2}{3}q_{AK\bar{K}}{}^5:\tfrac{4}{9}q_{A\eta\pi}{}^5$$

$$\approx 6:1:2.$$

The absolute ratio $g_{A\rho\pi}/g_{\rho\pi\pi}\approx 0.1$ was obtained earlier.[7] For the other octet member, the $K\pi$ resonance, we predict the following branching ratios:

$$\Gamma_{K\pi K\pi}:\Gamma_{K\pi K\eta}:\Gamma_{K\pi K^*\pi}\approx 25:1:5,$$

and it would be interesting to test these against the data when they become available. The prediction for the total decay widths,

$$\Gamma_{A_2}:\Gamma_{K\pi}\approx\tfrac{3}{4},$$

is supported by the experiments. With the assignment for the f^0 stated previously, the coupling $\mathfrak{L}$ alone gives

$$\Gamma_{f\pi\pi}:\Gamma_{A\rho\pi}\approx 6,$$

which is certainly not satisfied! However, it could be that the contribution from $\mathfrak{L}'$ (which will not affect the previous results) may be very significant, and of course it is then possible to fit the data. We know that $f^0\not\to\omega+\pi$ simply by isospin conservation; the small 4π mode of the f^0 can then be imagined to proceed via a virtual 2ρ intermediate state, but it is difficult to arrive at a reliable numerical estimate from this mechanism.

Table I. Physical properties of 2^+ resonances.

Resonances	Mass (MeV)	Width (MeV)	Modes	
A_2	1310	80	$\rho\pi$	70%
			$K\bar{K}$	10%?
			$\eta\pi$	20%?
$K\pi$	1430	100?	$K\pi, K\rho?, \cdots$	
f^0	1250	100	$\pi\pi$	95%
			4π	5%

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EXPERIMENTAL DETERMINATION OF BRANCHING RATIOS OF VECTOR MESONS INTO LEPTON PAIRS*

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The decay of neutral vector mesons ($V^0 = \rho, \varphi, \omega$) into leptons ($l = \mu$ or e) has been studied in the reaction $\pi^- + p \to n + V^0$ with the subsequent decay $V^0 \to l^+ + l^-$ at the Brookhaven National Laboratory AGS using a system of scintillation counters and spark chambers. Previous experiments[1] have placed upper limits on the decay $V^0 \to l^+ + l^-$. Our measured branching ratios are a factor of 10 lower than the previously measured upper limits for the electron decay modes. The importance of measuring these leptonic decay modes has been recognized for some time.[2]

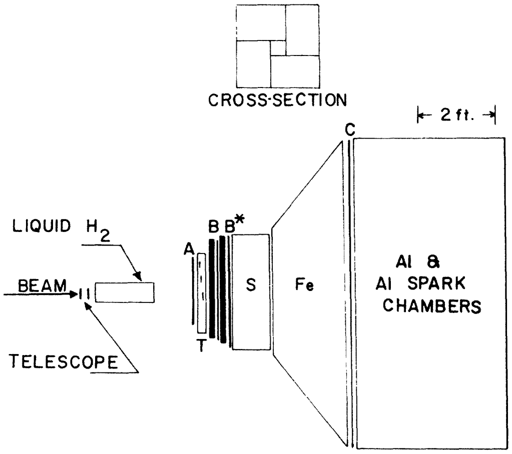

FIG. 1. Experimental arrangement.

Figure 1 shows the experimental arrangement. A pion beam of 3(±1%) BeV/c was incident on a liquid-hydrogen target. The experiment measured the opening angle of a pair of particles electronically identified as leptons. The distribution of opening angles is strongly correlated to the mass of the parent system. A 24-in. ×24-in.×4.5-in. spark chamber (T) registered the tracks of the lepton pair before appreciable material was encountered. All counters were segmented into a quadrant array about the beam axis as shown in the upper part of Fig. 1. Counters B and B^* were each preceded by $\frac{1}{4}$-in. lead sheets, and A, B, and B^* were interrogated for pulse height to identify an electron shower. The relative pulse-height distribution to discriminate against pions, to better than 1 part in 100, was determined by previous calibration over the energy range 0.5 to 2.0 BeV. (Thus, we discriminate against pairs of pions by a factor of more than 10^4.) The chambers marked S contained 8 radiation lengths of lead and scintillators, and the pulse height from these counters was proportional to the total energy of an electron passing through S. The pulses[3] from S were photographed from oscilloscope traces. A trigger required that signals from any two quadrants of A, B, and B^* simultaneously satisfied predetermined pulse-height levels. The remaining portion of the system separated muons from pions. This consisted of 4.4 in-