MAGNETIC DIPOLE TRANSITIONS AND g_R FACTORS IN DEFORMED EVEN-EVEN NUCLEI

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Up to now magnetic effects in even-even nuclei have not been satisfactorily explained. Nielsson and Prior[1] calculated the well-known lowering of the rotational g_R factors from the value Z/A assuming different pairing forces G_p and G_n for protons and neutrons, respectively. However, there has been no explanation[2] of the magnetic dipole transitions from the second 2^+ to the first 2^+ state generally observed in even-even nuclei.[3] It is the aim of this note to show the intimate connection between both observations and, in addition, to develop a plausible picture for understanding these effects.

The counterplay of the pairing and the quadrupole force determines the shape of the nucleus,[4,5] i.e., a spherical or a deformed shape. The pairing force makes the nuclei prefer spherical shapes while the quadrupole force leads to deformations. One therefore expects that for a definite nucleus, a larger pairing force will decrease the deformation. Consequently, since G_p is 30% larger than G_n, one expects less deformation for protons than for neutrons. It will be shown now that this picture has the consequence that the g factor becomes a g tensor, i.e., the magnetic moment $\vec{\mu}$, and the total angular momentum, $\vec{I}$, point in different directions.[6]

In the intrinsic coordinate system the g factor for a rotation of the system around the σ axis ($\sigma = 1, 2, 3$) is given by

$$g\sigma = \frac{\vec{I}_{p\sigma}}{\vec{I}_\sigma} = \frac{\omega \cdot \mathcal{J}_{p\sigma}}{\omega \cdot \mathcal{J}_\sigma}, \qquad (1)$$

where $\vec{I}_{p\sigma}$ and $\vec{I}_\sigma$ are the σ components of the angular momentum of the protons and of the total angular momentum of the system, respectively. ω is the angular velocity, and $\mathcal{J}_p$ and $\mathcal{J}$ are the moments of inertia of the protons alone and of the total system (protons + neutrons). They are given, according to the rotation-vibration model[7,8] of the nucleus, by the Bohr formulas[9]

$$\mathcal{J}_{1/2}(a_\nu) = B[3a_0^2 + 2a_2^2 \pm 2(6)^{1/2} a_0 a_2], \qquad (2)$$

where the a_ν are the shape parameters defined by the radius $R = R_0[1 + a_0 Y_{20} + a_2(Y_{22} + Y_{2-2})]$ and B is the mass parameter. We assume that the proton ellipsoid and the neutron ellipsoid are strongly coupled, i.e., their principal axes coincide. They will only be a little different in their equilibrium shape because of their different deformation. Therefore the angular velocity ω is the same for protons and for the total system (protons + neutrons)—see Fig. 1. The deformations are denoted for protons by

$$a_0(p) = B_0(p) + a_0', \qquad (3a)$$

$$a_2(p) = 0 + a_2'; \qquad (3b)$$

and for the total system by

$$a_0 = B_0 + a_0', \qquad (3c)$$

$$a_2 = 0 + a_2'. \qquad (3d)$$

Only the axial symmetric equilibrium deformations are assumed to be different, i.e. $B_0(p) < B_0$. The vibrational coordinates[7,8] a_ν' are taken to be the same for protons and for the total system.[10]

Inserting (2) into (1) we obtain for the g fac-

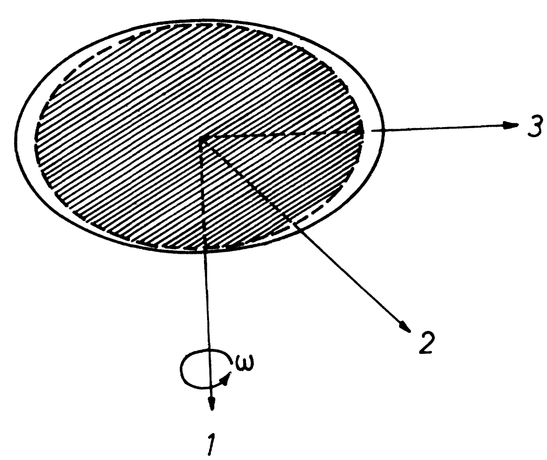

FIG. 1. Schematic picture of the mass (protons + neutrons) and proton ellipsoid. Solid line: shape of the mass distribution of the deformed nucleus. Dashed line: shape of the proton distribution of the deformed nucleus. Both ellipsoids are strongly coupled and rotate with the same angular velocity ω. The different deformations for protons and neutrons are related to their different pairing forces. The principal axes are numbered by 1, 2, and 3.

tors around the 1 and 2 axis

$$g_{1/2}=\frac{B_p[3a_0^2(p)+2a_2^2\pm 2(6)^{1/2}a_2a_0(p)]}{B[3a_0^2+2a_2^2\pm 2(6)^{1/2}a_2a_0]}. \qquad (4)$$

