Trieste.

Note added in proof.—We have just received a preprint from Princeton by J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa which deals with meson-baryon scattering in broken $\tilde{\mathrm{U}}(12)$.

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[1] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, to be published; B. Sakita and K. C. Wali, to be published; M. A. B. Bég and A. Pais, to be published.

[2] R. Delbourgo, to be published. Similar work in the context of $\mathrm{U}(6)\otimes\mathrm{U}(6)$ has been carried out by W. Rühl, to be published; and K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 264 (1965). Difficulties of physical interpretation for $\mathrm{U}(6)\otimes\mathrm{U}(6)$ have been emphasized by J. M. Charap and P. T. Matthews, to be published.

[3] R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters 14, 518 (1965).

[4] K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

[5] H. Harari and H. J. Lipkin, to be published.

[6] The baryon 364-fold B_{ABC} is fully symmetric, and the meson 143-fold $M_A{}^B$ is traceless. The indices $A, B, \cdots$ take the values $1, 2, \cdots, 12$. For details see reference 1.

[7] By considering single-particle pole contributions thereby further violating $\tilde{\mathrm{U}}(12)$, we obtain nonzero expressions for the two meson annihilation cross sections. They correspond to the F coupling of the mesons and the baryons. For the interesting case of two pseudoscalar mesons, there is a kinematical factor $(m-\mu)/\mu(2m^2-\mu^2)$. Using physical masses we obtain $\sigma(\pi^+\pi^-):\sigma(K^+K^-):\sigma(K^0\bar{K}^0)=5:1:\frac{1}{4}$, to be compared with the experimental ratios $3:1:\frac{2}{5}$. See the Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962). Also, recent compilations have been given by T. Ferbel et al., Phys. Rev. 137. B1250 (1965).

[8] By the two-meson decays we mean two "primary" mesons which do, of course, include the multipion resonances.

[9] For the physical η and ω particles we take the singlet-octet mixture $\frac{1}{2}(\varphi_1{}^1+\varphi_2{}^2)=3^{-1/2}(\varphi_1+\sqrt{2}\varphi_8)$. The orthogonal combination $\varphi_3{}^3=3^{-1/2}(\varphi_1-\sqrt{2}\varphi_8)$ is suppressed by $\tilde{\mathrm{U}}(12)$.

[10] Diminution of strange-particle production has at the SU(6) level been noted by H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964). Our conclusion (b) is nevertheless stronger.

PION PRODUCTION IN HIGH-ENERGY MUON-NUCLEON COLLISIONS

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Charged leptons are excellent probes of nuclear structure because their electromagnetic interactions with nucleons are not accompanied by a strong interaction. In the past the only experiments on the inelastic interactions of muons and nucleons were done by using cosmic rays, and the results[1,2] are not in agreement.

Recently, high-energy muon beams have been available,[3-8] and we report here our preliminary results[6] on the meson-nucleon inelastic interactions in nuclear emulsion exposed to 5-GeV/c negative muons of flux density 2×10^5/cm^2 at the Brookhaven AGS. The pion contamination in the muon beam was $\sim10^{-7}$. By area scanning, 136 inelastic interactions were found under very stringent criteria. The total inelastic muon-nucleon cross section[6,7] was found to be $\sim(3\pm0.25)$ μb/nucleon. Charged particles emitted from the muon interactions were identified by grain density and scattering measurements. The cross sections for an inelastic process in which charged pions and strange particles are produced are found to be approximately $(7\pm1.4)\times10^{-31}$ cm^2/nucleon and $(1\pm0.6)\times10^{-31}$ cm^2/nucleon, respectively. The average transverse momentum of pions produced is (244 ± 35) MeV/c, which checks with the previous values.[9]

The absence of strong coupling between the muon and the nucleon indicates that the incident muons will be scattered through a small angle from the original direction in the nuclear interactions. Figure 1 shows the distribution of the angular deflection in the laboratory system of muons producing nuclear interactions. An approximate theoretical angular distribution has been obtained by Kessler and Kessler[10] in which they made use of the virtual photon spectrum given by Weizsäcker and Williams.[11] The theoretical angular distribution is compared with the experiment, and the agreement is quite good. We may point out that the angular distribution is not very sensitive to the different theoreti-

