flux has a small deviation from the quantized value using the previously chosen function $\tilde{\chi}(x)$, we get equations for $\tilde{G}$ and $\tilde{F}$ containing a small vector potential $\vec{A}^{\sim}$. It is possible to take the residual vector potential $\vec{A}^{\sim}$ as a perturbation which leads to a body current. This current generates a magnetic field, opposite to the trapped one, and just this means the stability of the flux quantization for a thick cylinder. We turn to treat the case of a very thin cylinder whose wall thickness is much smaller than the penetration depth. In this case the vector potential can be given roughly by the following formula:

$$\vec{A}(x) \sim \mathrm{grad}[\chi_n(x) + \Delta\chi(x)],$$

for an arbitrary index n, and the remaining vector potential $\vec{A}^{\sim} = \mathrm{grad}\Delta\chi$ can be treated as a perturbation. For the case of $\Delta\chi = 0$ we get the field-free solutions for $\tilde{G}$ and $\tilde{F}$. By perturbation method starting from $\tilde{G}$ and $\tilde{F}$ corresponding to one of the allowed $\chi_n(x)$ and $\Delta\chi = 0$, a new solution can be built up, and the corresponding energy and thermodynamical potential are even functions of $\Delta\chi$. This treatment leads to a set of solutions labeled by n for a given flux value. The solution with the smallest thermodynamical potential corresponds to the stable one. As a consequence the current, energy, and thermodynamical potential expressed in terms of the one-particle Green function will be periodic functions of the flux, which is well known as the Little-Parks effect.[7]

The treatment of the flux quantization shows that <u>if a superconducting field-free state and Meissner effect exist for a thick (thin) enough ring, then the flux quantization (Little-Parks effect) occurs, independent of</u> (1) the concentration of the nonmagnetic or magnetic impurities (e.g. dirty superconductors), (2) the shape of the twofold connected sample. We should like to emphasize that <u>we have made use only of the gauge-invariant structure of the equations and no additional (rotational) symmetry has been assumed</u>. (For a real or dirty crystal the angular momentum is not a good quantum number.)

A detailed treatment of the electromagnetic behavior of superconductors based on the proposed Hamiltonian will be published later. The author expresses his sincere thanks to N. Menyhard for several detailed discussions and for her contribution in the elaboration of some points.

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SHIFT OF LANDAU LEVELS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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In mutually perpendicular strong electric and magnetic fields we have observed a resonance in the optical absorption in germanium below the direct-gap band edge, which we interpret as the transition between the zeroth Landau levels of the valence and conduction band, shifted to lower photon energies by the electric field. All measurements were made at 6°C.

The theory for optical absorption in semiconductors in crossed electric and magnetic fields was given by Aronov[1] in the effective-mass approximation for simple parabolic bands, and an experimental confirmation of some aspects of this theory was given by Vrehen and Lax,[2] for the case of relatively low electric field intensities. We have now extended our measurements to high electric fields, making use of the high electric fields present in the depletion layer of a reverse-biased planar *p-n* junction, a technique developed by Frova and Handler[3,4] for the study of the Franz-Keldysh effect. Both a reverse-biased dc voltage and a small ac voltage are applied to the junction. A monochromatic infrared beam is passed through the junction in a direction normal to the plane of the junction; the transmitted intensity I and its modulation dI/dV (V the voltage over the junction) are measured. Using the quantity dV/dx, where x is the thickness of the depletion layer, which can be obtained from the electrical characteristics of the junction, the quantity $(1/I)(dI/dx)$ can be derived. For a step junction this quantity equals $\alpha(E)-\alpha(0)$, where E is the maximum field in the junction region. For a gradual junction some corrections are necessary to obtain the absolute value of the absorption coefficient. In the present case the value of the maximum field E was determined by comparing our experimental data for the Franz-Keldysh effect with those of Frova and Handler.[4] Since the diode is a relatively thick structure ($\approx 200\ \mu$), only the absorption of photons having energy less than the direct-gap band edge can be studied in this way.

