dence, Rhode Island, 1957), Ser. II, Vol. 6, p. 319.

[3] H. J. Bhabha, Rev. Mod. Phys. 17, 200 (1945).

[4] E. P. Wigner, Phys. Rev. 51, 106 (1937).

[5] F. Gürsey and L. Radicati, Phys. Rev. Letters 13, 173 (1964); A. Pais, Phys. Rev. Letters 13, 175 (1964).

[6] M. Konuma, K. Shima, and M. Wada, Progr. Theoret. Phys. (Kyoto), Suppl. 28, 44 (1963).

[7] M. Gell-Mann, Phys. Letters 8, 214 (1964).

[8] P. Roman and J. J. Aghassi, Phys. Letters 14, 68 (1965).

$\tilde{\text{U}}$(12) PREDICTIONS FOR $p\bar{p}$ ANNIHILATION INTO MESONS

R. Delbourgo, Y. C. Leung, M. A. Rashid, and J. Strathdee
International Centre for Theoretical Physics, Trieste, Italy

(Received 15 March 1965)

The noncompact symmetry group $\tilde{U}(12)$ which, with appropriate prescriptions,[1] can be regarded as a relativistic generalization of SU(6), preserves the successful aspects of that group. At the same time, through its incorporation of momentum effects, it is applicable to a wider class of processes. Already the $\tilde{U}(12)$-invariant three-point functions have been studied extensively,[1,2] and attention is shifting to the four-point functions. Blankenbecler, Goldberger, Johnson, and Treiman,[3] in their treatment of baryon-meson elastic scattering have shown that $\tilde{U}(12)$, in addition to preserving those consequences of SU(6) which could be drawn for the case of forward scattering,[4] both good and bad, leads to a number of new relationships and, in particular, to predictions about particle polarizations which seem to be at variance with experiment. A preliminary examination of the associated nucleon-antinucleon annihilation problem has been carried out by Harari and Lipkin,[5] again leading to predictions that are not fulfilled, in this case the relative magnitudes for the production of $\pi^+\pi^-$, K^+K^-, and $K^0\bar{K}^0$.

Here we shall consider the annihilation problem. The first point to notice is that the amplitude for $p\bar{p}$ annihilation into two mesons,[6]

$$\bar{B}^{ABC}(-p')B_{ADE}(p)M_B^{\;\;D}(k_1)M_C^{\;\;E}(k_2),$$

vanishes at threshold $p'=p$. This is a direct consequence of the Bargmann-Wigner equations which are necessary for the physical interpretation of $\tilde{U}(12)$ states.[1] It is, no doubt, here that our prescriptions[7] differ from those of Harari and Lipkin. Having seen that the two-meson decays[8] are suppressed by $\tilde{U}(12)$, we can proceed to the consideration of three-meson decays for which the amplitude survives at threshold. It is given by

$$\bar{B}^{ABC}(-p)B_{DEF}(p)M_A^{\;\;D}(k_1)M_B^{\;\;E}(k_2)M_C^{\;\;F}(k_3).$$

Remarkably, the nucleon-antinucleon part of this structure contains only the nonstrange mesons π, η, ρ, and ω. The amplitudes A_{JI}, where J and I denote, respectively, the spin and isospin of the nucleon-antinucleon system, are easily extracted. They are

$$A_{00}=\sum\nolimits_{\text{perm}}\bar{N}\{[\eta(1)\eta(2)-\omega_\alpha(1)\omega_\alpha(2)-\pi_i(1)\pi_i(2)+\rho_{\alpha i}(1)\rho_{\alpha i}(2)]\eta(3)-\tfrac{1}{3}\epsilon_{\alpha\beta\gamma}\epsilon_{jkl}\rho_{\alpha j}(1)\rho_{\beta k}(2)\rho_{\gamma l}(3)\}N,$$

$$A_{01}=\sum\nolimits_{\text{perm}}\bar{N}\{[\eta(1)\eta(2)-\omega_\alpha(1)\omega_\alpha(2)-\pi_i(1)\pi_i(2)+\rho_{\alpha i}(1)\rho_{\alpha i}(2)]\pi_l(3)$$
$$-\tfrac{1}{3}\epsilon_{\alpha\beta\gamma}\epsilon_{jkl}\omega_\alpha(1)\rho_{\beta j}(2)\rho_{\gamma k}(3)\}\tau_l N,$$

$$A_{10}=\sum\nolimits_{\text{perm}}\bar{N}\{[\eta(1)\eta(2)-\omega_\beta(1)\omega_\beta(2)-\pi_i(1)\pi_i(2)+\rho_{\beta i}(1)\rho_{\beta i}(2)]\omega_\alpha(3)-\tfrac{1}{3}\epsilon_{\alpha\beta\gamma}\epsilon_{jkl}\rho_{\beta j}(1)\rho_{\gamma k}(2)\pi_l(3)\}\sigma_\alpha N,$$