Since the mass parameters B_p and B will be proportional to the proton and nucleon densities, respectively, we have $B_p/B=Z/A$. Inserting (3) into (4) and expanding in the small quantities, we obtain for the dominant terms

$$g_+\equiv\tfrac{1}{2}(g_1+g_2)=(Z/A)(1-2f), \qquad (5)$$

$$g_-\equiv\tfrac{1}{2}(g_1-g_2)=(Z/A)(1-2f)\tfrac{2}{3}(6)^{1/2}fa_2/B_0; \qquad (6)$$

$$f\equiv[B_0-B_0(p)]/B_0.$$

Note that for $B_0(p)\to B_0$, i.e. for equal deformation for protons and neutrons, $g_-=0$ and $g_+=Z/A$. Therefore the lowering of the g_R factor from Z/A indicates a smaller proton deformation. In addition, simultaneously with the lowering of g_+ occurs a nonvanishing value for g_- which is, due to the a_2 dependence, responsible for magnetic transitions between the γ and ground-state rotational bands. In addition it indicates that the g factor depends on the axis of rotation and, consequently, the usual g factor has now become a tensor. The magnetic moment is given in terms of the intrinsic components $\bar{\mu}_\nu$ by

$$\mu_\sigma=\sum_\gamma D_{\sigma\nu}{}'(\theta_i)\bar{\mu}_\nu;$$
$$\bar{\mu}_1=-2^{-1/2}(g_+I_++g_-I_-),$$
$$\bar{\mu}_{-1}=2^{-1/2}(g_-I_++g_+I_-),$$
$$\bar{\mu}_0=g_0I_0; \qquad (7)$$

where

$$I_\pm\equiv I_1\pm iI_2.$$

We use for our calculations the wave functions of the rotation-vibration model[7,8] $|IKn_2n_0\rangle$, which have been proven to give a good description of the low-energy spectra.[11] The g_R factor for the first rotational 2^+ state and the $M1$-$E2$ mixing parameter for the $2^{+\prime}\to 2^+$ transition are given by

$$g_R=(\langle IKn_2n_0|\bar{\mu}_0|IKn_2n_0\rangle/I)_{M=I}$$
$$\delta=\pm\left[\frac{T(E2)}{T(M1)}\right]^{1/2}=\pm\frac{\sqrt{3}}{10}\frac{E}{(\hbar c)}\left[\frac{B(E2|I'\to I)}{B(M1|I'\to I)}\right]^{1/2}. \qquad (8)$$

Here the + or − sign has to be chosen, depending on the sign of the reduced matrix elements. E is the transition energy. The quadrupole

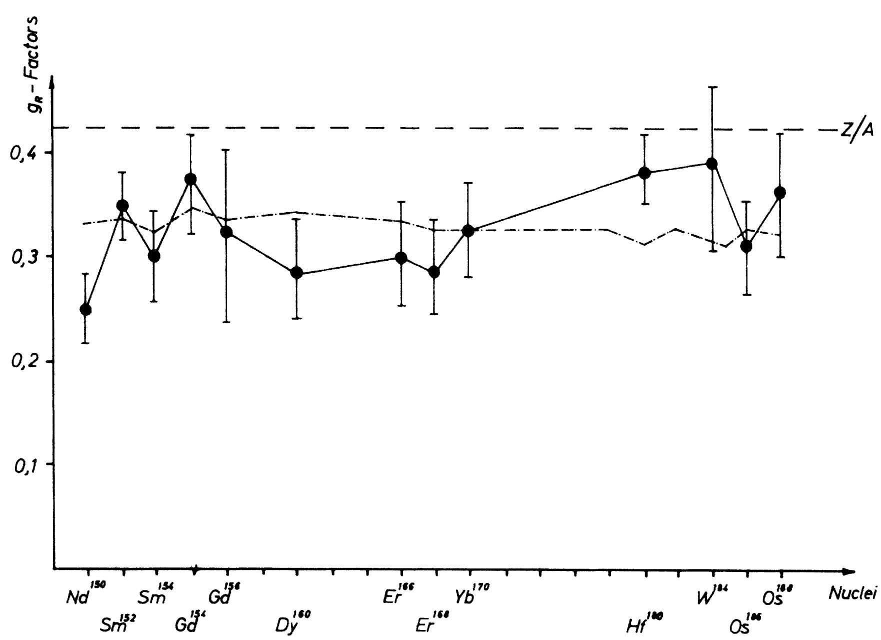

FIG. 2. Experimental g_R factors, [see E. Bodenstedt, Fortschr. Physik 10, 321 (1962); and P. Kienle et al., to be published] compared with the theoretical predictions using Formulas (9) and (10) of the text.