In Fig. 1, $\alpha(E)-\alpha(0)$ is plotted as a function of $\epsilon_g - \hbar\omega$, where ϵ_g is the value of the direct energy gap and $\hbar\omega$ is the photon energy, for $E = 2.1\times 10^4$ V/cm, for $H = 0$ and for $H = 64$ kOe in both the longitudinal and transverse configurations. In the crossed-field case a resonance can clearly be observed, which is absent in the other two cases. Figure 2 presents $\alpha(E)-\alpha(0)$ for $H = 96$ kOe and for various values of the electric field. With increasing electric field the resonance shifts to lower photon energies and its intensity decreases, which is consistent with Eqs. (1) and (2) below.

According to Aronov's theory the energy ϵ

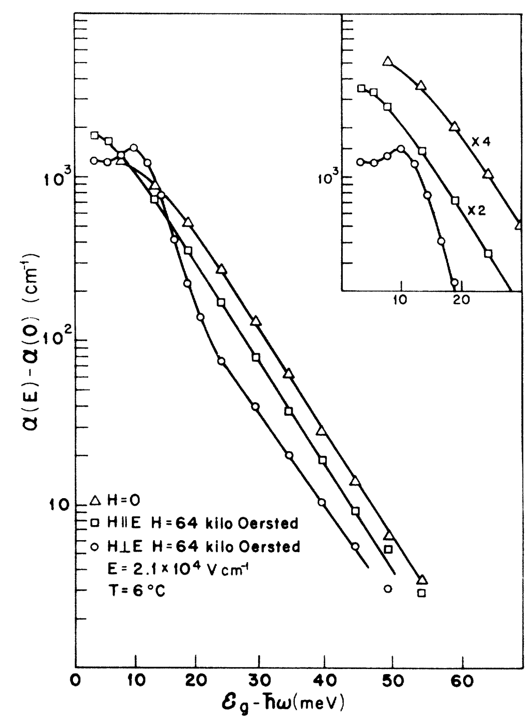

FIG. 1. Electric-field-induced optical absorption below the direct gap in germanium at 6°C, for $E = 2.1\times 10^4$ V/cm, $H = 0$, and $H = 64$ kOe in the longitudinal and transverse configuration. In crossed fields a resonance shows up approximately 10 meV below the gap. In the inset the absorption curves for $H \| E$ and $H = 0$ have been multiplied by factors of 2 and 4, respectively, to show the resonance more clearly.

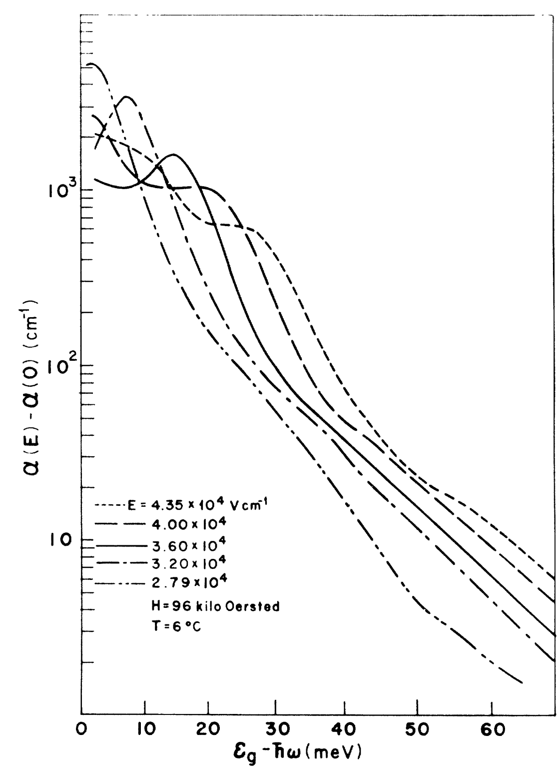

FIG. 2. Electric-field-induced optical absorption below the direct gap in germanium at 6°C, for $H \perp E$, $H = 96$ kOe, and for various values of E. With increasing E the resonance shifts to lower photon energies and decreases in intensity.

