PARTICLE-MIXTURE THEORY AND APPARENT CP VIOLATION IN K-MESON DECAY*

H. Ezawa,† Y. S. Kim, S. Oneda, and J. C. Pati

Department of Physics and Astronomy, University of Maryland, College Park, Maryland
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A recent experiment of Christenson et al.[1] reveals that either (a) CP is violated in weak interactions,[2] or[3] (b) there exists some subtle mechanism[4] consistent with CP conservation that leads to the two-pion-decay mode of the $(K^0,\bar{K}^0)$ system even at "long" times. In this note we wish to propose a simple model of the latter category based on the idea of particle mixtures.

The model introduces a set of partner particles $(K'^0,\bar{K}'^0)$ of $(K^0,\bar{K}^0)$, with the requirement that the primed particles mix with the unprimed ones rather weakly[5] (or semiweakly), depending upon their mass difference, which is required to be much less than (or of the order of) 1 MeV. Furthermore, the primed particles, somehow, are inhibited from having very strong interactions with the usual strongly interacting particles such as $(\bar{N}N)$ systems, etc., so that not only their production cross section is very low, but also their decay to pions and leptons is much slower than that of the $(K^0,\bar{K}^0)$ system.

We will discuss how these possibilities may be realized under a variety of circumstances, corresponding to different parity and charge-conjugation properties of the primed particles. At the moment it is rather obscure which one of these, if any, should correspond to a "natural" cause for their origin. We will presume in the following that the primed particles also correspond to spin-0 fields like the unprimed ones.

Since we are interested in the two-pion-decay mode subject to CP conservation, we will concentrate our attention on the mixing between $CP=+1$ states K_1 and K_1' (a similar discussion will apply to the mixing between $CP=-1$ states K_2 and K_2'). In the model given above, due to the mixing between K_1 and K_1', the mass (and lifetime) eigenstates will be two linear superpositions of K_1 and K_1'. Denoting these eigenstates by $\tilde{K}_1$ and $\tilde{K}_1'$, we may write

$$K_1=(1+\lambda^2)^{-1/2}\{\tilde{K}_1+\lambda\tilde{K}_1'\},$$
$$K_1'=(1+\lambda^2)^{-1/2}\{\tilde{K}_1'-\lambda\tilde{K}_1\}. \tag{1}$$

Thus if the mixing parameter λ is small, a particle produced as K_1 will propagate primarily as $\tilde{K}_1$, which is the usual short-lived K_1, and will, therefore, die out quickly. It will, however, have a small component of $\tilde{K}_1'$, the intensity of which is proportional to λ^2.[6] By the model given above, K_1' will decay mainly through its K_1 component, and hence its decay rate to two pions will be slower by a factor $\simeq\lambda^2$ than that of $\tilde{K}_1$. Thus in this model, the low intensity of K_1 is directly related to its long lifetime (both are given in terms of the single parameter λ), and one could expect to see the low-intensity two-pion-decay mode of $\tilde{K}_1'$ at times much longer than the short lifetime of $\tilde{K}_1$. This could explain the experiment of Christenson et al.[1]

To relate λ to the strength of the mixing interaction g, we have to transform the part of the Lagrangian[7] given by

$$\tfrac{1}{2}(\partial_\mu K_1)^2+\tfrac{1}{2}m_1^2K_1^2+\tfrac{1}{2}(\partial_\mu K_1')^2 + \tfrac{1}{2}m_1'^2K_1'^2+gK_1K_1', \tag{2}$$

to the form

$$\tfrac{1}{2}(\partial_\mu\tilde{K}_1)^2+\tfrac{1}{2}\tilde{m}_1^2\tilde{K}_1^2+\tfrac{1}{2}(\partial_\mu\tilde{K}_1')^2+\tfrac{1}{2}\tilde{m}_1'^2\tilde{K}_1'^2. \tag{3}$$

