HEAVY-PROJECTILE INELASTIC SCATTERING ABOVE THE COULOMB BARRIER

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At energies above the Coulomb barrier, inelastic scattering can be induced by either nuclear or Coulomb forces. It is the purpose of this Letter to show that closed-form expressions, having the attractive feature of being easily applied without the use of computers and being applicable over a wide range of angles, can be obtained for the differential cross sections for both the direct nuclear and Coulomb excitation processes. These expressions will then permit simple, quick identification, from the angular distributions obtained, of the nature of the excitation process.

The Coulomb process has been studied by Alder et al.[1] for energies below the Coulomb barrier. The nuclear process has been treated by the distorted-wave Born approximation for light projectiles.[2] Because of strong absorption the formulas obtained for the two processes are not directly applicable to heavy-ion scattering at energies above the Coulomb barrier. Here it is shown, however, that the Born approximation, if modified to take absorption into account according to the WKB method,[3] can be used to describe either process.

The characteristic feature of the experimental data is a large peak in the angular distribution at the Rutherford angle for a grazing collision. The position and width of the peak are functions of the bombarding energy. The position of the peak as a function of energy is given sufficiently well by the zero-range diffraction model,[4] so that our main task will be concerned with the width and its energy dependence.

Nuclear Excitations.—We consider for simplicity processes in which only the target or the projectile is left in an excited state. The resulting formulas will hold equally well if the excited state is bound or in the continuum. Thus we must restrict ourselves to breakup reactions in which the disintegration energy is much less than the bombarding energy and where the ratio of the masses of the fragments is equal to the ratio of their charges. In this case the center of mass of the fragments will nearly follow the classical Rutherford orbit, and the WKB method will be applicable.

Let $\chi_i(\xi), J_i$ be the initial wave function and spin for the particle undergoing excitation, and $\chi_f(\xi), J_f$ be the final wave function and spin. (ξ represents the internal coordinates). The interaction in the exit channel is taken to be a scalar function of the space coordinates. In the adiabatic approximation[5]

$$V(\vec{r}, \xi) = V(\vec{r}, \vec{R}), \tag{1}$$

where $\vec{R}$ describes the nuclear surface:

$$\vec{R}(\theta, \varphi) = R_0 + \sum \xi_{l,m} Y_{l,m}{}^*(\theta, \varphi). \tag{2}$$

The $\xi_{l,m}$ are operators for the collective motion and are the deformation parameters for the l, m multipoles. To first order in the deformation,

$$V(\vec{r}, \xi) \approx V(\vec{r}, R_0) + \frac{\partial V}{\partial R_0} \sum \xi_{l,m} Y_{l,m}{}^*(\theta, \varphi), \tag{3}$$

where $V(\vec{r}, R_0) \equiv U(\vec{r})$ is taken to be the optical potential for the relative motion of the colliding particles, while the remaining part of $V(\vec{r}, \xi)$ is responsible for the excitation.

When strong absorption is present the transition amplitude for a collective excitation of definite multipole order L may be well approximated by[6]

$$f_L{}^M(\theta) \approx i(4\pi)^{1/2} \langle \chi_f \| \xi_L \| \chi_i \rangle Y_{LM}(\pi/2, 0) \sum (2l+1)^{1/2} \times \exp(2i\bar{\sigma}_l) \frac{\partial \bar{\eta}_l}{\partial l} Y_{l,M}(\theta, 0). \tag{4}$$

Here σ_l is the Coulomb phase shift and η_l the nuclear reflection coefficient for the elastically scattered lth partial wave. (The bar indicates the average over the incident and exit channels.) We now introduce some further approximations:

(1) The reflection coefficient is assumed to have the Woods-Saxon l dependence:

$$\bar{\eta}_l = \{1 + \exp[(L_0 - l)/\delta]\}^{-1}, \tag{5}$$

where

$$L_0 = [\bar{k}R(\bar{k}R - 2\bar{n})]^{-1/2},$$

$$\delta = \bar{k}\bar{d}\left[1 - \left(\frac{\bar{n}}{\bar{k}R}\right)\right]\left[1 - \frac{2\bar{n}}{\bar{k}R}\right]^{-1/2},$$

$\bar{n}$ is the Coulomb parameter, $\bar{d}$ the diffuseness, R the sum of the radii of the colliding particles, and k the c.m. momentum. Frahn and Venter have demonstrated[7] that this form reproduces almost perfectly the experimental data on elastic scattering of nuclear particles at medium and high energies.

