kas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).

[14]Correspondingly, it should be noted that the strength of the direct coupling of K' to systems such as $(\bar{N}\Lambda)$, etc., may be arbitrarily small (without altering the mixing parameter λ), provided $|\delta m|$ is sufficiently small. To have a rough estimate of the strength of the K' coupling compared to the strong K-meson coupling constant, assume that K is pseudoscalar and has the same charge-conjugation properties as K. Thus if we take the K and K' couplings of the form $(f\bar{N}i\gamma_5\Lambda K + \text{H.c.})$ and $(f'\bar{N}i\gamma_5\Lambda K' + \text{H.c.})$, respectively, g [the strength of (KK') mixing] is expected to be roughly $ff'm^2$, where m is of the order of baryon mass. In that case the ratio of the amplitudes for productions of K' via K pole and direct production of K is given by $ff'm^2/m_K^2 - m_K'^2$. If we equate this to $\lambda \approx (2.3\times10^{-3})^{1/2}$ [Eq. (8)], we obtain $f' \approx 2m_K \delta m m^{-2} \times (1/20f) \approx 5x\times10^{-5}$ (putting $\delta m = x$ MeV, and $f \approx 1$). This is indeed so small that we may completely ignore direct K' production unmediated by the K pole. See references 16 and 17.

[15]Compare with the strength of the $(K\pi)$ vertex as given by, for instance, Y. S. Kim and S. Oneda, Phys. Letters 8, 83 (1964).

[16]It is possible that every usual particle of our world has a primed partner almost exactly identical to it. The primed particles have strong and electromagnetic interactions among themselves exactly similar to those between the unprimed ones. Thus it is possible that the primed and unprimed partners are almost degenerate in mass. However, the primed particles interact only semiweakly, weakly, or superweakly with the unprimed particles and presumably can have weak interactions much weaker than those of ours.

[17]For example, for case (ib) the $\Lambda NK'$ interaction should be of the form $\bar{N}\gamma_\alpha\Lambda\partial_\alpha K' + \text{H.c.}$, leading to a factor proportional to the baryon mass difference in the production matrix element which suppresses the couplings. For case (ii$b$), the coupling could be of the form $\partial_\mu\bar{N}i\gamma_5\sigma_{\mu\nu}\partial_\nu\Lambda K' + \text{H.c}$. Because it involves higher derivatives, it may be an induced interaction rather than a primary one. If we take a point of view that observed particles are the bound states of some fundamental entities, then the coupling will be proportional to the overlap integral of the wave functions of K', N, and Λ. For instance, charge-conjugation property of K' opposite to that of K may give rise to a large difference in their wave functions.

[18]This, of course, can be regarded only as an accident in our model, since, a priori, there is no direct relationship between the $\tilde{K}_1'$ and K_2 lifetimes, except that they should be of the same order of magnitude. It is remarkable, however, that if we use the rough estimate of λ^2 given by Eq. (8), the lifetime of K_1' comes out to be almost equal to that of K_2 (within a factor of 1.5).

[19]The mixing parameter λ for K_2 is expected to be the same as that for K_1, since the $(K_1 - K_2)$ mass difference is negligible or may at most be comparable to the (K^0, K'^0) mass difference.

[20]By "short-lived" K_2, we mean the usual K_2 with lifetime $\approx 5.6\times10^{-8}$ sec.

[21]This is, of course, still subject to the time-honored assumption of an exponential decay law.

[22]This is due to the (K^+, K'^+) and (K^-, K'^-) mixing. K'^+ and K'^- are the isotopic partners of K^0 and $\bar{K}'^0$.

[23]The intensity of the regenerated $\tilde{K}_1$ per incident K_2 of about 500 MeV/c for absorber of 3 cm of iron is nearly 3×10^{-4} (this can be made smaller by decreasing the thickness of the absorber), while the intensity of $\tilde{K}_1'$ relative to that of K_2 is λ^2, and the rate of $\tilde{K}_1' \to 2\pi$ compared to that of $\tilde{K}_1 \to 2\pi$ is also λ^2.

[24]We thank Professor G. A. Snow for drawing our attention to such an experiment.

[25]Since in this case the $K_2 \to 2\pi$ amplitude will interfere with the regenerated $K_1 \to 2\pi$ amplitude.