$\tilde{K}_1$ and $\tilde{K}_1'$ are related to K_1 and K_1' by Eq. (1). It follows that λ satisfies the equation

$$g\lambda^2+(m_1'^2-m_1^2)\lambda-g=0. \tag{4}$$

For small λ, one obtains

$$\lambda\simeq g/(m_1'^2-m_1^2). \tag{5}$$

The masses of $\tilde{K}_1$ and $\tilde{K}_1'$ are given by

$$\tilde{m}_1^2=m_1^2-g\lambda,$$
$$\tilde{m}_1'^2=m_1'^2+g\lambda. \tag{6}$$

We may now proceed to obtain an estimate of λ using the experiment of Christenson et al.[1] Their measured ratio R gives the number of (π^+,π^-) decay events compared to the total number of all charged K_2^0 decays (branching ratio $\approx\frac{3}{4}$) during a time interval[8] (t_1-t_2) which is short compared to the lifetime of K_2^0. Since the branching ratio of $\tilde{K}_1'$ to the (π^+,π^-) mode is nearly

$\frac{2}{3}$,[9] we have[10]

$$R=\frac{(\text{Number of }\tilde{K}_1'\text{ at }t=0)\{\exp(-t_1/\tau_1')-\exp(-t_2/\tau_1')\}(\frac{2}{3})}{(\text{Number of }K_2\text{ at }t=0)\{\exp(-t_1/\tau_2)-\exp(-t_2/\tau_2)\}(\frac{3}{4})}$$
$$\simeq\left(\frac{\text{Number of }\tilde{K}_1'\text{ at }t=0}{\text{Number of }K_2\text{ at }t=0}\right)\frac{\exp(-t_1/\tau_1)}{\exp(-t_1/\tau_2)}\left(\frac{\tau_2}{\tau_1}\right)\left(\frac{\tau_1}{\tau_1'}\right)\left(\frac{8}{9}\right)$$
$$\simeq(8/9)\times 629\lambda^4. \tag{7}$$

We have put λ^2 for the ratio[11] of the number of $\tilde{K}_1'$ to K_2 at $t=0$. The second factor is taken to be unity,[12] the third factor corresponds to the experimental[13] ratio of the lifetimes of the so-called K_2 and K_1 mesons, while the fourth factor is again λ^2, since by the model $\tilde{K}_1'$ decays to two pions mainly through its K_1 component.[14]

Using the value $R\simeq 45/(2.3\times10^4)$ given by reference 1, we obtain from Eq. (7)

$$\lambda^2\simeq 2.3\times10^{-3}. \tag{8}$$

To obtain a numerical value for g, one must know the mass difference between K_1 and K_1'. Since the experiment of Christenson et al.[1] gives this with an upper limit of 1 MeV, we put $\delta m = m_1-m_1'=x$ MeV, where $|x|$, for our purpose, is of the order of, or much less than, unity. Correspondingly, we obtain

$$g^2\simeq 3.24\times10^{-8}x^2 m_K^{\ 4}. \tag{9}$$

We next ask what significance we can attach to this value for the strength of the (KK') mixing interaction. We notice that g can be almost arbitrarily small,[14] depending upon how small $|x|$ is [still keeping λ constant through Eq. (5)]. If $|x|$ is of the order of unity ($|\delta m|\simeq 1$ MeV), g^2 has a strength intermediate between strong and weak interactions; if $|x|\simeq10^{-3}$ ($|\delta m|\simeq 1$ keV), the strength of g^2 is characteristic of weak interactions,[15] while if $|x|\lesssim10^{-6}$ ($|\delta m|\lesssim 1$ eV), the strength of the mixing interaction is much weaker than that of weak interactions. We refer to these three situations as semiweak, weak, and superweak mixing, respectively. At the moment it is hard to say which of these, if any, should correspond to reality. Notice, however, that a priori there does not exist any relationship between the strength of the mixing interaction g and the mass difference δm; i.e., so far as one can say, g need not play a role directly in determining the (K_1,K_1') mass difference. However, Eq. (9) is a consistency condition required for our model to explain the Christenson et al. experiment.[1]