$$A_{11}=\sum\nolimits_{\text{perm}}\bar{N}\{[\eta(1)\eta(2)-\omega_\beta(1)\omega_\beta(2)-\pi_i(1)\pi_i(2)+\rho_{\beta i}(1)\rho_{\beta i}(2)]\rho_{\alpha l}(3)-\tfrac{2}{3}[i\epsilon_{jkl}\rho_{\alpha j}(1)\rho_{\beta k}(2)\omega_\beta(3)$$
$$-i\epsilon_{\alpha\beta\gamma}\rho_{\beta l}(1)\rho_{\gamma i}(2)\pi_i(3)-\rho_{\alpha i}(1)\rho_{\beta i}(2)\rho_{\beta l}(3)-\rho_{\alpha l}(1)\rho_{\beta i}(2)\rho_{\beta i}(3)]\}\sigma_\alpha\tau_l N, \qquad (1)$$

where η, ω, π, and ρ are defined by the expression for the matrix element of the meson field $M_A{}^B$ between the Dirac spinors for nucleon and antinucleon,

$$\bar{u}(-p)M(k)u(p)=\chi^\dagger[\eta(k)+\omega_\alpha(k)\sigma_\alpha+\pi_i(k)\tau_i+\rho_{\alpha i}(k)\sigma_\alpha\tau_i]\chi.$$

In summary, $\tilde{U}(12)$ makes the following predictions for annihilations at rest: (a) Two-meson processes ($\bar{B}B\rightarrow MM$) are forbidden; (b) three-meson $p\bar{p}$ annihilations are forbidden when they involve φ or the strange particles; (c) three-meson decays involving[9] 3π, $\eta 2\pi$, $\rho 2\pi$, and $\omega 2\pi$ are comparable; and (d) four- or more-meson decays, being described by amplitudes with four or more factors $M_A{}^B$, can involve strange particles.

The experimental situation with respect to $p\bar{p}$ annihilation into mesons is roughly summarized in Table I. One sees that (a) is quite well obeyed apart from the $\rho\pi$ contribution to the annihilation into three final pions. However, $p+\bar{p}\rightarrow\rho+\pi$ and all the other processes forbidden by $\tilde{U}(12)$ constitute only 10% of the total annihilation into mesons. Prediction (b) seems also to be well supported insofar as we take the poor statistics for $\bar{K}K^*\pi$ and $K\bar{K}\pi$ to indicate small cross sections.[10] Prediction (c) is apparently violated because $\eta 2\pi$ and 3π are significantly less than $\rho 2\pi$ and $\omega 2\pi$. As for (d), the majority of the $\bar{K}K\eta\pi$ processes can be associated with $\tilde{U}(12)$ amplitudes containing more than three meson factors.

Equation (1) shows that $\eta 2\pi$ and 3π are associated with the $J=0$ protonium states, whereas $\omega 2\pi$ and $\rho 2\pi$ are associated with the $J=1$ states. Hence a conceivable remedy for prediction (c) is to suppose that there exists an initial-state interaction [breaking $\tilde{U}(12)$] which greatly enhances the triplet relative to the singlet states.

The authors are grateful to Professor Abdus Salam and the International Atomic Energy Agency for the hospitality extended to them at the International Centre for Theoretical Physics,

Table I. $p\bar{p}$ annihilation into mesons.[a]

Ultimate decay products	% of listed meson products	Primary decay products	$\tilde{U}(12)$ prediction[b]
$\pi^+\pi^-$	1		no
$\pi^+\pi^-\pi^0$	5	$\rho\pi$, 4?	no
		3π, 1?	yes
$2\pi^+2\pi^-$	6	$\rho 2\pi$, 4	yes
		4π, 2	yes
$2\pi^+2\pi^-\pi^0$	35	$\rho\omega$, (small)	no
		$\eta 2\pi$, (small)	yes
		$\rho\rho\pi$, (small)	yes
		$\omega 2\pi$, 7	yes
		$\rho 3\pi$, 27	yes
$3\pi^+3\pi^-$	7	$\omega 3\pi$, 5?	
		$\rho 4\pi$, small	
		rest, 2?	
$3\pi^+3\pi^-\pi^0$	22	$\omega 4\pi$, 16	
		rest, 6	
$4\pi^+4\pi^-$	1		
$4\pi^+4\pi^-\pi^0$	2		
K^+K^-	small		no
$K^+K^-n\pi$	22	$K\bar{K}\pi$, ?	no
		$K^*\bar{K}\pi$, ?	no
		$K\bar{K}\omega$, small	no
		rest	yes

[a] The percentages shown here are drawn from various sources: The *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962); and T. Ferbel *et al.*, Phys. Rev. 137, B1250 (1965).

