netic fields a resonance has been observed in the optical absorption below the direct-gap band edge in germanium. This resonance can be interpreted as the transition between the zeroth Landau levels in the valence and conduction bands. The variation of the position and intensity of this resonance with E/H is in good agreement with Aronov's theory, which treats the optical absorption in crossed fields in the effective-mass approximation. With increasing E/H the intensity of the resonance decreases, until it can finally no longer be observed and the absorption curves approach those for the Franz-Keldysh effect ($H = 0$). Since the theory for this latter effect has been worked out in the effective-mass approximation,[10,11] and since the theory has been shown to hold well for germanium,[4] it seems likely that the whole phenomenon of optical absorption below the gap in crossed fields in germanium can be reasonably described in the effective-mass approximation, up to the highest electric fields attainable.

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PHONON EFFECTS ON NUCLEAR SPIN RELAXATION IN SUPERCONDUCTORS*

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Nuclear spin relaxation rates, in normal and superconducting aluminum, have been measured by Hebel and Slichter[1] and by Redfield and Anderson.[2] The ratio R_S/R_n of the relaxation rates in the superconductor to those in the normal metal, as found from their data, deviates from the ratio as calculated from the Bardeen-Cooper-Schrieffer (BCS)[3] theory, namely

$$\frac{R_S}{R_n} = 2\int_{\Delta_0(T)}^{\infty} dE\,[N_{\mathrm{BCS}}(E)]^2 \times\left(1+\frac{\Delta_0^2(T)}{E^2}\right)\left(-\frac{\partial f(E)}{\partial E}\right), \quad (1)$$

where $N_{\mathrm{BCS}}(E)$ is the BCS density of states in the superconductor,

$$N_{\mathrm{BCS}}(E) = \frac{|E|}{[E^2-\Delta_0^2(T)]^{1/2}}; \quad |E| \geq \Delta_0(T). \quad (2)$$

$\Delta_0(T)$ is the energy gap, and $f(E)$ is the Fermi function at temperature T,

$$f(E) = (e^{\beta E}+1)^{-1}; \quad \beta = (kT)^{-1}. \quad (3)$$

R_S/R_n as given by Eq. (1) has a logarithmic divergence as the temperature T is raised to the critical value T_C, where $\Delta_0(T_C) = 0$. To make theory and experiment agree, Hebel and Slichter[1] and Hebel[4] introduced an ad hoc broadening of the energy levels and replaced $N_{\mathrm{BCS}}(E)$ in Eq. (1) by $N_S(E)$, where

$$N_S(E) = (2\delta)^{-1}\int_{-\delta}^{E+\delta} N_{\mathrm{BCS}}(E')dE' \quad (4)$$

is an average of the BCS density of states over the energy-level breadth function, which was chosen to be a rectangle of width 2δ and height $(2\delta)^{-1}$.

The purpose of this paper is to show that, indeed, an effective broadening of the energy levels is brought about by absorption processes of thermal phonons. More accurately, the gap function has a (negative) imaginary part, which has the effect of removing the singularity in the density of states at finite temperature.

Eliashberg[5] and Nambu[6] have shown that if the correct, retarded electron-phonon interaction is used in place of the constant effective potential V of the BCS theory, a complex energy-dependent gap function results. Calculations of the gap function for strong-coupling superconductors, such as lead, were done by Schrieffer, Scalapino, and Wilkins[7] for zero temperature and more recently by Scalapino, Wada, and Swihart[8] for the finite-temperature case.

We shall return later to an approximate evaluation of the gap function. Let us first derive the expression for R_S/R_n for a superconductor with a complex gap. The expression corresponding to Eq. (1) is

$$\frac{R_s}{R_n}=2\int_0^\infty dE\left\{\left[\mathrm{Re}\left(\frac{E}{[E^2-\Delta^2(E,T)]^{1/2}}\right)\right]^2+\left[\mathrm{Re}\left(\frac{\Delta(E,T)}{[E^2-\Delta^2(E,T)]^{1/2}}\right)\right]^2\right\}\left(-\frac{\partial f(E)}{\partial E}\right). \quad (5)$$

Because of the Fermi function, the main contribution to this integral comes from small values of E. We may therefore approximate $\Delta(E,T)$ by its value at the "branch point," defined by

$$\Delta_1(T)=\mathrm{Re}\{\Delta(E=\Delta_1(T),T)\}, \quad (6)$$

$$\Delta_2(T)=\mathrm{Im}\{\Delta(E=\Delta_1(T),T)\}, \quad (7)$$

and the lower limit of the integral is now $\Delta_1(T)$.

