duce the transformation law

$$\xi_{\dot{\alpha}} \to \eta^{\alpha}, \quad \eta^{\alpha} \to \xi_{\dot{\alpha}},$$

while from $\psi^c \to -\gamma_4\psi^c$ we get

$$\eta_{\dot{\alpha}} \to -\xi^{\alpha}, \quad \xi^{\alpha} \to -\eta_{\dot{\alpha}}.$$

Therefore, if one allows both $\psi\bar{\psi}$ and $\psi^c\bar{\psi}$ to describe the mesons, then in terms of two-component spinors one must allow for an ambiguity of sign in passing from a second-rank spinor, say, $M^{\alpha\beta}$, to its parity conjugate $M_{\dot{\alpha}\beta}$. This is just the ambiguity exploited by Charap and Matthews to obtain their two solutions.[13]

[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964).

[2]R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964). K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 13, 698 (1964); 14, 48 (1965). S. Okubo and R. E. Marshak, Phys. Rev. Letters 13, 818 (1964); 14, 156 (1965). W. Rühl, Phys. Letters 13, 349 (1964); J. M. Charap and P. T. Matthews, to be published.

[3]See, for example, Feynman, Gell-Mann, and Zweig, reference 2.

[4]A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965). See also B. Sakita and K. C. Wali, to be published.

[5]W. Rühl, to be published.

[6]V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. U. S. 34, 211 (1946).

[7]See, for example, D. Lurié and A. J. Macfarlane, Phys. Rev. 136, B816 (1964). If one drops this requirement, then both parities are possible for the mesons. See Charap and Matthews, reference 2, who find two solutions to the Bargmann-Wigner equations for the case of $\tilde{U}(6)\otimes\tilde{U}(6)$ symmetry corresponding to either positive or negative parity for the meson 35-plet.

[8]There is no difficulty in passing from this notation to the two-component spinor notation used by Charap and Matthews. See, for instance, H. Umezawa, Quantum Field Theory (North-Holland Publishing Company, Amsterdam, 1956), p. 51 ff. We follow Umezawa's notation and metric in this paper.

[9]Here we make use of the fact that, for $\tilde{U}(2)\otimes\tilde{U}(2)$, there exists a matrix C with the property that $\psi^c = C(\bar{\psi})^T$ transforms as ψ under the group. $\bar{\psi}$ and $\bar{\psi}^c$ are defined by $\bar{\psi} = \psi^\dagger\gamma_4$, $\bar{\psi}^c = (\psi^c)^\dagger\gamma_4$. The matrix C has the properties $C^{-1}\gamma_\mu C = -\gamma_\mu{}^T$, $C^\dagger = C^{-1}$, $C^T = -C$, and, in the special representation for which

$$\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix},$$

has the form

$$\begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}.$$

[10]We note in passing that if the parity operation is defined as $\psi \to i\gamma_4\psi$ [C. N. Yang and J. Tiomno, Phys. Rev. 79, 498 (1950)], then $\psi^c \to i\gamma_4\psi^c$ and $\Phi^{(i)} \to \gamma_4\Phi^{(i)}\gamma_4$ for all i from 1 to 4.

[11]Note that Eq. (3) is just the decomposition of $[(2,1)\oplus(1,2)]\otimes[(2^*,1)\oplus(1,2^*)]$ into irreducible representations of $\tilde{U}(2)\otimes\tilde{U}(2)$ in the language of four-component spinors.

[12]The pseudovector and tensor fields are thereby "eliminated" by being set equal to $im^{-1}\partial_\mu\varphi_5$ and $im^{-1}\times(\partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu)$, respectively. This is analogous to the case of the Dirac equation itself where ξ and η are linked by the equations of motion. See W. Rühl, reference 5.

[13]See Charap and Matthews, reference 2, Eqs. (4.4) and (4.5).

