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ROTATIONAL INSTABILITY IN A PENNING-TYPE DISCHARGE*

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The study of "anomalous diffusion" or enhanced loss of plasma across a magnetic field has been the object of many theories and experiments in a wide variety of discharges for many years (excellent reviews are given by Hoh,[1] Lehnert,[2] and Boeschoten[3]). In spite of the variety of plasma sources used, there are many telltale signs suggesting that this loss is of the same nature generally. In particular, the ratio of the ion current collected by a probe at the wall to that at the center decreases with increasing magnetic field in a characteristic manner until the critical field is reached, after which it increases. At this critical field there is an increase in low- and high-frequency noise. Also, the power needed by the plasma to maintain a constant density decreases until the critical field is reached and increases thereafter.

In searching for a unifying feature one is led to consider the effect of the radial electric field which is always present. Theories by Hoh,[4] Guest and Simon,[5] and Bingham[6] show that the $E\times B$ rotation can be unstable to fluting in the $m=1$ mode even in the absence of a directed discharge current for rather modest values of electric field (a few V/cm) when the field is inward. The theory of Kadomtsev and Nedospasov[7] adequately explains these phenomena in terms of an unstable $E\times B$ helical rotation in discharges with directed currents (positive column, arcs, etc.) as shown by the experiments of Paulikas and Pyle.[8] (A similar rotation was found in an arc discharge by Morse.[9]) In Penning-type discharges, however, one cannot appeal to this theory since there is no directed current. Using his theory Hoh was able to find agreement between his calculated critical fields and those observed by Chen and Cooper[10] and by Briffod and co-workers[11-13] at Saclay. In the cold-cathode experiments by the group at Saclay no rotation was found and, on the strength of the noise they observed,[13] they attributed the loss to some high-frequency microinstability. Using a hot-cathode, Penning-type discharge, Chen and Cooper apparently found a low-frequency oscillation, which they attributed to an asymmetric rotation due to the radial electric field, superimposed on incoherent hash. However, they examined the incoherent hash for longitudinal correlations since they felt the coherent rotation could not cause enhanced diffusion.

It is the purpose of this Letter to present the results of an investigation in a low-pressure, hot-cathode, Penning-type discharge which show a strong relation between this rotation and the effects normally considered to indicate anomalous diffusion.

The hydrogen discharge is 66 cm long and ~4.1 cm in diameter. There is one hot cathode, a hollow cylinder about an inch in diameter causing a slightly hollow discharge, and one cold cathode, a solid aluminum cylinder. The cylindrical anodes just in front of the cathodes are only a few centimeters long and so do not extend the length of the tube. In the center of the discharge tube (made of glass) were placed four probes separated radially by 90°, 90°, and 60°, and extending a centimeter into the discharge. The floating potentials of these probes were displayed simultaneously using a four-trace scope plug-in enabling one to look for $m=1$, 2, and 3 rotations of the plasma column. Further down the tube two probes were immersed from opposite sides so that one penetrated to the center and the other to a depth of 1 cm. With these the ratio of ion saturation currents to the probes was measured. Also, the discharge voltage was monitored for constant discharge current (100 mA in all cases, giving a density $\sim10^{10}$/cc, $T_e \sim 5$ V).

It was found that for a wide range of magnetic fields (50-500 G), the low-frequency noise contained a strong, nearly coherent signal in the 20- to 50-kc/sec range. The frequency and amplitude have been plotted in Fig. 1 for a pressure of about 1 μ, showing a change in character at 100 G. The ratio of ion saturation currents and discharge power (at 100 mA) were plotted versus magnetic field for the same pressure as for Fig. 1 showing the existence of what is normally called the critical field at the value where the noise undergoes its drastic changes.

The signal of Fig. 1 was accompanied by harmonics as seen in Fig. 2, where the spectrum is shown for various magnetic fields above and below the critical field. Note that below the critical field the growth appears to turn to damping, with the lowest frequency damped the heaviest. The third peak is now dominant, and it is this frequency which is plotted in Fig. 1 below 100 G.

By observing the four probe traces simultaneously it was found that the signal corresponded to an $m=1$ rotation from 100 to 500 G, and

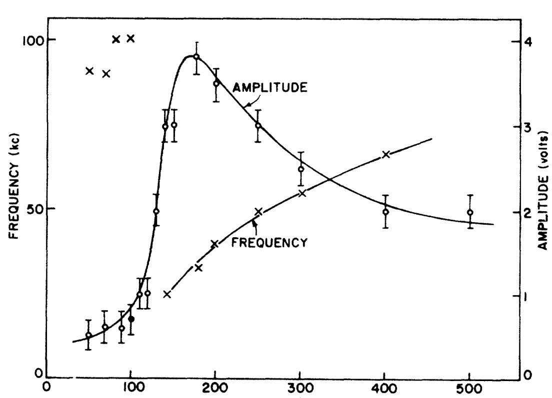

FIG. 1. Frequency and amplitude of the $m=1$ rotation. Discharge current = 100 mA, pressure $\approx 1\ \mu$, temperature ~5.