ALGEBRA OF CURRENT COMPONENTS AND THEIR MOMENTS: AN INTERPRETATION OF SU(6)

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It has been emphasized by Gell-Mann[1,2] that the algebra generated by current components may serve as a useful tool in understanding the structure of the system of hadrons. We assert that the $U(6)\otimes U(6)$ algebra of current components is useful independently of the question as to whether the dynamics of strong interactions is (approximately) invariant under the group $U(6)\otimes U(6)$ or SU(6), in understanding the hadron parameters of electromagnetic and leptonic weak interactions.[3] The purpose of this note is to support this assertion through some illustrative examples and provide a rational explanation of the so-called SU(6) symmetry,[4] which has been much discussed lately.

While the predictions of SU(6) in the regime of low-energy phenomena are startling,[5] the attempts[6] to reconcile the essentially nonrelativistic SU(6) and relativity have met with grave difficulties: The prescription of reference 6 for constructing S-matrix elements does lead to covariant S-matrix elements which possess

SU(6) symmetry in the static limit, but is in violent disagreement with one of the basic notions of quantum theory.[7] A theory invariant under a group which contains SU(3) and Poincaré group in any but the most trivial way will face catastrophic consequences.[8,9] The idea of intrinsically broken U(6) invariance[10] may fare better in this respect, but a convincing demonstration that a theory of this variety can reproduce the nonrelativistic SU(6) results has been frustrated by the lack of a reliable computational scheme. We suggest that many (perhaps all[11]) of the successes of the nonrelativistic SU(6) can be explained in terms of the algebra of currents and the highly convergent nature (in the dispersion-theory sense) of the relevant form factors; as for the dynamics of strong interactions, we assume only the unitary symmetry. We shall not assume that strong interactions are SU(6) invariant.

We shall assume that the integrals of components of vector and axial-vector currents

$$V_\mu^{(\lambda)}(t)=-i\int d^3x\,\mathfrak{V}_\mu^{(\lambda)}(\vec{x},t),$$

$$A_\mu^{(\lambda)}(t)=-i\int d^3x\,\mathfrak{A}_\mu^{(\lambda)}(\vec{x},t),\quad \mu=0,1,2,3, \tag{1}$$

generate the algebra[2] $U(6)\otimes U(6)$ at equal times. Here $V_\mu^{(\lambda)}$ and $A_\mu^{(\lambda)}$ transform like the state $|8,\lambda\rangle$ for $\lambda=1,\cdots,8$, and $|1,0\rangle$ for $\lambda=0$, with the phase convention of deSwart.[12] The commutation relation between two $A_i^{(\lambda)}$, $i=1,\cdots,3$ is

$$[A_i^{(\lambda)},A_j^{(\mu)}]=-i\epsilon_{ijk}\frac{4}{\sqrt{3}}\begin{pmatrix}8&8&1\\\lambda&\mu&0\end{pmatrix}A_k^{(0)}+i\epsilon_{ijk}\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}8&8&8_s\\\lambda&\mu&\nu\end{pmatrix}A_k^{(\nu)}-\delta_{ij}\sqrt{3}\begin{pmatrix}8&8&8_a\\\lambda&\mu&\nu\end{pmatrix}V_0^{(\nu)}. \tag{2}$$

We take the matrix element of Eq. (2) between the $\frac{1}{2}^+$ baryon octet states $|\alpha\rangle$ and $|\beta\rangle$ of zero momenta.[13] We insert the complete set of states between the two operators on the left of Eq. (2), to obtain

$$\sum_{C,\gamma}\langle\alpha|A_i^{(\lambda)}|C_\gamma\rangle\langle C_\gamma|A_j^{(\mu)}|\beta\rangle-\begin{pmatrix}\lambda\leftrightarrow\mu\\i\leftrightarrow j\end{pmatrix}$$
$$=i\epsilon_{ijk}\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}8&8&8_s\\\lambda&\mu&\nu\end{pmatrix}\langle\alpha|A_k^{(\nu)}|\beta\rangle-\delta_{ij}\sqrt{3}\begin{pmatrix}8&8&8_a\\\lambda&\mu&\nu\end{pmatrix}\langle\alpha|V_0^{(\nu)}|\beta\rangle+\cdots, \tag{3}$$