for the transition between the zeroth Landau levels for simple parabolic bands in crossed fields is given by

$$\epsilon = \epsilon_g + \tfrac{1}{2}\hbar\omega c - \tfrac{1}{2}(m_1 + m_2)c^2E^2/H^2. \qquad (1)$$

m_1 and m_2 are the effective masses for the conduction and valence band;

$$\omega_c = \frac{eH}{c}\,\frac{m_1+m_2}{m_1 m_2};$$

c is the velocity of light, e the charge of the electron, and E and H the electric and magnetic field, respectively. The absorption coefficient for this particular transition is found as

$$\alpha = AH\exp(-a^2/2)(\hbar\omega - \epsilon)^{-1/2}, \qquad (2)$$

where A is a constant and

$$a = eEL/\hbar\omega_c\ ,$$

in which L is the radius of the Landau orbit, $L = (\hbar c/eH)^{1/2}$, and

$$\omega_c{}^* = eH/c(m_1 + m_2).$$

Taking for m_1 and m_2 the electron and light hole mass, respectively (see below), one finds that the parameter a ranges from about 1.6 to 2.6 for the field values in Fig. 2. Equation (2) then explains why the intensity of the resonance decreases so rapidly with increasing electric field. The resonance can be observed over a limited range of E/H values only. For values of E/H which are too small, the resonance occurs above the gap and cannot be seen; for values of E/H which are too large, the intensity becomes too weak.

In Fig. 3 we plot $\epsilon - (\hbar\omega_c/2)$ as a function of $(E/H)^2$. ϵ is now the experimentally determined energy of the resonance, and $\hbar\omega_c/2$ is taken as the sum of the magnetic energies for the zeroth Landau levels in the conduction and valence band. For the conduction band an effective mass $m_1 = 0.036\ m_0$ was used,[5] and for the valence band the average shift of the $n = 0$ levels in a magnetic field was taken from Goodman.[6] In accordance with Eq. (1) the experimental points lie on a straight line having an intercept for $(E/H) = 0$ at 808 MeV, which corresponds closely to the value of the energy gap at the temperature of the measurements, $\epsilon_g = 810 \pm 1$ meV at 6°C.[7]

Since the absolute values of E have an estimated uncertainty of ±10%, the slope of the straight line in Fig. 3 has an uncertainty of

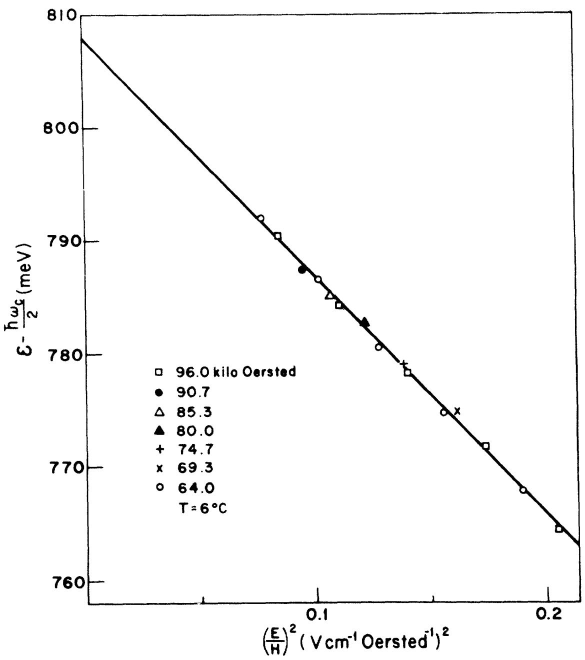

FIG. 3. ϵ, the energy at which the resonance occurs, diminished by $\hbar\omega_c/2$, the sum of the magnetic energy of the zeroth Landau levels in the conduction and valence band, as a function of $(E/H)^2$.