SU(6) THEORY AND THE ANTIPROTON-PROTON ANNIHILATION AT REST INTO TWO MESONS*

Freeman J. Dyson† and Nguyen-huu Xuong

Department of Physics, University of California, San Diego, La Jolla, California
(Received 29 March 1965)

Most of the success of SU(6) theory[1] has been confined to static questions, such as the identification of multiplets, mass formulas, electromagnetic properties, etc. The applicability of SU(6) symmetry notions in the relativistic domain is still doubtful.[2] However, Johnson and Treiman[3] have successfully applied SU(6) notions to high-energy forward elastic scattering of mesons and baryons. In a similar spirit, we want to study the rates of the antiproton-proton annihilation at rest into two mesons:

$$\bar{p} + p \to M + M. \tag{1}$$

It has been shown experimentally that the annihilation of antiprotons and protons at rest proceeds predominantly from s capture.[4] Because of parity conservation, the two outgoing mesons must be in a p state. The proton and antiproton belong, respectively, to the $\underline{56}$ and $\underline{56}^*$ multiplets, $B^{\alpha\beta\gamma}$ and $\overline{B}_{\alpha\beta\gamma}$, whose product gives

$$\underline{56}\times\underline{56}^*=\underline{1}\oplus\underline{35}\oplus\underline{405}\oplus\underline{2695}.$$

The mesons belong to the $\underline{35}$ multiplet $[M=2^{-1/2}\times(P+\sigma\cdot V)_\beta{}^\alpha$, where P represents the pseudoscalar and V the vector mesons]. The operator ($\sigma\cdot$grad) which generates P-wave mesons can also be represented as a $\underline{35}$ multiplet $[Q=(q\cdot\sigma)_\beta{}^\alpha]$. The right-hand side of Eq. (1) can be written as a product $\{[\underline{35}\times\underline{35}]$ symmetric$\times\underline{35}\}$, which contains one singlet, five $\underline{35}$ multiplets, three $\underline{405}$ multiplets, and one $\underline{2695}$ multiplet. At first sight, then, one would need 10 parameters to express the amplitudes of Reaction (1). Fortunately, the use of charge-conjugation invariance reduces these parameters to only two.

Let us consider the operator R defined by

$$R=C\exp(-i\pi J_y),$$

where C is the charge-conjugation operator, and $\exp(-i\pi J_y)$ represents a rotation of 180 degrees about the y direction of spin space. This operator leaves invariant the two $\overline{p}p$ states with $J=0,1$ and $J_z=0$, and transforms M into its transpose M^{T}. The matrix Q has the property

$$\exp(\tfrac{1}{2}i\pi\sigma_y)Q\exp(-\tfrac{1}{2}\pi\sigma_y)=-Q^{\mathrm{T}}.$$

Therefore, the amplitude of reaction (1) must obey the relation

$$A=-A^{\mathrm{T}}. \qquad (2)$$

Equation (2) restricts the amplitude to be of the form

$$A=\tfrac{1}{2}g_1B^{\alpha\beta\gamma}\overline{B}_{\alpha\beta\delta}[M_\epsilon{}^\delta M_\varphi{}^\epsilon Q_\gamma{}^\varphi-Q_\epsilon{}^\delta M_\varphi{}^\epsilon M_\gamma{}^\varphi]$$

$$+\tfrac{1}{2}g_2B^{\alpha\beta\gamma}\overline{B}_{\alpha\delta\epsilon}M_\beta{}^\delta(M_\varphi{}^\epsilon Q_\gamma{}^\varphi-Q_\varphi{}^\epsilon M_\gamma{}^\varphi), \qquad (3)$$

where g_1 and g_2 are parameters. We can write (3) as

$$A=g_1B^{\alpha\beta\gamma}\overline{B}_{\alpha\beta\delta}[MN-NM]_\gamma{}^\delta$$