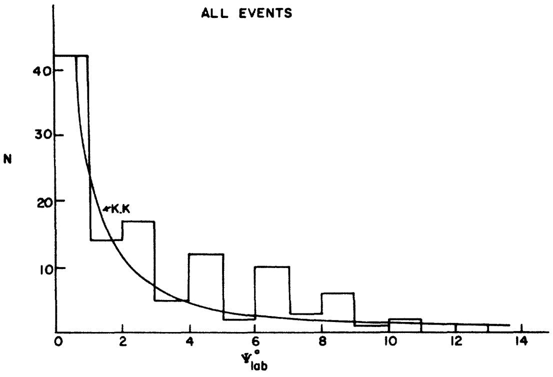

FIG. 1. Distribution of the angular deflection of muons producing nuclear interactions. Only those events where the deflected muons were identified were considered.

cal expressions representing the virtual photon fields as given by Heitler[12] or by Kessler and Kessler.[10] The angular distribution of secondary pions produced in inelastic interactions in the c.m. system of the target nucleon and the photon which is virtually associated with the incoming muon shows enhancement in the backward direction. Since there is a close relationship between the production of pions by either muons or real photons, the angular distribution of pions was compared with photopion production experiments[13,14] and with the theory.[15] The general form was found to be the same within the statistical errors.

Multiple-meson production by muon-nucleon interaction is a very complicated phenomenon to express theoretically. There is no rigorous theory which can explain satisfactorily the very limited experimental data. In fact as far as we know, prior to the present investigations there have not been any systematic studies in meson production by a well-defined monoenergetic primary beam of muons. The energy spectrum of single pions arising from muon-nucleon interactions is shown in Fig. 2. Fowler[16] calculated the energy spectrum of single pions produced by 10-GeV muons for a nucleus with $A = 90$. For these calculations, he combined the data of Watson et al.[13] in the total charged photon-meson cross section with the methods of Dalitz and Yennie.[17] He also considered the effect of initial nucleon momentum distribution averaged over pion angles in his calculations along with the effect of pion absorption in the same nucleus. This calculated curve shown in Fig. 2 has a general agreement with the experimental data except at the higher values of kinetic energies. This is expected since the curve was calculated for higher muon energy. A similar agreement between photoproduction experiments and pion production by electrons scattered at small angle has been reported by Panofsky and co-workers.[18]

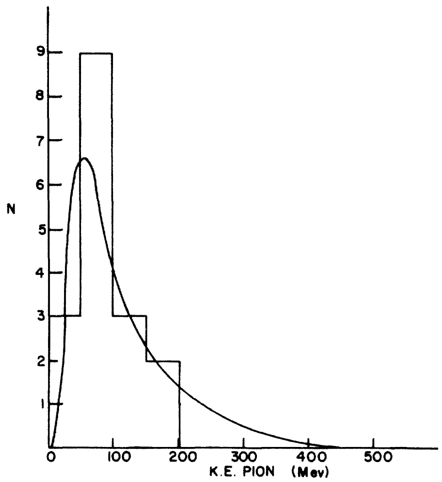

FIG. 2. Energy spectrum of pions produced from the interaction of muons with nucleons. In the calculated curve the effect of pion absorption and nucleon motion was taken into consideration.

It has been shown recently[19,20] that the total cross section for multiple meson production by muons can be expressed in terms of two unknown Lorentz- and gauge-invariant functions L and L'. The process is represented by a Feynman diagram shown in Fig. 3. An incident muon with momentum p_1 $(\vec{p}_1, E_1)$ collides with the rest nucleon p $(0, M)$ and emits a virtual photon of q $(\vec{q}, \epsilon)$ and has momentum p_2 $(\vec{p}_2, E_2)$ in the final state. The differential cross section as a function of the transferred energy and the square of four-momentum transfer q^2 is

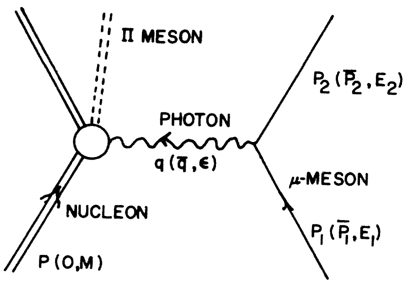

FIG. 3. Feynman diagram for an inelastic interaction between a muon and a nucleon.

