SPURION THEORY OF BROKEN $U_{\mathcal{L}}(12)$ SYMMETRY*

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Recently, the implications of intrinsically broken $U_{\mathcal{L}}(12)$ symmetry for strong and weak vertices[1,2] and for scattering amplitudes[3] have been discussed in some detail. In this scheme the formal $U_{\mathcal{L}}(12)$ invariance is broken down to $U(3)\otimes SL(2,c)$ by use of the Bargmann-Wigner equations[4] for free particles. Although this prescription gives reasonable results for vertex functions involving particles with small spin values, it leads to obvious contradictions with the unitarity condition, especially if four-particle amplitudes are included.[3,5]

We may want to disregard these basic difficulties and consider the $U_{\mathcal{L}}(12)$ scheme as a first approximation for the amplitudes. However, we find then that this scheme gives results which are too restrictive to be compatible with experimental findings like, for example, the polarization in the reaction $K^- + p \to \Xi^- + K^+$, and similar processes.[3] Also, for weak interactions, the assumption of a nonleptonic interaction transforming like a component of the representation $\underline{143}$ of $SU(12)_{\mathcal{L}}$ does not give good results for parity-conserving decay amplitudes. On the other hand, we find that the inclusion of a spurion like $S\propto\gamma_5\otimes 1$ improves the situation.[2]

It is the purpose of this note to propose a $U_{\mathcal{L}}(12)$ theory where, in lowest order, the formal symmetry is broken not only by the imposition of Bargmann-Wigner equations, but simultaneously by the insertion of a spurion. This spurion transforms like a component of the representation $\underline{143}$ which is an SU(3) singlet. More specifically, we write the spurion in the form

$$S=(\Gamma+\Gamma_5\gamma_5+\Gamma_\mu\gamma_\mu+\Gamma_{5\mu}i\gamma_\mu\gamma_5 + \tfrac{1}{2}\Gamma_{\mu\nu}\sigma_{\mu\nu})\otimes 1. \tag{1}$$

Here the Lorentz-covariant coefficients Γ_5, Γ_μ, etc., are functions of the independent momentum and spin variables appearing in the amplitude into which the spurion S is being inserted. Later we will see that in most cases of interest the number of irreducible terms in Eq. (1) is considerably reduced for amplitudes on the mass shell.

The insertion of the spurion S into the mass term for the baryons and the mesons does not give rise to any splitting, because we find

$$\bar{\Psi}_{ABC}\Psi^{ABC'}S_{C'}{}^{C}\propto\bar{\Psi}_{ABC}\Psi^{ABC}; \tag{2}$$

the same is true for the mesons. Using S in second order, we obtain a separation of the multiplets with different spin, but it may be more reasonable to break SU(3) in this same order by using a spurion S_8, which is given by Eq. (1) with 1 replaced by λ_8. For example, we have then for the baryon mass term,

$$\bar{\Psi}_{ABC}\Psi^{AB'C'}\{a_0\delta_{B'}{}^{B}\delta_{C'}{}^{C}+a_1\delta_{B'}{}^{B}(\lambda_8)_{C'}{}^{C} + a_2S_{B'}{}^{B}S_{C'}{}^{C}+a_3S_{B'}{}^{B}(S_8)_{C'}{}^{C}\}. \tag{3}$$

Here the second-order terms can be written as

$$(\sigma_{\mu\nu})_{B'}{}^{B}(\sigma_{\mu\nu})_{C'}{}^{C} \text{ and } (\sigma_{\mu\nu})_{B'}{}^{B}(\sigma_{\mu\nu}\lambda_8)_{C'}{}^{C},$$

respectively, other forms being reducible. The Ansatz (3) gives mass splittings corresponding to the formula[6]

$$M=M_0+M_1J(J+1)+M_2Y+M_3[I(I+1)-\tfrac{1}{4}Y^2], \tag{4}$$

which is satisfied with great accuracy by the experimental masses.