where C stands for the SU(3) dimensionality and other kinematical variables, $\gamma=(I,I_z,Y)$. The omitted term on the right of (3) refers to the uninteresting baryonic charge axial-vector current. The momentum of the state $|C_\gamma\rangle$ is zero, and the spin-parity of this state is $\frac{1}{2}^+$ or $\frac{3}{2}^+$. The summation includes the integration over masses. We assume that this integral is highly convergent, and further, that it is a good approximation to replace the integral by the sum over a few low-lying excitations (particles and resonances).[14] In this approximation, Eq. (3) may be viewed as a self-consistency equation for the Gamow-Teller matrix element for the nucleon and various transition moments. The over-all scale of these constants is determined by

$$\langle\alpha|V_0^{(\nu)}|\beta\rangle=\sqrt{3}\begin{pmatrix}8&8&8_a\\\beta&\nu&\alpha\end{pmatrix}, \tag{4}$$

which follows from the fact that $V_0^{(\nu)}$ are the generators of SU(3).

We look for a solution of Eq. (3) assuming that only the $\frac{1}{2}^+$ octet contributes in the sum over intermediate states. We find that there is none. Next, we include the $\frac{3}{2}^+$ decuplet states in $|C_\gamma\rangle$ and write

$$\langle\alpha|A_k^{(\nu)}|\beta\rangle=\bar{u}(0)\sigma_k u(0)\sum_\xi\begin{pmatrix}8&8&8_\xi\\\beta&\nu&\alpha\end{pmatrix}G_\xi,$$

$$\langle 10_\gamma|A_k^{(\nu)}|\beta\rangle=\bar{u}_k(0)u(0)\begin{pmatrix}8&8&10\\\beta&\nu&\gamma\end{pmatrix}G^*, \tag{5}$$

where $u_\mu(0)$ is the Rarita-Schwinger wave function for a spin-$\frac{3}{2}$ particle at rest. In this case, we find the unique solution

$$G_s=\left(\frac{5}{3}\right)^{1/2},\quad G_a=\frac{2}{\sqrt{3}},\quad G^*=2,$$

or

$$-G_A=\left(\frac{1}{3}\right)^{1/2}\left[G_a+\left(\frac{9}{5}\right)^{1/2}G_s\right]=\frac{5}{3},$$

$$\frac{D}{F}=\left(\frac{9}{5}\right)^{1/2},\quad \frac{G_s}{G_a}=\frac{3}{2}. \tag{6}$$

We have reproduced the results of SU(6).[15]

The reason for this is clearly not because the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet belong to a supermultiplet of the group SU(6) which is an invariance of the Hamiltonian; we did not assume, nor did we make use of, this. It is because the assumption that the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet saturate the sum over intermediate states in (3) amounts to saying that these states form bases of an irreducible representation of the algebra U(6) generated by $V_0^{(\nu)}$ and $A_k^{(\nu)}$.

As a further illustration, we consider the commutation relation between the magnetic-moment operators

$$\mathfrak{M}_i^{(\lambda)}=\tfrac{1}{2}\int d^3x\,\epsilon_{ijk}x_j\mathcal{V}_k^{(\lambda)}(x);$$

$$4\langle\alpha|[\mathfrak{M}_i^{(\lambda)},\mathfrak{M}_j^{(\mu)}]|\beta\rangle$$
$$=\langle\alpha|i\epsilon_{ijk}\left(\frac{5}{3}\right)^{1/2}\begin{pmatrix}8&8&8_S\\ \lambda&\mu&\nu\end{pmatrix}\int d^3x\,x_k[x_m\mathfrak{A}_m^{(\nu)}(x)]$$
$$-\sqrt{3}\begin{pmatrix}8&8&8_a\\ \lambda&\mu&\nu\end{pmatrix}[\tfrac{2}{3}\int d^3x\,x^2\delta_{ij}\mathcal{V}_0^{(\nu)}(x)$$
$$+\tfrac{1}{3}\int d^3x(x^2\delta_{ij}-3x_ix_j)\mathcal{V}_0^{(\nu)}(x)]+\cdots|\beta\rangle. \qquad (7)$$