The expression in the curly brackets in Eq. (5) then becomes

$$1+\frac{E^2+\Delta_1^2+\Delta_2^2-[(E^2-\Delta_1^2+\Delta_2^2)^2+4\Delta_1^2\Delta_2^2]^{1/2}}{2[(E^2-\Delta_1^2+\Delta_2^2)^2+4\Delta_1^2\Delta_2^2]^{1/2}}+\frac{(\Delta_1^2-\Delta_2^2)E^2-(\Delta_1^2+\Delta_2^2)}{(E^2-\Delta_1^2+\Delta_2^2)^2+4\Delta_1^2\Delta_2^2}. \quad (8)$$

One term is immediately integrated to give $2f(\Delta_1)$. For the other terms we use the fact, which will be shown later, that $\Delta_2(T)\ll\Delta_1(T)$, and consider $\partial f/\partial E$ as a slowly varying function compared to the last two terms in (8). The result is

$$\frac{R_s}{R_n}=2f(\Delta_1)\left\{1+\frac{\Delta_1(T)}{kT}[1-f(\Delta_1)]\ln\left(\frac{2\Delta_1(T)}{|\Delta_2(T)|}\right)\right\}, \quad (9)$$

where terms of order $\Delta_2(T)$ were neglected.

We now turn to the evaluation of $\Delta_1(T)$ and $\Delta_2(T)$. The equation for the gap and renormalization functions, at finite temperature,[8,9] can be put in the form

$$\Delta(\omega,T)=Z^{-1}(\omega,T)\int_0^{\omega_C}d\omega'\,\mathrm{Re}\left(\frac{\Delta(\omega',T)}{[\omega'^2-\Delta^2(\omega',T)]^{1/2}}\right)\{[\int d\omega_q\,\Phi(\omega_q)K_\omega(\omega'+\omega_q)-U_C]\tanh\tfrac{1}{2}\beta\omega'+\int d\omega_q\,\Phi(\omega_q)[f(\omega')+n(\omega_q)][K_\omega(\omega'+\omega_q)+K_\omega(\omega'-\omega_q)]\}, \quad (10)$$

$$[1-Z(\omega,T)]\omega=\int_0^{\omega_C}d\omega'\,\mathrm{Re}\left(\frac{\omega'}{[\omega'^2-\Delta^2(\omega',T)]^{1/2}}\right)\{[\int d\omega_q\,\Phi(\omega_q)S_\omega(\omega'+\omega_q)\tanh\tfrac{1}{2}\beta\omega'+\int d\omega_q\,\Phi(\omega_q)[f(\omega')+n(\omega_q)][S_\omega(\omega'+\omega_q)+S_\omega(\omega'-\omega_q)]\}. \quad (11)$$

Here $Z(\omega,T)$ is the renormalization function of the single-electron excitation energies due to electron-phonon interactions, U_C is the Coulomb pseudopotential designed to take account of the electron-electron scattering outside the band $\pm\omega_C$ about the Fermi surface, and $\Phi(\omega_q)$ is given by

$$\Phi(\omega_q)=\textstyle\sum_\lambda \alpha_\lambda^2(\omega_q)g_\lambda(\omega_q), \quad (12)$$

where $\alpha_\lambda(\omega_q)$ is the electron-phonon coupling for phonons of polarization λ and $g_\lambda(\omega_q)$ their energy spectrum. The kernels K_ω and S_ω are

$$K_\omega(\omega'\pm\omega_q)=\lim_{\delta\to 0}\left\{\frac{1}{\omega'\pm\omega_q+\omega+i\delta}+\frac{1}{\omega'\pm\omega_q-\omega-i\delta}\right\}, \quad (13)$$

$$S_\omega(\omega'\pm\omega_q)=\lim_{\delta\to 0}\left\{\frac{1}{\omega'\pm\omega_q+\omega+i\delta}-\frac{1}{\omega'\pm\omega_q-\omega-i\delta}\right\}. \quad (14)$$