$$+g_2B^{\alpha\beta\gamma}\overline{B}_{\alpha\delta\epsilon}M_\beta{}^\delta O_\gamma{}^\epsilon, \qquad (4)$$

with

$$N=\tfrac{1}{2}(MQ+QM)=2^{-1/2}(q\cdot\sigma P+q\cdot V),$$

and

$$O=\tfrac{1}{2}(MQ-QM)=2^{-1/2}i\sigma\cdot(q\times V).$$

Many reactions depend only on one parameter g_1. The ratio of the rates of these reactions can, therefore, be predicted.[5] Let us define, for example, $R(\pi^+\pi^-)$ to be the rate of the reaction $\overline{p}+p\to\pi^++\pi^-$, and $R[K_1(K_2\pi)^*]$ to be the rate of the reaction $\overline{p}+p\to K_1+K^{*0}\to K_1+K_2+\pi^0$. We get

$$R(\pi^+\pi^-)/R(K^+K^-)=\tfrac{5}{2}, \qquad (5)$$

$$R(K^+K^-)/R(K^0\overline{K}^0)=16/1, \qquad (6)$$

$$R[K_1(K_1\pi^0)^*]:R[K_1(K_2\pi^0)^*]:R[K_1(K^\mp\pi^\pm)^*]:$$

$$R[K^\mp(K_1{}^0\pi^\pm)^*]:R(K^+K^-)=18:1:36:180:187. \qquad (7)$$

In Table I, we present some experimental rates of $\overline{p}p$ annihilation at rest into two mesons. Relation (5) is more or less verified, but Relations (6) and (7) are strongly contradicted by experiment. The reactions $\overline{p}+p\to\rho+\pi$ depend on both g_1 and g_2. We have

$$R(\rho^+\pi^-):R(\rho^-\pi^+):R(\rho^0\pi^0):R(\pi^+\pi^-)$$

$$=4(1+\lambda):4(1+\lambda):4\lambda:10, \qquad (8)$$

with

$$\lambda=[6+(g_2/g_1)]^2/144.$$

Relation (8) would agree with experiment if

$$(g_2/g_1)^2\sim450,$$

but then

$$R(\rho^0\rho^0):R(\rho^0\omega^0):R(\pi^+\pi^-)=42:7:1, \qquad (9)$$

which disagrees with experiment.

<u>We conclude that a simple calculation, using nonrelativistic SU(6) theory for the rates of antiproton-proton annihilation at rest into two mesons, gives results which disagree strongly with experiment</u>. One cannot attribute this disagreement to the mass differences of the mesons because Relations (6) and (7) are between reactions of particles with similar masses. There are three possible sources of disagreement:

Table I. Measured rates of $\bar{p}p$ annihilation at rest into two mesons.

Reaction	Rate in 10^{-3} of all annihilation	Reference number
$\bar{p}+p\rightarrow\pi^{+}+\pi^{-}$	3.95 ± 0.38	a
$K^{+}+K^{-}$	1.31 ± 0.38	a
$K^{0}+\bar{K}^{0}$	0.56 ± 0.08	a
$K_1^{0}+K^{0*}\rightarrow K_1^{0}+(K_1+\pi^{0})^{*}$	~0	b
$K_1^{0}+K^{0*}\rightarrow K_1^{0}+(K_2+\pi^{0})^{*}$	0.20 ± 0.05	b
$K_1^{0}+K^{0*}\rightarrow K_1^{0}+(K^{\pm}+\pi^{\mp})^{*}$	0.47 ± 0.07	b
$K^{\mp}+K^{\pm*}\rightarrow K^{\mp}+(K_1^{0}+\pi^{\pm})^{*}$	0.31 ± 0.06	b
$\rho^{+}+\pi^{-}$	8 ± 3	c
$\rho^{-}+\pi^{+}$	9 ± 3	c
$\rho^{0}+\pi^{0}$	9 ± 3	c
$\rho^{0}+\rho^{0}$	~0	d
$\rho^{0}+\omega^{0}$	6 ± 3	d

[a]See reference 4.