(2) For $l \gg M$ and for angles satisfying

$$(4l)^{-1} \ll \theta \ll \pi - (4l)^{-1}, \tag{6}$$

the spherical harmonics may be approximated by the leading term in their asymptotic expansion for large l:

$$Y_{l,M}(\theta,0) \approx \frac{(2\pi)^{-1}}{(\sin\theta)^{1/2}}\Big[\exp\{i[(l+\tfrac{1}{2})\theta+(2M-1)\pi/4]\} + \exp\{-i[(l+\tfrac{1}{2})\theta+(2M-1)\pi/4]\}\Big]. \tag{7}$$

(3) For large $|l+i\eta|$,[8]

$$2d\sigma/dl \approx \varphi \equiv 2\arctan(\bar{n}/l). \tag{8}$$

Here φ is the classical scattering angle for a particle of angular momentum $l\hbar$ in a pure Coulomb field. Since $\partial\bar{\eta}_l/\partial l$ is sharply peaked at L_0, which corresponds to the angular momentum in a grazing collision, we may approximate $\exp(2i\bar{\sigma}_l)$ by

$$\exp(2i\bar{\sigma}_l) \approx \exp(2i\bar{\sigma}_{L_0})\exp[i\theta_0(L_0-l)], \tag{9}$$

where $\theta_0 = 2\arctan(\bar{n}/L_0)$ is the Rutherford scattering angle for a grazing collision.

(4) The l summation is replaced by an integration from $-\infty$ to $+\infty$. This is admissible because of the extreme localization in l of $\partial\bar{\eta}_l/\partial l$.

With these approximations, Eq. (4) leads to

$$\frac{d\sigma}{d\theta} = \frac{(2J_f+1)}{(2J_i+1)}\frac{(2L_0+1)}{2\pi}|\langle\chi_f\|\xi_L\|\chi_i\rangle|^2 \times\sum_M\{|A_M|^2+|B_M|^2+2\,\mathrm{Re}(A_M B_M^*)\}, \tag{10}$$

where

$$A_M = \frac{\pi\delta(\theta-\theta_0)}{\sinh[\pi\delta(\theta-\theta_0)]}Y_{L,M}(\pi/2,0) \times\exp\{-i[(2M-1)\pi/4+(L_0+\tfrac{1}{2})\theta]\}, \tag{11}$$

$$B_M = \frac{\pi\delta(\theta+\theta_0)}{\sinh[\pi\delta(\theta+\theta_0)]}Y_{L,M}(\pi/2,0) \times\exp\{i[(2M-1)\pi/4+(L_0+\tfrac{1}{2})\theta]\}. \tag{12}$$

There is thus a smooth contribution to $d\sigma/d\theta$ due to $|A|^2+|B|^2$, peaked at $\theta=\theta_0$, and an oscillatory cross term proportional to $\sin[(2L_0+1)\theta]$. Obviously, if

$$2\pi\delta\theta_0 \gg 1, \tag{13}$$

a condition well satisfied in heavy-ion experiments, only the $|A|^2$ term need by considered. Thus we may write, finally,

$$\frac{d\sigma}{d\theta} = C_1\left[\frac{\pi\delta(\theta-\theta_0)}{\sinh[\pi\delta(\theta-\theta_0)]}\right]^2, \tag{14}$$

where C_1 is independent of θ. Thus it is seen that $d\sigma/d\theta$ is independent of L, the angular-momentum transfer, for such heavy-ion scattering. This result was obtained for transfer reactions at $L=0$ by Frahn and Venter.[9] We further note that $d\sigma/d\theta$ is symmetric about θ_0, and that the half-width is given approximately by

$$\Delta \approx 3/\pi\delta. \tag{15}$$

Coulomb Excitations.—Following Alder et al.,[1] we write the differential cross section for Coulomb excitations via an electric transition of order λ as

$$\left(\frac{d\sigma}{d\Omega}\right)_{E\lambda} = 4\left(\frac{Z_p eM}{\hbar^2}\right)^2\frac{k_f}{k_i}\frac{B(E\lambda)}{(2\lambda+1)^3} \times\sum_\mu|\langle\vec{k}_f|r^{-\lambda-1}Y_{\lambda\mu}(\theta,\varphi)|\vec{k}_i\rangle|^2, \tag{16}$$