$f(\omega)$ is the Fermi function, and $n(\omega_q)$ is the Bose-Einstein function

$$n(\omega_q) = (e^{\beta\omega_q} - 1)^{-1}. \quad (15)$$

For values of ω just above the Fermi level, $K_\omega(\omega' + \omega_q)$ in the first term in Eq. (10) may be approximated by $2/\omega_q$, since the main contribution to the integral over ω_q comes from large ω_q where $g_\lambda(\omega_q)$ is large. In a similar way, $S_\omega(\omega' + \omega_q)$ in the first term in Eq. (11) may be approximated by $-2\omega/(\omega' + \omega_q)^2$. We then have at zero temperature

$$Z(\omega, 0) = \frac{m^*}{m} = 1 + 2\int_0^\infty d\omega' \, \mathrm{Re}\left(\frac{\omega'}{[\omega'^2 - \Delta^2(\omega', 0)]^{1/2}}\right) \times \int d\omega_q \, \Phi(\omega_q) \frac{1}{(\omega' + \omega_q)^2}, \quad (16)$$

where m^*/m is the ratio of the effective mass to the "bare" mass of the electron due to electron-phonon interaction. This ratio in a superconductor is not much different from the ratio in the normal state (in particular for weak-coupling superconductors, such as aluminum). We may also define

$$N_0 V = m/m^* [\int d\omega_q \, \Phi(\omega_q) K_\omega(\omega' + \omega_q) - U_c], \quad (17)$$

which corresponds to the BCS parameter. Using these definitions and retaining only the largest contributing terms we find from Eqs. (10) and (11)

$$\Delta_2(T) = -\Gamma^2(T) \times \mathrm{Re}\left(\frac{1}{\{\Delta_1^2(T) - [\Delta_1(T) + i\Delta_2(T)]^2\}^{1/2}}\right), \quad (18)$$

where

$$\Gamma^2(T) = \pi \frac{m}{m^*} \int d\omega_q \, \omega_q \, \Phi(\omega_q) n(\omega_q). \quad (19)$$

Solving Eq. (18), we find

$$\Delta_2(T) = -[\tfrac{1}{2}\Gamma^2(T)]^{2/3} [\Delta_1(T)]^{-1/3}. \quad (20)$$

Since aluminum has a single Debye frequency,[10] it is convenient to express $\Gamma^2(T)$ in terms of the experimental value for m^*/m, the Debye energy, ω_D, and the temperature; that is,

$$\Gamma^2(T) = (\pi^4/15)(m^*/m - 1)(m/m^*)(kT)^4 \omega_D^{-2}, \quad (21)$$

where a Debye spectrum $g(\omega_q)$ and a constant coupling α were assumed. For aluminum m^*/m is found from specific-heat measurements[11,12] to be 1.5 to 1.6. The Debye energy[10] is 34 meV, the transition temperature[1] $T_c = 1.172$°K, and the ratio $2\Delta_0/k_c = 3.5$, where Δ_0 is (real part of) the gap at 0°K at the top of the Fermi level. For $\Delta_1(T)$ we take the BCS temperature-dependent gap; small corrections in $\Delta_1(T)$ will have no effect on the results. With these values we find

$$\Delta_2(T)/\Delta_0 = -1.5 \times 10^{-2} [\Delta_0/\Delta_1(T)]^{1/3} (T/T_c)^{8/3}. \quad (22)$$

With this value for $\Delta_2(T)$ in Eq. (9) the ratio of the nuclear spin relaxation rates, between superconducting and normal aluminum, is plotted in Fig. 1 and compared with the experimental results of references 1 and 2; the agreement seems to be quite good. The values for $\Delta_1(T)$ were taken from a table by Muhlschlegel.[13] We should note that due to the logarithmic dependence of R_S/R_n on $\Delta_1(T)/\Delta_2(T)$, the results are rather insensitive to variations in $\Delta_2(T)$ (if the electron-phonon coupling, for example, is varied).