[b]N. Barrash, P. Franzini, J. Steinberger, T. H. Tan, R. Plano, and P. Yager, to be published.

[c]G. B. Chadwick, W. T. Davies, M. Derrick, C. J. B. Hawkins, J. H. Mulvey, D. Radojicic, C. A. Wilkinson, M. Cresti, S. Limentani, and R. Santangelo, Phys. Rev. Letters 10, 62 (1963).

[d]M. Cresti, A. Grigoletto, S. Limentani, A. Loria, L. Peruzzo, R. Santangelo, G. B. Chadwick, W. T. Davies, M. Derrick, C. J. B. Hawkins, P. M. D. Gray, J. H. Mulvey, P. B. Jones, D. Radojicic, and C. A. Wilkinson, Proceedings of the Sienna International Conference on Elementary Particles (Società Italiana di Fisica, Bologna, Italy, 1963), Vol. I, p. 263.

(1) The rates of $\bar{p}p$ annihilation into three or more mesons are much bigger and could influence by competition the ratio between the rates of annihilation into two mesons.

(2) Mesons in motion could have different SU(6) transformation properties from mesons at rest; i.e., one would need a relativistic SU(6) theory.

(3) The annihilation goes via a process which violates SU(6). Our calculation can be applied to $\bar{p}n$ reactions by rotating the $I=1$, $I_3=0$ $\bar{p}p$ state in isotopic spin space. In this way we deduce[6]

$$R(\bar{p}+n\rightarrow\varphi+\pi^{-})=0.$$

This reaction has been observed by Barnes et al.[7]; they find $R(\bar{p}+n\rightarrow\varphi+\pi^{-})=(0.58\pm0.18)\times10^{-3}$. This violation cannot be explained by the influence of the rates of $\bar{p}n$ annihilation into three or more mesons.

We want to thank Professor J. Steinberger for letting us know the results of his group before publication. One of us (F. J. D.) would like to thank the Physics Department of the University of California, San Diego, for its hospitality. The other (N.-h. X.) would like to thank Professor O. Piccioni for his advice and support.

*Work done under the auspices of the U. S. Atomic Energy Commission.

†On leave of absence from the Institute for Advanced Study, Princeton, New Jersey.

[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964).

[2]See, for example, R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters 14, 518 (1965).

[3]K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965); R. Good and N.-h. Xuong, Phys. Rev. Letters 14, 191 (1965).

[4]R. Armenteros, L. Montanet, D. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, J. Ballam, C. Ghesquière, B. Gregory, D. Rahm, P. Rivet, and F. Solmitz, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 351.

[5]The mass differences of the mesons are more or less taken into account by using the real masses and momenta of the mesons in every reaction.

[6]This prediction can be easily understood, if we look at the decomposition of SU(6) into SU(4)⊗ U(2). Then φ would have strange-spin 1, while π and the $\bar{p}n$ state would have strange-spin zero. Conservation of strange spin [i.e., charge independence in the U(2) symmetry] would then forbid the reaction $\bar{p}+n\rightarrow\varphi+\pi^{-}$. See, for example, H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964).

[7]V. E. Barnes, K. W. Lai, P. Anninos, L. Gray,

P. Hagerty, E. Harth, T. Kalogeropoulos, S. Zenone, V. Dore, G. Moneti, and V. Valente, Proceedings of the International Conference on High-Energy Physics, Dubna, 1964 (to be published).

MESON SYMMETRY*

Richard C. Arnold

Department of Physics, University of California, Los Angeles, California

(Received 25 March 1965)

In this Letter we present evidence for a broen chiral symmetry in the mass spectrum of mesonic states, and an interpretation of all well-established mesons in terms of a single family of Regge trajectories associated with the representation $(\underline{3},\underline{3}^*)\oplus(\underline{3}^*,\underline{3})$ of the group $SU(3)_L \otimes SU(3)_R$. The mesons are interpreted as bound states, primarily of baryon-antibaryon systems.