given by

$$\frac{d^2\sigma}{dq^2d\epsilon}=\frac{\alpha}{8\pi^2}\frac{1}{|\vec{p}_1|}\frac{1}{q^4}[L\{(E_1{}^2+E_2{}^2)^2q^2-2m^2\epsilon^2-\tfrac{1}{2}q^4\}$$
$$+L'(2m^2-q^2)], \tag{1}$$

where α is the fine-structure constant and m the mass of a muon. L and L' are functions of q^2 and ϵ, the transferred energy. Neglecting transitions not present in photoproduction,[19] we put $L'=0$, and the function L is expressed by the following equations:

$$L(\epsilon,q^2)=L_0\frac{\Lambda^2}{q^2+\Lambda^2}=(4\pi/\epsilon)\sigma_{h\nu} \text{ with } \Lambda^2=\infty,$$
$$=(4\pi/\epsilon)\sigma_{h\nu}\Lambda^2/(q^2+\Lambda^2)$$
$$\text{with finite } \Lambda, \tag{2}$$

where the value of $\Lambda^2=0.365$ (GeV)^2 corresponds to the radius of the proton charge distribution, i.e., 0.8×10^{-13} cm, obtained by the experiments on electron-proton scattering.[21] By considering $\sigma_{h\nu}$ to be independent of ϵ, one can calculate the integral spectrum of energy transferred to the target particle. But the energy-transfer distribution curve does not give much distinction between point charge and extended charge distribution of the nucleon. However, the calculation for four-momentum transfer (q) gives such a differentiation. In relativistic approximation, the square of four-momentum transfer q^2 is given by $q^2\approx4E_1(E_1-\epsilon)\sin^2(\theta/2)$ in which θ is a deflection angle of the incident muon.

From Eq. (1) we get the integral spectrum of the square of the transferred four-momentum for different values of Λ^2 and $q_{\max}{}^2$, which are shown in Fig. 4. Theoretical curve A with finite charge distribution and with the approximate kinematic upper limit of $q_{\max}{}^2=100$ (GeV/c)^2 fits the data much better ($\chi^2=12.2$) than curve C ($\chi^2=152.8$) with point-charge nucleon and with the same $q_{\max}$ value. The q value greater than 1 (GeV/c) has been observed,[8] and the more we increase the $q_{\max}$ value, the poorer the fit with the data for the point-charge nucleon.

Electron scattering data[18,22] on single-pion production have shown no evidence of contribution from longitudinally polarized photons or from the other effects which have no analogy in photoproduction. Hand,[22] in a study of single-pion production by electrons, has used an analogous representation in terms of photoproduction cross sections and has found experimental agreement with a formula whose q^2 dependence is similar to ours. Our preliminary experimental results on multiple-pion production which extends up to a momentum transfer of ~1 GeV/c ($q^2=0.2$ F^{-2}) show no evidence of significant contribution of longitudinally polarized virtual photons in either the spectrum of produced pions or that of the q^2 distribution. Our results agree insofar as we have been able to compare them with electron-pion production results at lower four-momentum transfer, and we have found no evidence in our analysis of

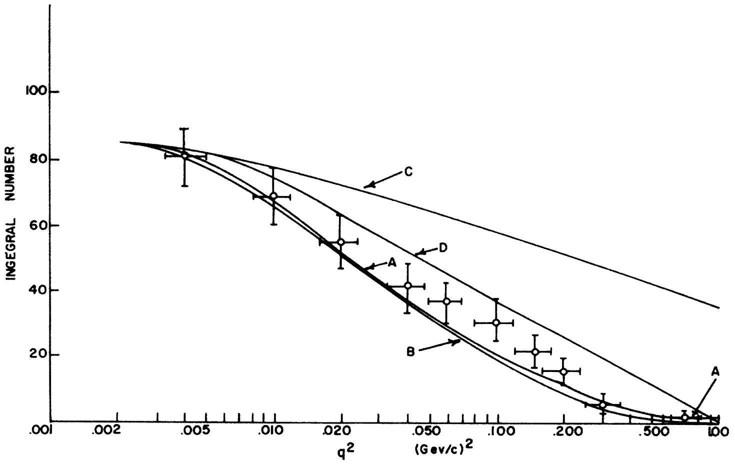

FIG. 4. The integral spectrum of the square of the transferred four momentum for different values of $q_{\max}$ and Λ. Curve A: $q_{\max}{}^2=100$ (GeV/c^2 and $\Lambda^2=0.365$ (GeV/c)^2; Curve B: $q_{\max}{}^2=1$ (GeV/c)^2 and $\Lambda^2=0.365$ (GeV/c)^2; Curve C: $q_{\max}{}^2=100$ (GeV/c)^2 and $\Lambda^2=\infty$; Curve D: $q_{\max}{}^2=1$ (GeV/c)^2 and $\Lambda^2=\infty$.

"anomalous" scattering.

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*An account of this research will be submitted by P. J. McNulty in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the Department of Physics, State University of New York at Buffalo.

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