RADIATION-INDUCED INSTABILITY OF ELECTRON PLASMA OSCILLATIONS

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It is well known that external radiation of frequency ω_0 incident upon a plasma is absorbed by inverse bremsstrahlung of electrons in the presence of ions.[1] However, when ω_0 is close to the plasma frequency, ω_p, the radiation energy is preferentially fed into electron plasma oscillations at a rate proportional to $I_0/nckT$, where I_0 is the radiation intensity, n the density of electrons, c the speed of light, and kT the electron thermal energy. We wish to point out in this Letter the possibility of electron plasma oscillations becoming unstable and growing in amplitude when they gain energy at a rate faster than they can dissipate it by their dominant damping mechanism; that is, when $I_0/nckT \gtrsim \gamma_L/\omega_p$, where γ_L is the plasma damping rate.

Monochromatic coherent external radiation of form $\frac{1}{2}\vec{E}_0 \exp[i(\vec{k}_0\cdot\vec{r}-\omega_0 t)]+\text{c.c.}$, where $\vec{k}_0\cdot\vec{E}_0 = 0$, will modulate the longitudinal polarization, $P_L(\vec{k},\omega)$, in the plasma at the sum and difference frequencies $\omega\pm\omega_0$:

$$P_L(\vec{k},\omega) = -E_L(\vec{k},\omega) = \chi_L(\vec{k},\omega)E_L(\vec{k},\omega) + \vec{E}_0\cdot\vec{\chi}_L^{NL}(\vec{k}_0,\omega_0;\vec{k}-\vec{k}_0,\omega-\omega_0)E_L(\vec{k}-\vec{k}_0,\omega-\omega_0) + \vec{E}_0\cdot\vec{\chi}_L^{NL}(\vec{k}_0,\omega_0;\vec{k}+\vec{k}_0,\omega+\omega_0)E_L(\vec{k}+\vec{k}_0,\omega+\omega_0). \quad (1)$$

E_L is the effective longitudinal field in the plasma $(=-P_L)$, $\chi_L = \chi_L^e + \chi_L^i$ is the usual equilibrium longitudinal susceptibility with a contribution from the electrons, χ_L^e, and a contribution from the ions, χ_L^i, $\vec{\chi}_L^{NL}$ is a nonlinear longitudinal susceptibility connecting the polarization to $\vec{E}_0$ and $E_L(\omega\pm\omega_0)$. A similar equation for $-E_L(\omega-\omega_0)$ follows from (1), which now couples to frequencies ω and $\omega-2\omega_0$. In fact, a chain of coupled equations is generated. Assume in the following that ω is near the plasma resonance frequency, $\omega_L = (\omega_p^2+3v^2k^2)^{1/2}$, and ω_0 slightly is above it. In the chain of equations we can then neglect all fields E_L propagating at frequencies other than ω and $\omega-\omega_0$ (strongly damped low-frequency ion-acoustic waves can still conceivably respond at the difference frequency $\omega-\omega_0$).[2] This leaves only two coupled homogeneous equations for $E_L(\vec{k},\omega)$ and $E_L(\vec{k},\omega-\omega_0)$.[3] The condition for a nontrivial solution is the vanishing of the following function, which is the nonlinear longitudinal dielectric constant $\epsilon_L^{NL}(\vec{k},\omega)$,

$$\epsilon_L^{NL}(\vec{k},\omega) = \epsilon_L(\vec{k},\omega) - \vec{E}_0\cdot\vec{\chi}_L^{NL}(\vec{k}_0,\omega_0;\vec{k},\omega-\omega_0)\vec{E}_0\cdot\vec{\chi}_L^{NL}(-\vec{k}_0,-\omega_0;\vec{k},\omega)[\epsilon_L(\vec{k},\omega-\omega_0)]^{-1}, \quad (2)$$

where $\epsilon_L(\vec{k},\omega) = 1+\chi_L(\vec{k},\omega)$ is the usual plasma dielectric constant. If we consider real frequencies, ω, only the real part $\epsilon_L^{NL}(\vec{k},\omega)$ must vanish (this determines the resonant frequency), and the imaginary part will give the effective damping.