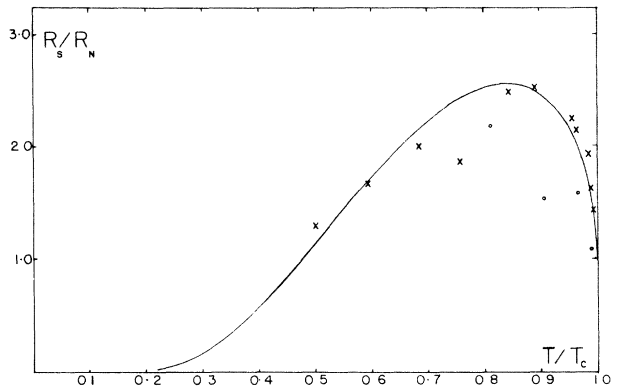

FIG. 1. Ratio of nuclear spin relaxation R_S/R_n between superconducting and normal aluminum, as function of the reduced temperature $t = T/T_C$ from Eq. (9). The circles represent experimental data by Hebel and Slichter, the crosses data by Redfield and Anderson.

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REORIENTATION EFFECT IN Cd^{114}†

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The reorientation effect arises from the dependence of the Coulomb excitation process on the static moments of nuclear states. Various authors[1-4] have suggested experiments in which this effect can be singled out so that the quadrupole moment of, e.g., the first 2^+ state in an even-even nucleus may be determined. The theoretical treatment of the process has so far only been carried out using second-[2,4] and third-order[3] perturbation expansions. The present paper describes the experimental determination of the reorientation effect in Cd^{114} by an evaluation of the measured cross sections for the scattering of O^{16} and He^4 ions in terms of Q_{2^+}, the quadrupole moment of the first 2^+ state. The analysis has been made using a computer program which avoids the perturbation expansion.

The Coulomb excitation of the 0.5578-MeV state in Cd^{114} has been determined by two methods:

(a) The particles backscattered from a thin target into an annular solid-state detector were recorded in coincidence with the de-excitation gamma radiation. If the efficiency of the gamma counter, the gamma-ray angular distribution, and the conversion coefficient are known, the quantity $R^{(a)}$ = number of de-excitations of the first 2^+ state per backscattered particle can be determined through this experiment. In terms of the cross sections $d\sigma_i$, $R^{(a)}$ is expressed in the following way:

$$R^{(a)} = \sum_{i=2}^{7} f_i d\sigma_i \Big/ \sum_{i=1}^{7} d\sigma_i .$$

The index i refers to the different nuclear states indicated in Fig. 1, and the quantity f_i denotes the fraction of the decays of level i which cascade through the 0.5578-MeV level. The experimental technique used in this method has been described by de Boer, Goldring, and Winkler.[5] The O^{16} and He^4 bombarding energies were chosen in such a way as to yield similar values for the adiabaticity parameter

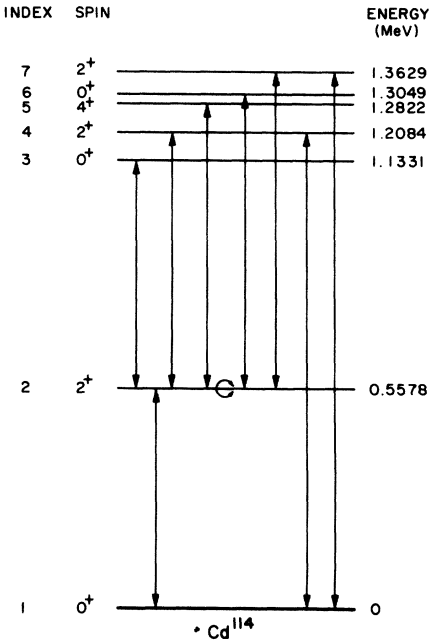

FIG. 1. Energy spectrum of Cd^{114}, taken from R. K. Smither, Phys. Rev. 124, 183 (1961). The arrows indicate the $E2$ matrix elements included in the computer analysis. Their values are given in Table III.