±20%. From this slope and Eq. (1) one finds $m_1 + m_2 = (0.074 \pm 0.015)m_0$, where m_0 is the electron mass. Taking $m_1 = 0.036m_0$ one deduces $m_2 = (0.038 \pm 0.015)m_0$, which is of the order of magnitude of the light hole mass $0.043m_0$.[8] The simple theory of Aronov cannot directly be applied to germanium where the valence band is degenerate. m_2 should therefore be considered as a parameter which should not necessarily take on the value of either the light or the heavy hole mass. In fact, a perturbation calculation[9] indicates that m_2 should be somewhat larger than the light hole mass. Anyhow, the experiments indicate m_2 to be close to the light hole mass, and this value has therefore been used for the evaluation of the parameter a above.

For heavy-hole transitions both the electric-field-induced shift and the parameter a are much larger than for light-hole transitions. It is likely that these heavy-hole transitions are responsible for the exponential tail in the absorption observed for photon energies below the resonance.

The results presented above may be summarized as follows. In crossed electric and magnetic fields a resonance has been observed in the optical absorption below the direct-gap band edge in germanium. This resonance can be interpreted as the transition between the zeroth Landau levels in the valence and conduction bands. The variation of the position and intensity of this resonance with E/H is in good agreement with Aronov's theory, which treats the optical absorption in crossed fields in the effective-mass approximation. With increasing E/H the intensity of the resonance decreases, until it can finally no longer be observed and the absorption curves approach those for the Franz-Keldysh effect ($H = 0$). Since the theory for this latter effect has been worked out in the effective-mass approximation,[10,11] and since the theory has been shown to hold well for germanium,[4] it seems likely that the whole phenomenon of optical absorption below the gap in crossed fields in germanium can be reasonably described in the effective-mass approximation, up to the highest electric fields attainable.

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PHONON EFFECTS ON NUCLEAR SPIN RELAXATION IN SUPERCONDUCTORS*

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Nuclear spin relaxation rates, in normal and superconducting aluminum, have been measured by Hebel and Slichter[1] and by Redfield and Anderson.[2] The ratio R_s/R_n of the relaxation rates in the superconductor to those in the normal metal, as found from their data, deviates from the ratio as calculated from the Bardeen-Cooper-Schrieffer (BCS)[3] theory, namely

$$\frac{R_s}{R_n} = 2\int_{\Delta_0(T)}^{\infty} dE[N_{\rm BCS}(E)]^2 \times\left(1+\frac{\Delta_0^2(T)}{E^2}\right)\left(-\frac{\partial f(E)}{\partial E}\right), \quad (1)$$

where $N_{\rm BCS}(E)$ is the BCS density of states in the superconductor,

$$N_{\rm BCS}(E) = \frac{|E|}{[E^2-\Delta_0^2(T)]^{1/2}}; \quad |E| \geq \Delta_0(T). \quad (2)$$

$\Delta_0(T)$ is the energy gap, and $f(E)$ is the Fermi function at temperature T,

$$f(E) = (e^{\beta E}+1)^{-1}; \quad \beta = (kT)^{-1}. \quad (3)$$

R_s/R_n as given by Eq. (1) has a logarithmic divergence as the temperature T is raised to the critical value T_c, where $\Delta_0(T_c) = 0$. To make theory and experiment agree, Hebel and Slichter[1] and Hebel[4] introduced an ad hoc broadening of the energy levels and replaced $N_{\rm BCS}(E)$ in Eq. (1) by $N_S(E)$, where

$$N_S(E) = (2\delta)^{-1}\int_{-\delta}^{E+\delta} N_{\rm BCS}(E')dE' \quad (4)$$

is an average of the BCS density of states over the energy-level breadth function, which was chosen to be a rectangle of width 2δ and height $(2\delta)^{-1}$.