where Z_pe is the charge of the projectile and M its reduced mass. $B(E\lambda)$ is the reduced transition probability. Taking absorption into account as prescribed by the WKB method,[3] and working in the angular-momentum representation,[6] one obtains

$$\langle\vec{k}_f|r^{-\lambda-1}Y_{\lambda\mu}(\theta,\varphi)|\vec{k}_i\rangle \approx \sum_l(2l+1)^{1/2}\exp(2i\bar{\sigma}_l) \times\bar{\eta}_l Y_{l,\mu}(\theta,0)M_{l,l-\lambda}^{-\lambda-1}. \tag{17}$$

The $M_{l,l'}^{-\lambda-1}$ are the radial integrals calculated with radial Coulomb wave functions.

Assuming again a Woods-Saxon form for $\bar{\eta}_l$, one naturally considers this expression in two regions; the first for $\theta \geq \theta_0$, where the rapid falloff of $\bar{\eta}_l$ completely dominates; the other for $\theta < \theta_0$, where the most rapidly varying factor is the product of $\exp(2i\bar{\sigma}_l)$ and $\exp[\pm i(l+\frac{1}{2})\theta]$ (occurring in the asymptotic expression for $Y_{l,\mu}$). In the first region, only values of l near L_0 contribute because of the form of $\bar{\eta}_l$. Since, in general, L_0 is large, we can thus use the large-l approximation for $M_{l,l+\mu}{}^{-\lambda-1}$, namely,

$$M_{l,l+\mu}{}^{-\lambda-1} \approx \frac{\bar{k}^{\lambda-2}}{4\bar{n}^{\lambda}} \frac{2\pi}{\Gamma(\frac{1}{2}(\lambda+1-\mu))} \xi^{\lambda} \left(\frac{2l\xi}{\bar{n}}\right)^{-(\lambda+\mu+1)/2} \exp\left[-\left(\frac{l}{\bar{n}}+\frac{\pi}{2}\right)\xi\right], \tag{18}$$

where $\xi = n_f - n_i$. Here we have retained in the sums of Alder et al. over both subscripts of M only those terms in which $\mu = -\lambda$. This approximation is valid when $(2l\xi/\bar{n}) \gg 1$, so that terms with $l_f - l_i > -\lambda$ can be neglected.

With the above approximation and application of the four approximations described for nuclear processes in the preceding section, we may write

$$\frac{d\sigma}{d\theta} = C\sum_M\{|\bar{A}_M|^2 + |\bar{B}_M|^2 + 2\,\mathrm{Re}(\bar{A}_M\bar{B}_M{}^*)\}, \tag{19}$$

where now

$$\bar{A}_M = \csc[\pi\delta\xi/\bar{n} + i\pi\delta(\theta-\theta_0)]Y_{\lambda,M}(\tfrac{1}{2}\pi,0) \times\exp\{i[(L_0+\tfrac{1}{2})\theta + (2M-1)\pi/4]\}, \tag{20}$$

$$\bar{B}_M = \csc[\pi\delta\xi/\bar{n} - i\pi(\theta+\theta_0)]Y_{\lambda,M}(\tfrac{1}{2}\pi,0) \times\exp\{-i[(L_0+\tfrac{1}{2})\theta + (2M-1)\pi/4]\}, \tag{21}$$

and C is a constant independent of θ. Again we find the sum of a smooth and an oscillating part in the cross section. The condition that the oscillation be sufficiently damped for this case is the same as for elastic scattering,[7] namely

$$2\pi\delta\theta_0 \gg 1.$$

If this condition is satisfied (and again this is usually the case for heavy-ion scattering), then $d\sigma/d\theta$, as a function of angle for $\theta \geq \theta_0$, takes the simple form

$$\frac{d\sigma}{d\theta} \approx C_2\{\cosh^2[\pi\delta(\theta-\theta_0)] - \cos^2(\pi\delta\xi/n)\}^{-1} \tag{22}$$

$$(\theta \geq \theta_0).$$

Again C_2 is independent of θ. The "right" half-width for such a distribution is seen to be approximately

$$\Delta = \xi/\bar{n}.$$

In the second region, where $\theta \leq \theta_0$, and l is therefore large, the rapid oscillation of the aforementioned factor allows one to invoke the stationary-phase approximation. Thus, in Eq. (17) we consider only these terms in the sum in the neighborhood of the point of stationary phase given by