We next ask if the nature of the mixing could partially be related to the quantum numbers of the primed particles. For example, given the spin of the primed particles as zero, they could be either (i) scalar (0^+), or (ii) pseudoscalar (0^-), and they could have either (*a*) normal ($C|K'^0\rangle=|\bar{K}'^0\rangle$), or (*b*) abnormal ($C|K'^0\rangle=-\bar{K}'^0\rangle$) charge-conjugation properties. Under possibility (ii*a*), K' has exactly the same quantum numbers as K. With our usual experience it is then hard to guess[16] why the primed particles behave so differently from the unprimed ones. Under possibilities (i*b*) and (ii*b*), the assignment of abnormal charge-conjugation property restricts the form of K' coupling[17] and may conceivably be related to their semiweak properties. Similarly, if the primed particles happen to be scalar rather than pseudoscalar, the KK' mixing is naturally required to be weak or superweak. We should reserve any further speculation along these lines unless experiments urge their necessity.

Finally, we turn to the important question: How can the mixing effect proposed here be tested independently of the experiment of Christenson et al.? We propose the following possible tests.

(I) Study of the long-lived component of K_1. – First, by studying the number of two-pion decays of the $(K^0,\bar{K}^0)$ system as a function of time at times much larger than the short K_1 lifetime, one can associate a lifetime with the particle decaying to two pions. If this lifetime is found to be different from that of K_2 outside of experimental error, it will definitely establish that K_2 was not the source of the two-pion decays. However, if it is found to be the same[18] as the K_2 lifetime (within experimental error), this experiment cannot distinguish between the possibility of CP violation and the mixing effect proposed here.

(II) Study of the long-lived component of K_2. –

In the model proposed here, not only K_1 but also K_2 will have a low-intensity long-lived component. Its relative intensity will be lower and lifetime will be longer, both by a factor $\simeq\lambda^2$,[19] than those of the "short-lived"[20] K_2. By concentrating on a region which is at least 15-20 times longer than the usual K_2 lifetime, if one sees an intensity of K_2-type decays much higher than the exponential decay law will permit, one will confirm[21] the particle-mixture hypothesis proposed here, since the possibility of CP violation does not play any role in this case. Thus, although this is a harder experiment, it will yield a definitive answer on the idea of mixing presented here.

(III) Study of the long-lived components of $K^{\pm}$.—One will also expect low-intensity long-lived components of $K^{\pm}$.[22] The mixing parameter λ in this case, however, is not directly related to that of K_1 and K_2. This is because, given a strength of the mixing interaction (g), λ^2 is inversely proportional to the square of the mass difference between the mixing partners [Eq. (5)]. Even if the neutral partners are almost degenerate, the charged partners may differ from each other by a few MeV. Thus there is a possibility that the intensity of the long-lived components of $K^{\pm}$ may be too low to be observed. If the mixing of the neutral partners is semiweak ($|\delta m|\simeq 1$ MeV), one can hope to see the long-lived components of $K^{\pm}$ as well. It will therefore be interesting to study the intensity of $K^{\pm}$-type decays at long times.

(IV) Possible interference effects in regeneration experiments.—In a regeneration experiment, we would expect to see two-pion decays not only from the regenerated $\tilde{K}_1$ but also from the long-lived component $\tilde{K}_1'$. Since the two amplitudes can be comparable[23] in magnitude, one should expect to see an interference effect[24] between them, except for the possibility that the mass difference between $\tilde{K}_1$ and $\tilde{K}_1'$ may be so large (1 eV-1 MeV) compared to the inverse of the $\tilde{K}_1$ lifetime as to make the oscillations too rapid to be observable. Thus if the interference effect is observed by studying the time distribution of the two-pion decays beyond the absorber, it can be understood on the basis of the mixing model proposed here and will support the idea of superweak mixing (it is, of course, also consistent with the possibility of CP violation[25]). On the other hand, the absence of a noticeable interference effect will not rule out the model proposed here; it only sets a lower limit on the mass difference between K_1 and K_1'. (It will, however, rule out the explanation of the experiment of Christenson *et al.* on the basis of CP violation.)