Further quantitative discussion requires a knowledge of the susceptibility, $\vec{\chi}_L^{NL}$. This, together with the structure of Eq. (1), may be obtained from Poisson's equation, $i\vec{k}E_L(\vec{k},\omega) = 4\pi e\langle n(\vec{k},\omega)\rangle$, and an evaluation of the average density response, $\langle n(\vec{k},\omega)\rangle$, using a suitable microscopic theory. A Vlasov-type equation for the electron distribution function in an external field is an example. The random-phase approximation gives only the usual χ_L, but if second-order terms in the effective field are included, one may pick out the appropriate nonlinear susceptibility. In the language of Feynman diagrams, Fig. 1 shows the coupling we are considering. In a forthcoming paper, we will present a Green's-function derivation. We find $\vec{\chi}_L^{NL}$ can be related quite generally

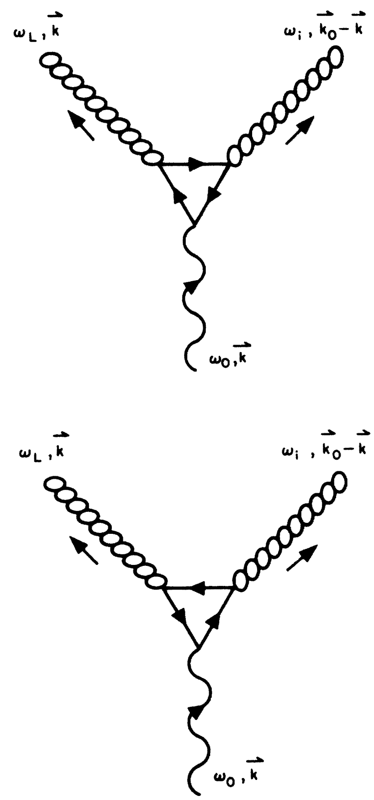

FIG. 1. Feynman diagrams for nonlinear coupling between plasma mode, ion mode, and external field. Wavy lines represent the external transverse field, light lines electrons, and braided lines longitudinal fields.

in the collisionless approximation to the equilibrium longitudinal susceptibility χ_L of a nonrelativistic two-component plasma. When one mass, m_e, is much lighter than the other, m_i (neglecting terms of order v/c),

$$\vec{E}_0\cdot\vec{\chi}_L^{NL}(\vec{k}_0,\omega_0;\vec{k},\omega-\omega_0)$$
$$=(-ie\vec{k}\cdot\vec{E}_0/2m_e\omega_0^2)[\chi_L^e(\vec{k},\omega-\omega_0)-\chi_L^e(\vec{k},\omega)]$$
$$=\vec{E}_0\cdot\vec{\chi}_L^{NL*}(-\vec{k}_0,-\omega_0;\vec{k},\omega). \tag{3}$$

Otherwise there will be another contribution involving χ_L^i, with m_i replacing m_e. For the high-temperature electron-ion plasma with ω near the plasma frequency, and ω_0 slightly above it, the dominant term of Eq. (3) when $k/k_D \ll 1$ is simply

$$\vec{E}_0\cdot\vec{\chi}_L^{NL}(\vec{k}_0,\omega_0;\vec{k},\omega-\omega_0)=\frac{-ie\vec{k}\cdot\vec{E}_0}{2m_e\omega_0^2}\frac{k_D^2}{k^2}. \tag{4}$$

Equation (2) may then be stated simply as

$$\epsilon_L^{NL}(\vec{k},\omega)=\epsilon_L(\vec{k},\omega)-\frac{\Lambda^2}{(k^2/k_D^2)\epsilon_L(\vec{k},\omega-\omega_0)}, \tag{5}$$

where $\Lambda^2=\frac{1}{4}(\omega_p^4/\omega_0^4)\cos^2\theta(I_0/nckT)$, $I_0\equiv E_0^2c/4\pi$, and θ is the angle between $\vec{E}_0$ and $\vec{k}$. With $k \parallel E_0$, $\Lambda^2 \approx I_0/4nckT$. In what follows, Λ^2 will always be $\ll 1$.