$$l_\theta \approx \bar{n}\cot(\tfrac{1}{2}\theta). \tag{23}$$

The functions η_l (here slowly varying) and $M_{l,l-\lambda}{}^{-\lambda-1}$ can now be removed from the sum and replaced by their values at $l = l_\theta$. The result of this approximation, and those mentioned previously, is simply related to the Rutherford amplitude, and the cross section for $\theta < \theta_0$ is given finally by

$$\frac{d\sigma}{d\theta} \approx C_3 \sin\theta[\sin\tfrac{1}{2}\theta]^{-3}\exp(-2\xi\csc\tfrac{1}{2}\theta) \tag{24}$$

$$(\theta < \theta_0).$$

Results.—We have thus obtained simple, closed-

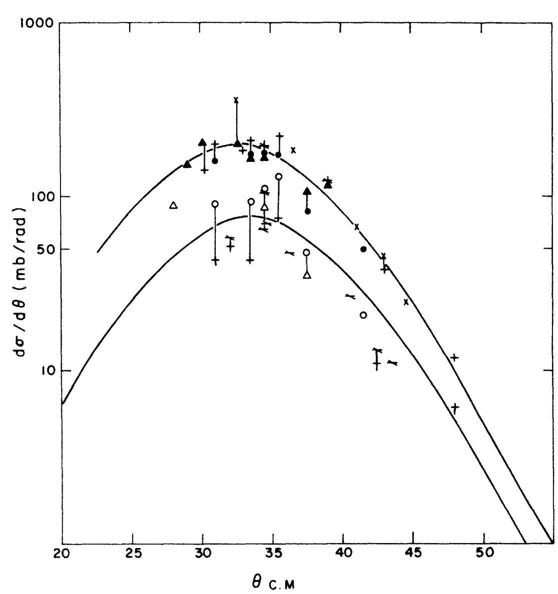

FIG. 1. The angular distribution, $d\sigma/d\theta$, for inelastic scattering of C^{12} on Pb^{208} ($E_{\rm lab} = 125.6$ MeV). The upper set of experimental points is for $Q = -2.7$ MeV, the lower for $Q = -4.2$ MeV. The curves are the theoretical predictions for a direct nuclear process. See Table I.

Table I. The parameters relevant to the experimental graphs shown in the figures, along with the half-widths and "right" half-widths predicted by the theory for the nuclear and Coulomb processes, respectively. (The Δ value in parentheses corresponds to $Q=-2.7$ MeV.) The quoted values for the radii are calculated from the relationship given in reference 4.

Reaction	E_{lab} (MeV)	n	θ (degrees)	d (fm^{-1})	δ	R (fm)	r_0 (fm)	Δ Nuc (rad.)	½Δ Coulomb (rad.)	Reference
$C^{12}+Pb^{208}$	125.6	24.03	33	0.24	1.930	13.51	1.64	0.495	0.010(0.005)	a
Li^6+Au^{197}	60.6	11.75	34	0.31	1.259	12.76	1.67	0.759	0.009	b
Li^6+Au^{197}	33.0	16.00	82	0.31	0.926	13.52	1.77	1.031	0.033	c

[a] See reference 10.

[b] K. H. Wang and J. A. McIntyre, Proceedings of the Third Conference on Reactions Between Complex Nuclei, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, California, 1963).

[c] C. E. Anderson, ibid., p. 67.

form expressions for the differential cross section and the half-width for nuclear and Coulomb excitations. With these expressions one has a simple test available with which to determine the process giving rise to the excitation, especially since the theoretical nuclear-excitation cross section is symmetric while the Coulomb is not. It should be stressed that aside from the over-all normalization and θ_0, which are fitted to each particular experiment, there are no adjustable parameters in these formulas. (The diffuseness is taken from elastic scattering.[7])

In Fig. 1 is shown $d\sigma/d\theta$ for $C^{12}+Pb^{208}\rightarrow(C^{12}+Pb^{208})^*$ for two final bound states of excitation. Parameters relevant to this graph and to those of Figs. 2 and 3 are given in Table I. Comparison of the half-widths leads immediately to the application of the formula for nuclear processes, Eq. (14), which is seen to fit remarkably well in both cases. Figures 2 and 3 show the experimental data for the reaction $Au^{197}(Li^6,$