To sum up, it seems that Experiment (II) on the study of the long-lived component of K_2 is by far the most definitive of the above four.

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†Address from 1 February 1964: Department of Physics, University of Illinois, Urbana, Illinois.

[1]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

[2]N. Cabbibo, Phys. Letters 12, 137 (1964); R. Sachs, Phys. Rev. Letters 13, 286 (1964); L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964); T. N. Truong, Phys. Rev. Letters 13, 358 (1964); S. L. Glashow, Phys. Rev. Letters 13, 35 (1965).

[3]We will assume that (a) and (b) do not both occur.

[4]J. Bernstein, N. Cabbibo, and T. D. Lee, Phys. Letters 12, 146 (1964); T. D. Lee, to be published; J. S. Bell and J. K. Perring, Phys. Rev. Letters 13, 348 (1964).

[5]We later specify more precisely what we mean by weak and semiweak mixing.

[6]Since λ^2 is small, the long-lived component will be masked by the short-lived one at "small" times.

[7]We adopt the same notation for state vectors as for field operators.

[8]t_1 is measured from the time of production of K^0 and is of the order of K_2 lifetime.

[9]We are disregarding the radiative decay of $\tilde{K}_1'$ to $\tilde{K}_1+2\gamma$. This will be negligible compared to its two-pion-decay mode if $\tilde{K}_1'$ is very close in mass to $\tilde{K}_1$ $\times(|\delta m|<1$ keV). Of course, we could also consider $\tilde{K}_1$ to be heavier than $\tilde{K}_1'$, which is quite consistent with the model, and forbids the above radiative decay. It will allow the slow decay of $\tilde{K}_1$ to $\tilde{K}_1'+2\gamma$, but that will be only a small correction to the "short" lifetime of $\tilde{K}_1$.

[10]We are using the notation τ_1, τ_1', and τ_2 for the lifetimes of $\tilde{K}_1$, $\tilde{K}_1'$, and the usual K_2, respectively. We are also denoting the usual K_2 by K_2, rather than by $\tilde{K}_2$.

[11]This is because the number of K_2 is equal to that of K_1 at $t=0$, and the amount of $\tilde{K}_1'$ in K_1 is given by λ^2. See also reference 14.

[12]This is because both τ_1' and τ_2 are large and are of the same order of magnitude, while t_1 is nearly equal to τ_2. Thus the approximation that $\exp(-t_1/\tau_1')/\exp(-t_1/\tau_2)$ is nearly equal to unity again will not alter the order-of-magnitude estimate of λ^2.

[13]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Bar-

kas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).

[14]Correspondingly, it should be noted that the strength of the direct coupling of K' to systems such as $(\bar{N}\Lambda)$, etc., may be arbitrarily small (without altering the mixing parameter λ), provided $|\delta m|$ is sufficiently small. To have a rough estimate of the strength of the K' coupling compared to the strong K-meson coupling constant, assume that K is pseudoscalar and has the same charge-conjugation properties as K. Thus if we take the K and K' couplings of the form $(f\bar{N}i\gamma_5\Lambda K + \text{H.c.})$ and $(f'\bar{N}i\gamma_5\Lambda K' + \text{H.c.})$, respectively, g [the strength of (KK') mixing] is expected to be roughly $ff'm^2$, where m is of the order of baryon mass. In that case the ratio of the amplitudes for productions of K' via K pole and direct production of K is given by $ff'm^2/m_K^2 - m_K'^2$. If we equate this to $\lambda \approx (2.3\times 10^{-3})^{1/2}$ [Eq. (8)], we obtain $f' \approx 2m_K\delta m m^{-2} \times (1/20f) \approx 5x \times 10^{-5}$ (putting $\delta m = x$ MeV, and $f \approx 1$). This is indeed so small that we may completely ignore direct K' production unmediated by the K pole. See references 16 and 17.