The condition $\mathrm{Re}\,\epsilon_L^{NL}(\vec{k},\omega)=0$ gives a small shift[4] in the resonant frequency: $\omega=\omega_L[1+O(\Lambda^2)]$. Of greater interest is the damping rate,

$$\frac{\gamma_L^{NL}}{\omega_p}=\mathrm{Im}\,\epsilon_L^{NL}(\vec{k},\omega_L)$$
$$=\frac{\gamma_L}{\omega_p}-\Lambda^2\,\mathrm{Im}\frac{k_D^2}{k^2}\epsilon_L^{-1}(\vec{k},\omega_L-\omega_0), \tag{6}$$

where $\gamma_L/\omega_p=\mathrm{Im}\,\epsilon_L(\vec{k},\omega_L)$ is the damping in the absence of the external field. For $k \ll k_D$, $\mathrm{Im}(k_D^2/k^2)\epsilon_L^{-1}(\vec{k},\omega_L-\omega_0)=f(u)$ is a well-known positive function[5] of positive $u=(\omega_0-\omega_L)/v_ik$, where $v_i=(m_e/m_i)^{1/2}v$ is the thermal velocity. $f(u)$ rises from zero at $u=0$, to a maximum of 0.6 at $u=1.7$, after which it exponentially approaches zero as $u\to+\infty$. It has a half-width of roughly $\Delta u \simeq 1$. Hence, the negative damping term in Eq. (6) can be appreciable only when

$$\omega_0-\omega_L=(1.7\pm0.5)v_ik. \tag{7}$$

Physically, this requirement is a frequency-matching condition for the external radiation to excite an electron plasma wave and a strongly damped ion-acoustic wave [whose frequency lies in the range indicated by the right-hand side of (7)]. It is a stringent requirement on the monochromaticity of the external field, the density homogeneity of the plasma,[6] and the wave number, k, of the plasma mode. When $k_0/k_D \ll k/k_D \ll 1$, one may show from Eq. (7) that the plasma mode receiving the maximum negative damping, $-0.6\Lambda^2$, has a wave number given by

$$\frac{k}{k_D}=\left[\frac{2}{3}\left(\frac{\omega_0}{\omega_p}-1\right)+\left(\frac{1.7}{3}\right)^2\frac{m_e}{m_i}\right]^{1/2}-\frac{1.7}{3}\left(\frac{m_e}{m_i}\right)^{1/2}. \tag{8}$$

ω_0/ω_p should be chosen small enough so that $k/k_D \ll 1$ and the plasma mode is weakly damped, but large enough to allow the radiation to penetrate the plasma. Although the laws of reflection and transmission at a sharp boundary do not strictly apply to diffuse plasma boundaries,

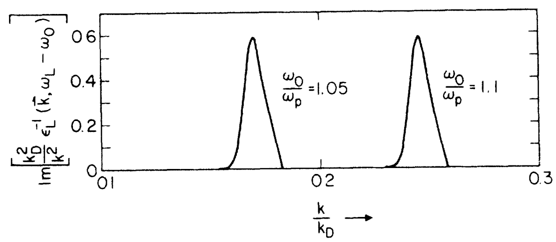

FIG. 2. (Negative damping/sec)/$\Lambda^2\omega_p = \mathrm{Im}(k_D^2/k^2) \times \epsilon_L^{-1}(\vec{k}, \omega_L - \omega_0)$ as a function of k/k_D for $\omega_0/\omega_p = 1.1$, 1.05 in a fully ionized hydrogen plasma.

they are an indication of the ease of penetration of radiation. The transmission coefficient, given by $\tau = 4n(\omega_0)/[n(\omega_0)+1]^2$ [where $n(\omega_0) = (1 - \omega_p^2/\omega_0^2)^{1/2}$] is 80% for $\omega_0/\omega_p = 1.1$, corresponding to $k/k_D \cong \frac{1}{4}$. Hence, it should be possible to choose a frequency ω_0 which both penetrates the plasma and stimulates a weakly damped plasma mode. Around the optimum k/k_D given in Eq. (8) there will be a small range, $\Delta k/k_D$, of wave numbers which receive comparable negative damping. When the ion Doppler width $v_i k$ is greater than γ_L, this range is $\Delta k/k_D = \frac{1}{3}(m_e/m_i)^{1/2}$. In Fig. 2 we plot