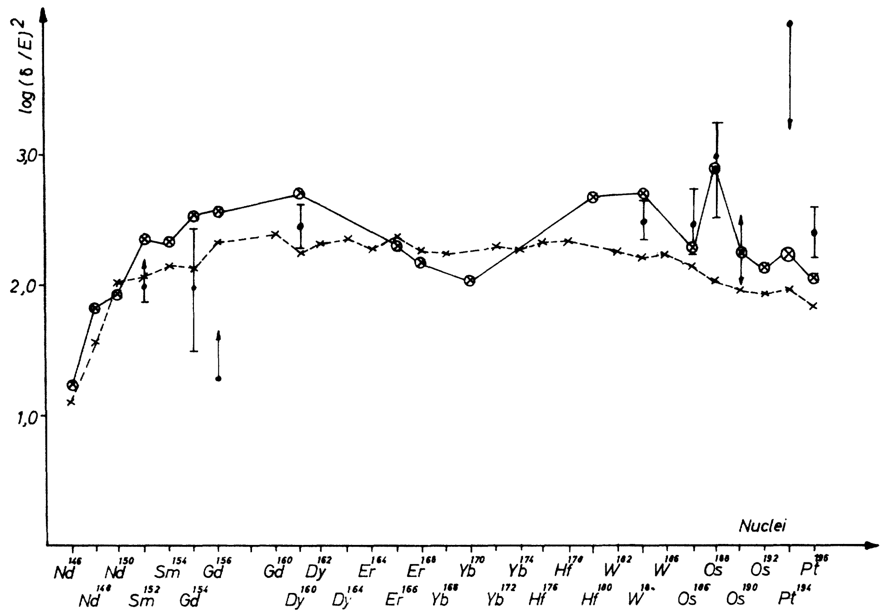

FIG. 3. The values for $\log(\delta/E)^2$ for various nuclei. The dotted curve gives theoretical predictions using Formulas (9) and (10) of the text. The full line gives theoretical values for $\log(\delta/E)^2$ using a value for the lowering factor f [see Eq. (6) of the text] which has been deduced from the experimental g_R factors shown in Fig. 2. It is noted that some structure for the magnetic dipole transitions seems to be quantitatively related to the structure of the g_R factors, especially in the Os region. It should be noted, however, that the experimental errors of the g_R factors (see Fig. 2) scatter the related predictions for $(\delta/E)^2$ appreciably.

operator is calculated, as usual, for a homogeneous charge distribution. The results are the following formulas:

$$(g_R)_{2^+} = g_+ = (Z/A)(1-2f),$$

$$\left(\frac{\delta}{E}\right)_{|I200\rangle \to |I'000\rangle} = +\left\{8.70\times 10^{-6}\frac{B_0^{\,2}(1-0.72B_0^{\,2})}{f^2(1-2f)^2}A^{10/3} \times \frac{(I'2I|022)^2}{(I'1I|112)^2}\,\frac{1}{I'(I'+1)}\right\}^{1/2}, \qquad (9)$$

where B_0 is the nuclear deformation parameter. No rotation-vibration interaction band mixing has been taken into account, since their effects are negligible. We note that as $f\to 0$, only quadrupole transitions occur $(\delta\to\infty)$.

Figure 2 shows the lowering of the g_R factors if the relation

$$\frac{B_0(p)}{B_0(n)} = \left(\frac{G_n}{G_p}\right)^{1/2} = \left(\frac{20}{30}\right)^{1/2};$$

$$B_0 = \frac{NB_0(n) + ZB_0(p)}{A} \qquad (10)$$

is used for determining the ratio of proton and neutron deformation.[12] Such a formula follows from the quasispin model.[13] In Fig. 3 the dotted line gives $(\delta/E)^2$ in a logarithmic scale under the same assumptions. A formula like (10) is crude, however, and therefore it seems more realistic to use for the parameter f those values which are deduced from the experimental g_R factor [using Eq. (5)] and then predicting the $M1$-$E2$ mixing parameter. This is shown by the full line of Fig. 3. Further, the sign of δ is, according to (9), always positive, in agreement with experiments.[14] The results indicate that the main effect of the lowering of the g_R factor and the $M1$ transitions can be understood in this simple, lucid way, and that even some structure of the former can be related to the structure of the latter. It seems desirable, however, to search for other kinds of experiments in order to prove further the idea of different proton and neutron deformations.

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perimental data.

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[6]This is similar for electrons, where the Landé factor is constructed from the orbital and the spin-g factors in a tensorial way.

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[10]This is, of course, only approximately true, since the different pairing forces also affect the restoring forces for the nucleons in an involved way.

[11]I is the total spin, K its projection on the intrinsic symmetry axis. n_2 and n_0 are the numbers of $a_2{}^1$ and $a_0{}^1$ phonons, respectively.

[12]The values for $G_p = 30A^{-1}$ and $G_n = 20A^{-1}$ are taken from reference 1.

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[14]The experimental values are taken from reference 3 and from H. Debrunner and W. Kündig, Helv. Phys. Acta 33, 395 (1960); R. W. Bauer and M. Deutsch, Phys. Rev. 128, 751 (1962); H. J. Körner, in Perturbed Angular Correlations, edited by E. Karlsson et al. (North-Holland Publishing Company, Amsterdam, 1964). T. Yamazaki, Nucl. Phys. 44, 353 (1963).