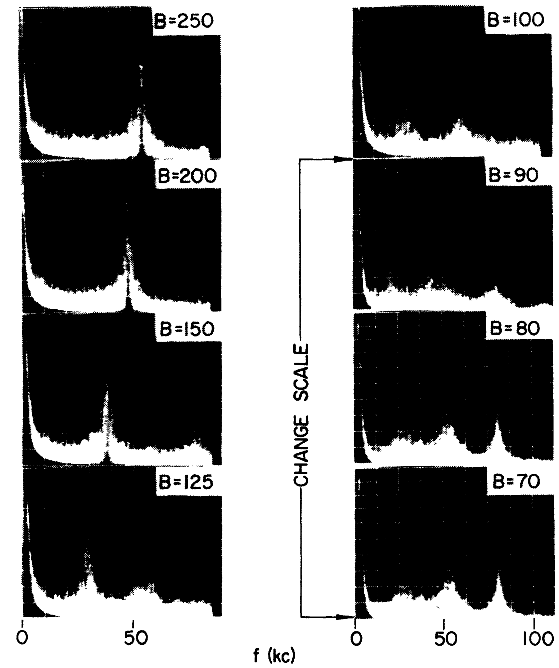

FIG. 2. Frequency spectrum of the floating potential near the edge of the discharge for magnetic fields above and below the critical field (~100 G).

an $m=3$ rotation below 100 G. The direction was that corresponding to an $E\times B$ drift with the E field inward, and the frequency was appropriate to an electric field ~1/cm. From the frequency variation, presumably proportional to the $E\times B$ rotational speed, it appears that the increase of E with B is more than linear. In the theory of Hoh, the critical field results when E reaches a certain value given by the parameters of the discharge.

In summary, we have observed (1) a dominant $m=1$ structure in the discharge noise above the critical field, (2) an increase in noise amplitude of the $m=1$ mode at the critical magnetic field above which the diffusion is "anomalous", (3) a dominant $m=3$ rotation below the critical magnetic field, and (4) damping of the $m=1$ flute below the critical field, indicating perhaps that the fluted pattern is a natural state of the column.

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NUCLEAR SUPERRADIANCE IN SOLIDS*

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A general theory of coherent spontaneous gamma-ray emission from an assemblage of isomeric nuclei in a perfect crystalline solid has been developed. The solid, characterized by internal energy states of the nuclei, by the lattice vibrations, and by the electromagnetic field, is treated as an integrated quantized system rather than as a number of noninteracting nuclei.[1-4] Transition probabilities are calculated by the usual methods of first-order time-dependent perturbation theory.

Coherent spontaneous emission of radiation from a gas has been discussed by Dicke.[5] It was shown that transitions exist for which the radiation rates, line shapes, and linewidths are all different from the corresponding quantities for an assemblage of noninteracting radiators. In particular, certain states were predicted that possess radiation rates much greater than normal because of correlations among the internal motions of the various molecules composing the system.

In a solid composed of N identical two-level nuclei in a perfect crystal lattice at a uniform and low temperature, correlations in the internal motions of the radiators are more probable than in the case of a gas. Furthermore, the interactions among members of the solid system are much stronger than in the gas, because of the coupling between neighbors in the lattice. The usual assumption[1-4] that each nucleus radiates independently of the states of other nuclei in the system is incompatible with the coupling of the nuclei through the common electromagnetic and phonon fields. Calculations of the spontaneous radiation rate for a solid system in which the nuclei are a priori assumed independent preclude the possibility of coherent spontaneous gamma emission by assumption. The present analysis is free from this inconsistency. Finally, the wavelength of the radiation is comparable with the spacing of nuclei in the lattice.

Using the method of Dicke,[5] the nuclear states are described by a vector model in which the vector orientation is quantized in energy space in analogy with fermion spin. The nuclei are assumed identical, in a uniform and field-free environment, with only two nondegenerate internal energy states coupled by a radiative transition. The lattice is assumed harmonic with nearest-neighbor interactions only; the phonon spectrum is approximated by the Debye model. The crystal is considered in the adiabatic approximation, and is assumed to be at rest with respect to the observer.

In the Hamiltonian for this system,

$$H = H_{\text{nuclei}} + H_{\text{lattice}} + H_{\text{radiation}} + H',$$

all terms except the interaction term H' are independent of the time. The latter is

$$H' = -\tfrac{1}{2}\sum_k [(\vec{a}_k \cdot \vec{e})R_{k+} + (\vec{a}_k{}^* \cdot \vec{e}^*)R_{k-}],$$

in which $\vec{a}_k{}^*$ and $\vec{a}_k$ are photon creation and destruction operators, respectively, k characterizes a mode of the electromagnetic field, $\vec{e}$ and $\vec{e}^*$ were defined in reference 5, and the nuclear excitation and de-excitation operators

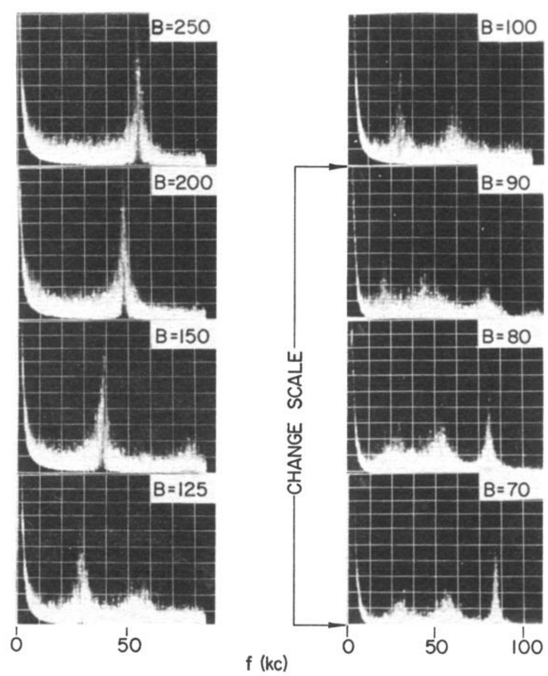

FIG. 2. Frequency spectrum of the floating potential near the edge of the discharge for magnetic fields above and below the critical field (~100 G).