$$\mathrm{Im}(k_D^2/k^2)\epsilon_L^{-1}(\vec{k}, \omega_L - \omega_0)$$

as a function of k/k_D for $\omega_0/\omega_p = 1.1$ and $\omega_0/\omega_p = 1.05$. We conclude that when $0.6\Lambda^2 \gtrsim \gamma_L/\omega_p$, plasma oscillations with wave number in a small range about the k/k_D given by Eq. (8) will begin to go unstable and grow. The damping may be estimated using the collisional conductivity value,[1] $\gamma_L/\omega_p = (6\sqrt{2}\pi^{3/2})^{-1}(k_D^3/n)\ln(kT/\hbar\omega_p)$. For k/k_D less than ~0.2 this damping dominates Landau damping. The growth criterion would appear to be met easily with low-density plasmas and microwave radiation of kW/cm^2 intensity. For example, if $n = 10^{13}$ electrons/cm^3, and $kT = 1$ eV, $\gamma_L/\omega_p \approx 10^{-3}$, whereas $\Lambda^2 \cong 10^{-2}$ for 1-kW, 1-cm waves. In the presence of net gain ($\gamma_L^{NL}/\omega_p < 0$), plasma oscillations spontaneously present due to density fluctuations will be amplified by a gain factor $\exp(+|\gamma_L^{NL}|t)$. With the plasma inside a microwave cavity direct detection of these growing waves may be possible.

For very high-density plasmas, such as those produced by a focused ruby laser,[7] the instability might be induced by the laser radiation, although density inhomogeneity and strong damping are obstacles.

In a forthcoming paper, one of us (M.V.G.) will give a fuller derivation of these results as well as the spectral properties of the density fluctuations, using quantum statistical mechanics.

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[1]J. Dawson and C. Oberman, Phys. Fluids 5, 517 (1962); C. Oberman, A. Ron, and J. Dawson, Phys. Fluids 5, 1514 (1962); D. F. DuBois, V. Gilinsky, and M. S. Kivelson, Phys. Rev. 129, 2376 (1963).

[2]Under certain circumstances, $E_L(\omega - 2\omega_0)$ must be retained too. If $\omega - \omega_0 = -\omega_i$, where ω_i is a frequency at which the ions respond strongly ($\omega_i \approx v_i k_i$), then $\omega - 2\omega_0 = -\omega - 2\omega_i$, and if $2\omega_i < \gamma_L$, the plasma mode at $-\omega_L$ will participate. In this case, Eq. (1) must include an E_0^2 term coupled to $E_L(\omega - 2\omega_0)$, and one has three coupled homogeneous equations to solve. The dielectric constant which must vanish will be more complicated in that case, but the instability is still present, and the order of magnitude of quantities we shall calculate is unchanged.

[3]Once inside the medium, ω_0 will be related to $\vec{k}_0$ by the dispersion relation $\omega_0^2 = \omega_p^2 + c^2k_0^2$. Since $\omega_0^2 \approx \omega_L^2$, k_0 usually will be of $O(kv/c)$. Therefore, in the following discussion we will write k for $|k - k_0|$.

[4]To calculate the shift precisely, one must also include an E_0^2 correction to $\chi_L(\vec{k}, \omega)$ in Eq. (1). This will also produce a Λ^2 correction to $\mathrm{Im}\epsilon_L(\vec{k}, \omega)$ which turns out to be negligible compared to the Λ^2 term in Eq. (5).

[5]$(k^2/k_D^2)\epsilon_L(\vec{k}, \omega_L - \omega_0) = 1 - \frac{1}{2}Z^1(-u/2)$, where Z^1 is a complex function which may be found in B. D. Fried and S. D. Conte, The Plasma Dispersion Function (Academic Press, Inc., New York, 1961).

[6]For example, assuming E_0 is perfectly monochromatic, $\Delta\omega_p/\omega_p = \Delta n/2n \cong (m/M)^{1/2}k/k_D$, where $(m/M)^{1/2}$ is the electron-to-ion mass ratio.

[7]J. M. Dawson, Phys. Fluids 7, 981 (1964); R. G. Meyerand and A. F. Haught, Phys. Rev. Letters 13, 7 (1964).