In Fig. 1 are represented the mesonic states known to date,[1] on a plot of angular momentum J vs $(\text{mass})^2 = t$.[2] According to theoretical ideas concerning bound states, we associated a Regge trajectory with each particle and resonance. Some of the high-ranking poles have apparently been observed in the region $t<0$ through high-energy crossed-channel reactions such as elastic scattering (P, ω, P'),[3] charge exchange (ρ, R),[4] and associated production (K^*). These, with Q [deduced from SU(3) symmetry or bootstrap dynamics[5]] and φ (a trajectory must exist associated with this particle although nothing is known about its intercept), are also indicated near $t=0$ in Fig. 1.

In general, as exchange potentials are intrinsic in crossing-symmetric relativistic problems, physical states will recur at intervals

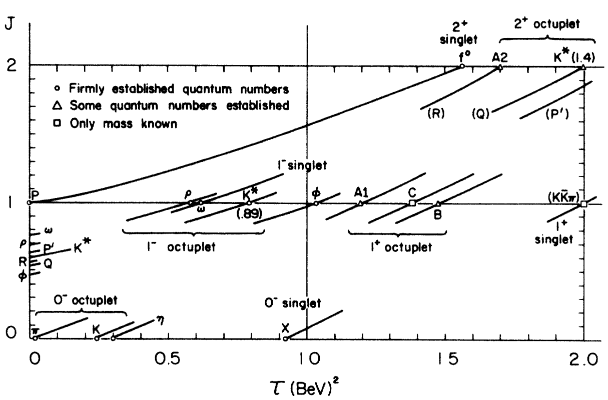

FIG. 1. Known meson states and $B=0$ Regge trajectories with suggested SU(3) classification.

$\Delta J = 2$ along a given trajectory. No two physical mesons are observed which would be expected to lie on the same trajectory, assuming the slopes are all less than unity (in units of BeV^{-2}), as suggested both by elastic scattering analyses[3] and theoretical estimates.[6] However, there is now evidence[7,8] that A_2 and $K^*(1.4\text{ BeV})$ have $J^P = 2^+$, which is suggestive of the possibility that they may be physical states lying on the (R, Q) trajectories roughly paralleling those of the known vector mesons.

Considering $B\bar{B}$ bound states in a bootstrap picture, in which the binding potentials are principally due to meson exchanges,[9] one would expect the exchange part of the effective potential (arising from baryon-number two states) to be weak compared to the direct terms. This leads to a picture in which the trajectories of the vector octet + singlet $[\rho, K^*(0.890),\ \varphi,\ \omega]$ are almost degenerate with a set of trajectories of opposite signature. This second set would be expected to have opposite G parity, when eigenstates of G are considered, since at least in the $\bar{N}N$ system $G = (-1)^{s+I+l}$, and the trajectories may be characterized by definite I, s, and parity.[10] We therefore associate the vector octet trajectories with another octet which is to appear at $J^P = 2^+$. Presumably this is $A_2(I^G = 1^-)$, $K^*(1.4)$ with $I=\frac{1}{2}$, and a third yet-to-be-discovered isosinglet $(2^+, 0^+)$ meson $f^{0\prime}$ with mass around 1.5 BeV, dynamical companion of the $(1^-, 0^-)$ φ meson.

The SU(3) singlet[11] Pomeranchuk trajectory, including f^0, will have as its dynamical companion the singlet ω trajectory. Both singlet and octet trajectory pairs have 1^-, 2^+, etc., states as physical possibilities, but not 1^+, 2^-, etc. We will use the terminology positive-parity trajectory for the former case, and negative for the latter. The approximate dynamical degeneracy of the two sets of trajectories with opposite G parity and signature will be termed exchange degeneracy.