[15]Compare with the strength of the $(K\pi)$ vertex as given by, for instance, Y. S. Kim and S. Oneda, Phys. Letters 8, 83 (1964).

[16]It is possible that every usual particle of our world has a primed partner almost exactly identical to it. The primed particles have strong and electromagnetic interactions among themselves exactly similar to those between the unprimed ones. Thus it is possible that the primed and unprimed partners are almost degenerate in mass. However, the primed particles interact only semiweakly, weakly, or superweakly with the unprimed particles and presumably can have weak interactions much weaker than those of ours.

[17]For example, for case (ib) the $\Lambda NK'$ interaction should be of the form $\bar{N}\gamma_\alpha\Lambda\partial_\alpha K' + \text{H.c.}$, leading to a factor proportional to the baryon mass difference in the production matrix element which suppresses the couplings. For case (ii$b$), the coupling could be of the form $\partial_\mu\bar{N}i\gamma_5\sigma_{\mu\nu}\partial_\nu\Lambda K' + \text{H.c}$. Because it involves higher derivatives, it may be an induced interaction rather than a primary one. If we take a point of view that observed particles are the bound states of some fundamental entities, then the coupling will be proportional to the overlap integral of the wave functions of K', N, and Λ. For instance, charge-conjugation property of K' opposite to that of K may give rise to a large difference in their wave functions.

[18]This, of course, can be regarded only as an accident in our model, since, a priori, there is no direct relationship between the $\tilde{K}_1'$ and K_2 lifetimes, except that they should be of the same order of magnitude. It is remarkable, however, that if we use the rough estimate of λ^2 given by Eq. (8), the lifetime of K_1' comes out to be almost equal to that of K_2 (within a factor of 1.5).

[19]The mixing parameter λ for K_2 is expected to be the same as that for K_1, since the (K_1-K_2) mass difference is negligible or may at most be comparable to the (K^0, K'^0) mass difference.

[20]By "short-lived" K_2, we mean the usual K_2 with lifetime $\approx 5.6\times 10^{-8}$ sec.

[21]This is, of course, still subject to the time-honored assumption of an exponential decay law.

[22]This is due to the (K^+, K'^+) and (K^-, K'^-) mixing. K'^+ and K'^- are the isotopic partners of K^0 and $\bar{K}'^0$.

[23]The intensity of the regenerated $\tilde{K}_1$ per incident K_2 of about 500 MeV/c for absorber of 3 cm of iron is nearly 3×10^{-4} (this can be made smaller by decreasing the thickness of the absorber), while the intensity of $\tilde{K}_1'$ relative to that of K_2 is λ^2, and the rate of $\tilde{K}_1' \to 2\pi$ compared to that of $\tilde{K}_1 \to 2\pi$ is also λ^2.

[24]We thank Professor G. A. Snow for drawing our attention to such an experiment.

[25]Since in this case the $K_2 \to 2\pi$ amplitude will interfere with the regenerated $K_1 \to 2\pi$ amplitude.

ALGEBRA OF CURRENT COMPONENTS AND THEIR MOMENTS: AN INTERPRETATION OF SU(6)

Benjamin W. Lee*

The Institute for Advanced Study, Princeton, New Jersey
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It has been emphasized by Gell-Mann[1,2] that the algebra generated by current components may serve as a useful tool in understanding the structure of the system of hadrons. We assert that the U(6) ⊗ U(6) algebra of current components is useful independently of the question as to whether the dynamics of strong interactions is (approximately) invariant under the group U(6) ⊗ U(6) or SU(6), in understanding the hadron parameters of electromagnetic and leptonic weak interactions.[3] The purpose of this note is to support this assertion through some illustrative examples and provide a rational explanation of the so-called SU(6) symmetry,[4] which has been much discussed lately.

While the predictions of SU(6) in the regime of low-energy phenomena are startling,[5] the attempts[6] to reconcile the essentially nonrelativistic SU(6) and relativity have met with grave difficulties: The prescription of reference 6 for constructing S-matrix elements does lead to covariant S-matrix elements which possess