Let us now consider the effects of the insertion of a single spurion S. At first we have an academic but instructive example: the scattering of singlet mesons by quarks. Here strict, formal $U_{\mathcal{L}}(12)$ invariance gives only one amplitude. With the spurion we obtain

$$F=\bar{\Psi}_A(p')[\delta_{A'}{}^{A}F_1(s,t)+S_{A'}{}^{A}F_2(s,t)] \times\Psi^{A'}(p)\varphi(k')\varphi(k), \tag{5}$$

where the only irreducible spurion terms may be written in the form $S=-i\gamma\cdot(k+k')\otimes 1$. Equation (5) is the most general SU(3)-invariant form of the amplitude. There are no further restrictions and no violations of unitarity.

More interesting is the meson-baryon vertex. As far as the spin-$\frac{1}{2}$ multiplet contained in the $\underline{364}$ tensor Ψ_{ABC} is concerned, we only have the two terms

$$\overline{\Psi}_{ABC}(p')\Psi^{ABC'}(p)\{g_0\Phi_{C'}{}^{C}(q) + g_1 2mP^{-2}(-i\gamma\cdot P)_D{}^C\Phi_{C'}{}^D(q)\}, \quad (6)$$

and all other substitutions of the spurion S are reducible. Here we have used the notation $q=p-p'$ and $P=p+p'$, with $p^2=p'^2=-m^2$ and, on the mass shell, $q^2=-\mu^2$. In terms of SU(3) tensors, we obtain from Eq. (6) the expression

$$g_5\,\mathrm{Tr}[\overline{B}i\gamma_5(MB)_{d+\frac{2}{3}f}] + [(g_E(P\cdot e)/2m]\,\mathrm{Tr}[\overline{B}(VB)_f] + (g_M/4m^2)\,\mathrm{Tr}[\overline{B}i(r\cdot e)(VB)_{d+\frac{2}{3}f}], \quad (7)$$

with

$$g_E \propto (1-q^2/2m\mu)g_0-g_1,$$

$$g_M=(1+q^2/4m^2)g_5 \propto (1+2m/\mu)g_0-(2m/\mu)g_1,$$

and

$$r_\mu=\epsilon_{\mu\nu\rho\sigma}P_\nu q_\rho\gamma_\sigma\gamma_5. \quad (8)$$

In Eq. (8) we should write $q^2=-\mu^2$ for the vertex on the mass shell. We see from Eq. (7) that the favorable d/f ratios of theories with broken SU(6) symmetry are preserved in our scheme.

Equations (7) and (8) may be used in order to obtain expressions for the electromagnetic form factors within the framework of a vector-meson dominance model. For this purpose we replace the coefficients of the vector-meson terms in Eq. (7) by

$$G_E(q^2)(1+q^2/4m^2)^{-1}=(1-q^2/2m\mu)G_0(q^2)-G_1(q^2),$$

$$G_M(q^2)(1+q^2/4m^2)^{-1} = (1+2m/\mu)G_0(q^2)-(2m/\mu)G_1(q^2), \quad (9)$$

where $G_{0,1}(q^2) \propto (q^2+\mu^2)^{-1}$. We find that the well-known SU(6) prediction[7] $\mu_n/\mu_p=-\frac{2}{3}$ remains valid in our scheme, because we have the same d/f ratio. Furthermore, we see from Eqs. (9) and (7) that the electric form factor of the neutron vanishes for all values of q^2,[8] and that $G_{E,M}(q^2=-4m^2)=0$. The latter result implies that annihilations at rest like $p+\bar{p}\to e^+ + e^-$ are forbidden in our approximation.[9] For the magnetic moment of the proton we do not have any more the relation $\mu_p=1+2m/\mu$, which follows only with formal $U_{\mathfrak{L}}(12)$ invariance.[1] Instead, we have $G_0(0)-G_1(0)=1$, and hence[10]

$$\mu_p=G_0(0)+2m/\mu. \quad (10)$$

Finally, we consider the amplitudes for the scattering of pseudoscalar mesons by baryons.[11] With strict formal $U_{\mathfrak{L}}(12)$ invariance there are four independent complex amplitudes $f_1(s,t),\cdots,f_4(s,t)$.[3] These are to be compared with the 14 amplitudes which one obtains on the basis of Lorentz invariance and SU(3) symmetry alone. In our spurion theory, we obtain at most seven new terms in addition to the four mentioned above. Four of these new amplitudes result from the contraction of the spurion indices with those of the baryon tensors, and the remaining three involve also contractions with indices of meson tensors. There are many reducibilities, and from Lorentz invariance and positive energy projections, we find that it is sufficient to consider the term

$$\gamma\cdot(k+k')\otimes 1$$

in the expansion (1) of the spurion.