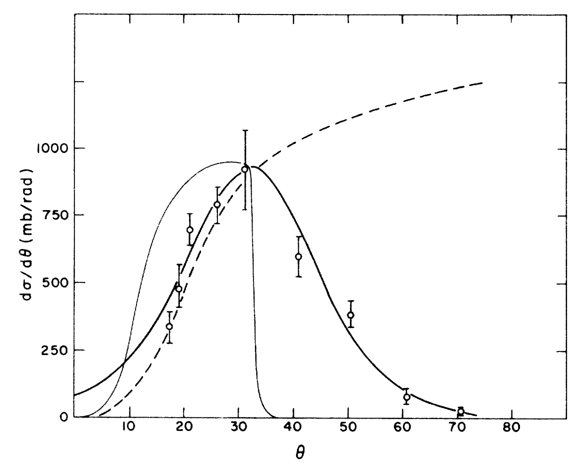

FIG. 2. The angular distribution, $d\sigma/d\theta$, for the breakup reaction $Au^{197}(Li^6,\alpha+d)Au^{197}$, $E_{lab}=60.6$ MeV. The heavy line is the nuclear-excitation cross section, Eq. (14), the light line the Coulomb, Eqs. (22) and (24), and the dashed line the Coulomb cross section, ignoring absorption [see R. L. Gluckstern and G. Breit, Proceedings of the Second Conference on Reactions Between Complex Nuclei, Gatlinburg, Tennessee, 1960 (John Wiley & Sons, Inc., New York, 1960)]. See Table I.

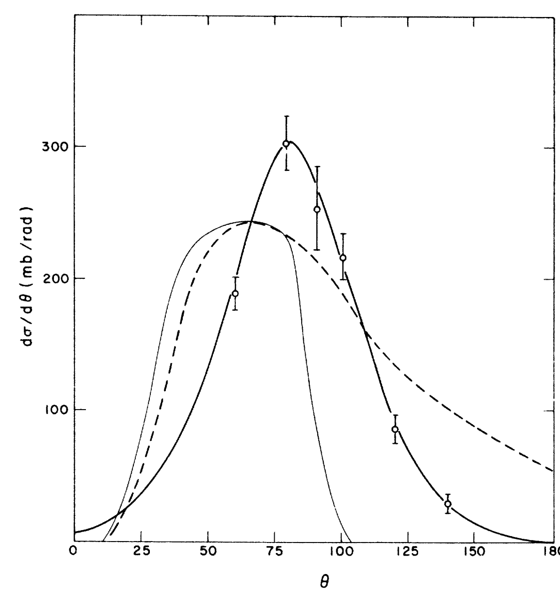

FIG. 3. The angular distribution, $d\sigma/d\theta$, for the breakup reaction $Au^{197}(Li^6,\alpha+d)Au^{197}$, $E_{lab}=33$ MeV. The heavy line is the nuclear-excitation cross section, Eq. (14), the light line the Coulomb, Eqs. (22) and (24), and the dashed line the Coulomb cross section ignoring absorption [see R. L. Gluckstern and G. Breit, Proceedings of the Second Conference on Reactions Between Complex Nuclei, Gatlinburg, Tennessee, 1960 (John Wiley & Sons, Inc., New York, 1960)]. See Table I.

$(\alpha+d)Au^{197}$ at two different energies. The heavy solid line, corresponding almost exactly to the data, is the nuclear excitation cross section, Eq. (14). The dashed line is an attempt, by Gluckstein and Breit,[10] to fit the data, assuming Coulomb excitation without absorption. The light line in these figures corresponds to the Coulomb cross section as given by Eqs. (22) and (24). We conclude that for all cases considered here the dominant process is nuclear in origin.

Finally, it should be pointed out that for nuclear processes, where the aforementioned assumptions are valid, $d\sigma/d\theta$ is a universal function of $\pi\delta(\theta-\theta_0)$ and the function $(d\sigma/d\theta)/(d\sigma/d\theta_0)$, plotted as a function of $\pi\delta(\theta-\theta_0)$, should be the same for all relevant reactions.

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[4]For reactions localized to a ring around the nucleus, the angular distribution will be peaked (for large values of the Coulomb parameter, n) at the Rutherford angle, θ_0, corresponding to a grazing collision. From this it is easy to deduce that $\theta_0 = 2\arcsin[\bar{n}/(\bar{K}R-\bar{n})]$.

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