[b] "Yes" and "no" indicate forbidden and allowed processes, respectively.

Trieste.

Note added in proof.—We have just received a preprint from Princeton by J. M. Cornwall, P. G. O. Freund, and K. T. Mahanthappa which deals with meson-baryon scattering in broken $\tilde{U}(12)$.

---

[1] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); R. Delbourgo, M. A. Rashid, A. Salam, and J. Strathdee, to be published; B. Sakita and K. C. Wali, to be published; M. A. B. Bég and A. Pais, to be published.

[2] R. Delbourgo, to be published. Similar work in the context of $U(6)\otimes U(6)$ has been carried out by W. Rühl, to be published; and K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 264 (1965). Difficulties of physical interpretation for $U(6)\otimes U(6)$ have been emphasized by J. M. Charap and P. T. Matthews, to be published.

[3] R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman, Phys. Rev. Letters 14, 518 (1965).

[4] K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

[5] H. Harari and H. J. Lipkin, to be published.

[6] The baryon 364-fold B_{ABC} is fully symmetric, and the meson 143-fold $M_A{}^B$ is traceless. The indices A, B, $\cdots$ take the values $1, 2, \cdots, 12$. For details see reference 1.

[7] By considering single-particle pole contributions thereby further violating $\tilde{U}(12)$, we obtain nonzero expressions for the two meson annihilation cross sections. They correspond to the F coupling of the mesons and the baryons. For the interesting case of two pseudoscalar mesons, there is a kinematical factor $(m-\mu)/\mu(2m^2-\mu^2)$. Using physical masses we obtain $\sigma(\pi^+\pi^-):\sigma(K^+K^-):\sigma(K^0\bar{K}^0) = 5:1:\frac{1}{4}$, to be compared with the experimental ratios $3:1:\frac{2}{5}$. See the Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962). Also, recent compilations have been given by T. Ferbel et al., Phys. Rev. 137. B1250 (1965).

[8] By the two-meson decays we mean two "primary" mesons which do, of course, include the multipion resonances.

[9] For the physical η and ω particles we take the singlet-octet mixture $\frac{1}{2}(\varphi_1{}^1+\varphi_2{}^2) = 3^{-1/2}(\varphi_1+\sqrt{2}\varphi_8)$. The orthogonal combination $\varphi_3{}^3 = 3^{-1/2}(\varphi_1-\sqrt{2}\varphi_8)$ is suppressed by $\tilde{U}(12)$.

[10] Diminution of strange-particle production has at the SU(6) level been noted by H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964). Our conclusion (b) is nevertheless stronger.

PION PRODUCTION IN HIGH-ENERGY MUON-NUCLEON COLLISIONS

P. L. Jain and P. J. McNulty*

Department of Physics, State University of New York at Buffalo, Buffalo, New York

(Received 22 January 1965; revised manuscript received 25 February 1965)

Charged leptons are excellent probes of nuclear structure because their electromagnetic interactions with nucleons are not accompanied by a strong interaction. In the past the only experiments on the inelastic interactions of muons and nucleons were done by using cosmic rays, and the results[1,2] are not in agreement.

Recently, high-energy muon beams have been available,[3-8] and we report here our preliminary results[6] on the meson-nucleon inelastic interactions in nuclear emulsion exposed to 5-GeV/c negative muons of flux density 2×10^5/cm^2 at the Brookhaven AGS. The pion contamination in the muon beam was $\sim10^{-7}$. By area scanning, 136 inelastic interactions were found under very stringent criteria. The total inelastic muon-nucleon cross section[6,7] was found to be $\sim(3\pm0.25)$ μb/nucleon. Charged particles emitted from the muon interactions were identified by grain density and scattering measurements. The cross sections for an inelastic process in which charged pions and strange particles are produced are found to be approximately $(7\pm1.4)\times10^{-31}$ cm^2/nucleon and $(1\pm0.6)\times10^{-31}$ cm^2/nucleon, respectively. The average transverse momentum of pions produced is (244 ± 35) MeV/c, which checks with the previous values.[9]

The absence of strong coupling between the muon and the nucleon indicates that the incident muons will be scattered through a small angle from the original direction in the nuclear interactions. Figure 1 shows the distribution of the angular deflection in the laboratory system of muons producing nuclear interactions. An approximate theoretical angular distribution has been obtained by Kessler and Kessler[10] in which they made use of the virtual photon spectrum given by Weizsäcker and Williams.[11] The theoretical angular distribution is compared with the experiment, and the agreement is quite good. We may point out that the angular distribution is not very sensitive to the different theoreti-