The amplitudes for the individual meson-baryon reactions can be written in the conventional form

$$\bar{u}(p')[A(s,t)-i\gamma\cdot\tfrac{1}{2}(k+k')B(s,t)]u(p). \quad (11)$$

For many of these amplitudes the $U_{\mathfrak{L}}(12)$-symmetry scheme gives rise to $\mathrm{Im}AB^*=0$, or even to $B(s,t)\equiv 0$, which results in the absence of polarization effects. With the inclusion of the spurion S in first order, we find that these restrictions are relaxed, and that, to some extent, it becomes possible for the amplitudes to comply with the unitarity condition, at least as far as two-particle meson-baryon states are concerned.[12] Also, polarization effects are now possible for reactions like $K^-+p\to\Xi^-+K^+$.

On the other hand, in the special case of forward scattering, we find that the insertion of the spurion does not generate any new amplitudes. We have the same number of terms as in the case of $U_{\mathcal{L}}(12)$ symmetry, and, consequently, the Johnson-Treiman relations[13]

$$F(\pi^+p)-F(\pi^-p)$$
$$=F(K^+n)-F(K^-n)=\tfrac{1}{2}[F(K^+p)-F(K^-p)]$$

between the elastic forward-scattering amplitudes are preserved in our theory.

We have proposed in this note a $U_{\mathcal{L}}(12)$ scheme where the intrinsic symmetry breaking due to the equations of motion is supplemented by a spurion which transforms like a component of the representation $\underline{143}$ of $SU(12)_{\mathcal{L}}$. The spurion is an SU(3) singlet, and it preserves the Lorentz invariance of the amplitudes. We obtain a relativistic formalism which reproduces the successes of models with SU(6) covariance, but which is less restrictive than the intrinsically broken $U_{\mathcal{L}}(12)$-symmetry scheme and avoids difficulties encountered in this scheme.

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[9]Y. Hara, to be published, discusses this point in the limit of $U_{\mathcal{L}}(12)$ symmetry.

[10]After this work had been completed, we received a preprint by M. A. Bég and A. Pais (see reference 5) where a generalization of the $U_{\mathcal{L}}(12)$-invariant vertex is considered which, as far as this vertex is concerned, gives a result similar to that of our spurion theory.

[11]In the following we consider only the octet of baryons.

[12]The details of these considerations, as well as the effects of the insertion of the spurion into amplitudes with a more complicated spin structure, will be discussed elsewhere.

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INTERPRETATION OF THE BASIC $\underline{20}$ IN $SU(6)/Z(3)$

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As is known, the eightfold way,[1,2] which emphasizes the adjoint representation $\underline{8}$ of SU(3), may be restated: The physical group is the adjoint group, i.e., the factor group $SU(3)/Z(3)$, where the center $Z(3)$ consists of elements commuting with all others in SU(3).[3-5] The two groups have the same local properties (Lie algebra or F-spin commutation relations,[1] but differ globally.[6] The adjoint group is selected because all its irreducible representations (IR's) of low dimension are physical, whereas many of SU(3) are not. In extending the SU(3) algebra to SU(6),[7,8] one may ask: Is the associated physical group SU(6) or some factor group, $SU(6)/Z(k)$, where $Z(k)$ ($k=2,3,6$) is a subgroup of the center, $Z(6)$? Two candidates are favored, the adjoint group $SU(6)/Z(6)$, and another, $SU(6)/Z(3)$. The first appears to be a natural generalization of $SU(3)/Z(3)$. The second is expected because the physical group must contain the direct product $SU(2)\otimes[SU(3)/Z(3)]$ as a subgroup. [In the static limit the J-spin group is SU(2), not $SU(2)/Z(2)$.]

In this Letter we (1) show that one must pick $SU(6)/Z(3)$ with IR's $\underline{1}, \underline{20}, \underline{35}, \underline{56}, \underline{56}^*, \underline{70}, \underline{70}^*$, $\cdots$ (the adjoint group does not have the representation $\underline{56}$ which contains the nucleon), and (2) speculate on the apparent absence of particles to fill the $\underline{20}$. We tentatively postulate the existence of $\underline{20}$ fundamental states, which