The omitted term comes from $A_k^{(0)}$, which is of no interest to us. The commutation relation (7) can be deduced in much the same way as is Eq. (2).[16]

We define

$$\langle\alpha|\mathfrak{M}_i^{(\lambda)}|\beta\rangle=\bar{u}(0)\sigma_iu(0)\sum_\xi\begin{pmatrix}8&8&8_\xi\\ \beta&\lambda&\alpha\end{pmatrix}M_\xi,$$
$$\langle 10_\gamma|\mathfrak{M}_i^{(\lambda)}|\beta\rangle=\bar{u}_i(0)u(0)\begin{pmatrix}8&8&10\\ \beta&\lambda&\gamma\end{pmatrix}M^*, \qquad (8)$$

and

$$\langle\alpha|\int d^3x\,x_i[x_j\mathfrak{A}_j^{(\nu)}(x)]|\beta\rangle$$
$$=\bar{u}(0)\sigma_iu(0)\sum_\xi\begin{pmatrix}8&8&8_\xi\\ \beta&\lambda&\alpha\end{pmatrix}A_\xi.$$

The over-all scale of Eq. (7) is set by

$$\langle\alpha|\int d^3x\,x^2\mathcal{V}_0^{(\nu)}(x)|\beta\rangle$$
$$=-\tfrac{1}{2}\sqrt{15}\langle r_n^2\rangle\begin{pmatrix}8&8&8_S\\ \beta&\nu&\alpha\end{pmatrix}$$
$$+\sqrt{3}(\langle r_p^2\rangle-\tfrac{1}{2}\langle r_n^2\rangle)\begin{pmatrix}8&8&8_a\\ \beta&\nu&\alpha\end{pmatrix}, \qquad (9)$$

where $\langle r_p^2\rangle^{1/2}$ and $\langle r_n^2\rangle^{1/2}$ are the rms charge radii of the proton and neutron. The electric quadrupole moment operator

$$Q_{ij}^{(\nu)}=\int d^3x(\tfrac{1}{3}\delta_{ij}x^2-x_ix_j)\mathcal{V}_0^{(\nu)}$$

has no matrix element between the $\frac{1}{2}^+$ baryon states $|\alpha\rangle$ and $|\beta\rangle$.

The structural similarity of Eqs. (3) and (7) allows us to read off the values of the magnetic moments, A_ξ, in terms of the proton charge radius. We see that[17]

$$\langle r_n^2\rangle=0,$$
$$2M_\xi/(\tfrac{2}{3}\langle r_p^2\rangle)^{1/2}=G_\xi=A_\xi/(\tfrac{2}{3}\langle r_p^2\rangle),$$
$$2M^*/(\tfrac{2}{3}\langle r_p^2\rangle)^{1/2}=G^*.$$

In terms of more familiar quantities, we have

$$\mu_p/\mu_n=-\tfrac{3}{2},$$
$$\langle N^*|\mathfrak{M}_i^{(3)}+3^{-1/2}\mathfrak{M}_i^{(8)}|p\rangle=(2/\sqrt{3})\mu_pu_i(0)u(0), \qquad (10)$$

and

$$\mu_p^2=\tfrac{1}{6}\langle r_p^2\rangle, \qquad (11)$$

$$-\frac{1}{G_A}\frac{dG_A}{dq^2}\bigg|_{q^2=0}=-\frac{10}{9G_A}\langle r_p^2\rangle-\frac{1}{8m^2}-\left(\frac{G_p}{G_A}\right)\frac{1}{m\mu}, \qquad (12)$$

where m and μ are masses of the proton and muon, $G_A(q^2)$ is the axial-vector form factor, and G_p is the induced pseudoscalar form factor in μ capture. The relations (10) are the well-known SU(6) results.[18] Equation (11) has been derived also by Dashen and Gell-Mann[19]; it agrees with experiment to within 20%. Equation (12) predicts the slope of the axial-vector form factor: With $G_p\simeq 8G_A$, we obtain $-G_A^{-1}\times dG_A/dq^2<0$, which contradicts our a priori notion. This is due to the rather large G_p/G_A, which comes in with a negative sign. It may be that the rather large value for the nuclear G_p is a reflection of a serious breakdown of the octet symmetry; presumably this is due to the small pion mass.[20]

The lessons we extract from this exercise are the following:

(A) The success of the nonrelativistic SU(6) is not necessarily predicated upon the existence of a group that contains SU(6) as a symmetry of the strong-interaction dynamics. It is sufficient that some low-lying SU(3) multiplets form bases of an irreducible representation of the current algebra. Coleman[9] attributes to Gell-Mann the remark that "a group might be useful for classifying particles even if it has no connection...with the approximate symmetries of the world." We hope to have shown the usefulness of a group which may not be a symmetry of the world.

(B) We may attribute the success of SU(6) to the correctness of the assumptions that (a) the current integrals and the moment operators have the proper commutators as Eqs. (2) and (3); (b) the current integrals and the moment operators turn the $\frac{1}{2}^+$ octet into the $\frac{3}{2}^+$ decuplet and into itself. The algebra of currents is deduced from a rather specific model, under a stringent dynamics-dependent hypothesis.[16] Nonetheless, the successful predictions such as (10) and (11) give some support for the postulated algebra.

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[1]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

[2]R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).

[3]In reference 2, the authors assume that "the total A spin is a good symmetry" of the strong interactions.

[4]F. Gürsey and L. Radicati, Phys. Rev. Letters 13, 299 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964).

[5]See reference 2, footnote 15.

[6]A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 267 (1965); B. Sakita and K. Wali, Phys. Rev. Letters 14, 404 (1965).

[7]M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 509 (1965); R. Blankenbecler, M. Goldberger, K. Johnson, and S. Treiman, Phys. Rev. Letters 14, 518 (1965); J. Cornwall, P. Freund, and K. T. Mahanthappa, Phys. Rev. Letters 14, 515 (1965).

[8]B. Sakita and L. Michel, to be published.

[9]S. Coleman, to be published.

[10]K. Bardakci, J. Cornwall, P. Freund, and B. Lee, Phys. Rev. Letters 13, 698 (1964); 14, 48,264 (1965).

[11]In order to discuss mass splittings, etc., in our scheme, we need consider the commutators of the energy-momentum tensors with the current components. In this way we can generate an infinite algebra. See also M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965).

[12]J. de Swart, Rev. Mod. Phys. 35, 916 (1963). We shall use his notation for Clebsch-Gordan coefficients, etc.

[13]To avoid ambiguities, we must deal with wave packets centered about zero momenta for the states $|\alpha\rangle$ and $|\beta\rangle$ in the manner of F. Ernst, R. Sachs, and K. C. Wali, Phys. Rev. 119, 1105 (1960). We shall understand always that such state vectors are used in our expressions. Details of calculation will be discussed elsewhere.

[14]Note that $\langle\alpha|A_i^{(\lambda)}|C_\gamma\rangle$ refers to the form factor at $q^2=-(m_C-m)^2$, where m_C is the mass of the state C. The assumption that the form factors $\to 0$ as $|q^2|\to\infty$ rapidly is consistent with this assumption.

[15]F. Gürsey, A. Pais, and L. Radicati, Phys. Rev. Letters 13, 299 (1964).

[16]We assume that the equal-time commutation relation of the currents does not contain any terms more singular than the delta function. This is a severe restriction on dynamics.

[17]The uniqueness of this solution can be verified by an elementary calculation.

[18]M. A. B. Bég, B. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

[19]R. F. Dashen and M. Gell-Mann, to be published. At the time of writing, I have not had the benefit of seeing this paper; the fact that they derived an equivalent relation was known to me at an earlier date through Professor M. Goldberger.

[20]In a dispersion treatment of $G_p(q^2)$ [see M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 346 (1958)], $G_p(0)\simeq \mathrm{const}/m_\pi^2$. In an SU(3)-symmetric treatment, we expect to obtain a formula $G_p(0)\simeq\mathrm{const}/\bar{m}^2$, where $\bar{m}^2$ is the mean-squared mass of the pseudoscalar octet. We further expect that the numerator in this expression is insensitive to the SU(3) breaking. If this view is correct, we must multiply the last term on the right of Eq. (12) by $(m_\pi^2/\bar{m}^2)\simeq(7.3)^{-1}$. We then obtain $-G_A^{-1}dG_A/dq^2|_{q^2=0}\simeq 0.18\ (\mathrm{fermi})^2$.